Superimposed Training on Redundant Precoding for Low-Complexity Recovery of Block Transmissions

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Abstract—**The adoption of orthogonal frequency-division multiplexing (OFDM) by wireless local area networks and audio/video broadcasting standards testifies to the importance of recovering block precoded transmissions propagating through frequency-selective FIR channels. Existing block transmission standards invoke bandwidth-consuming error control codes to mitigate channel fades and training sequences to identify the FIR channels. For low-complexity block-by-block processing, redundant precoders with cyclic prefix (CP) along with superimposed training sequence are optimally designed to enable least squares estimation of time-varying FIR channels and symbol recovery regardless of the underlying FIR frequency-selective multipath channel. Numerical results are also presented to access the performance of the proposed affine precoders.**

I. Introduction

Block transmissions relying on linear redundant filterbank precoding with cyclic prefixed or zero padded blocks have gained increasing interest recently for frequency-selective multipath effects (see e.g., [1], [2], [3] and references therein). Redundancy removes inter block interference (IBI) and also facilitates (even blind) acquisition of channel state information (CSI) at the receiver. It leads to data efficient low-complexity linear equalization (of the zero-forcing (ZF) or minimum mean squared error (MMSE) type) with guaranteed symbol recovery regardless of the zero locations of the underlying FIR channel [2].

When CSI is available at the transmitter (through a feedback channel) optimal precoders and decoders become available under various criterion [2]. However, rapid variations of the wireless channel render the CSI feedback to transmitter outdated, and motivate channel-independent precoders. On the other hand, because CSI is indispensable at the receiver, training sequences are utilized to acquire it. Blind schemes offer a bandwidth-efficient alternative when frequent re-training is required but they often are more complex and require longer data records than training based approaches.

Instead of long training sequences, inserting short known symbols is known as pilot symbol aided modulation (PSAM), and was originally developed for time and frequency offset synchronization [3],[4]. The inserted pilot symbols in PSAM are separated from the information symbols in the time domain, while the so-termed pilot tones (complex exponentials) are separated from the information symbols in the frequency domain. In the superimposed pilot (or spread-spectrum) pilot schemes of [5] and [6], a pseudo noise sequence is linearly added to information sequence. PSAM based channel estimation is analyzed for time-selective channels in [7] and for frequency-selective channels in [8]. The maximum likelihood (ML) joint symbol and channel estimation using superimposed pilot symbols is dealt with in [9].

This paper deals with linearly precoded information blocks with a superimposed pilot-block that can be modeled as an affine precoded transmission system. PSAM transmissions with pilot tones as well as transmissions with superimposed pilot symbols can be treated as special cases of affine precoded transmissions. In [10], affine precoding was discussed for ML joint symbol and channel estimation. However, the type of affine precoders suitable for optimal channel estimation and symbol recovery was not specified.

We focus on channel estimation and symbol recovery by affine precoders for relatively fast time-varying FIR channels. To enable low-complexity block-by-block processing, we first eliminate IBI by utilizing redundancy in the form of the cyclic prefix (CP). Affine precoders are designed to obtain the optimal least squares channel estimator. It is shown that pilot tones equally spaced and powered are optimal both in terms of bandwidth efficiency and in channel estimation error. Then, an approximate expression for symbol mean-squared error (MSE) of the ZF equalizer based on the channel estimate is derived. With this approximate expression, optimal power loading on information symbols and pilot symbols is investigated. To ensure the symbol recovery regardless of channel nulls, we incorporate the technique developed for generalized multi-carrier (GMC) CDMA [11] into our affine precoder. Simulations are presented to access the performance of the proposed affine precoding scheme.

II. Modeling and Problem Statement

The discrete-time baseband equivalent model considered in this paper is depicted in Fig. 1. The information sequence is parsed into blocks of size M. Each $M \times 1$ block $\boldsymbol{s}(i)$ is precoded by a $P \times M$ precoding matrix $\bar{\boldsymbol{A}}$ (with complex-valued entries) to mitigate the effects of frequency-selective channels, where $P > M$ ensures redundancy.

Preceded by a parallel-to-serial converter (P/S), the $P \times 1$ pilot vector $\bar{\boldsymbol{b}}$, which is also known to the receiver, is added to

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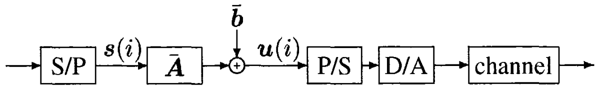

Fig. 1. Discrete-time baseband equivalent of an affine block precoded redundant transceiver.

the precoded information block to obtain

$$u(i) = \bar{A}s(i) + \bar{b}, \tag{1}$$

which is serialized, D/A converted, pulse-shaped and carrier-modulated for transmission through the channel. The encoding scheme in (1) will be henceforth referred to as affine precoding.

The channel is linear time-invariant over one received block but is allowed to vary from block-to-block. Since we only consider block-by-block processing, omitting time dependency, we express the finite impulse response (FIR) of the discrete-time baseband equivalent channel as $\{h(n)\}$. We assume that the maximum order of the channel is not greater than L; we choose the information block size M to satisfy $M \gg L$. Then, the $P \times 1$ received vector $\bar{x}(i)$ can be expressed as [1]

$$\bar{x}(i) = H_0 u(i) + H_1 u(i-1) + \eta(i), \tag{2}$$

where H_0 and H_1 are square Toeplitz convolution matrices with first column $[h(0), h(1), \ldots, h(L), 0, \ldots, 0]^T$ and first row $[h(0), 0, \ldots, 0]$ and with first column $\mathbf{0}$ and first row $[0, \ldots, 0, h(L), h(L-1), \ldots, h(1)]$, respectively; and $\eta(i)$ is a zero-mean additive white Gaussian noise (AWGN).

To enable low-complexity block-by-block processing at the receiver, we first eliminate inter block interference (IBI) by utilizing the so-called cyclic prefix (CP), which is employed by orthogonal frequency division multiplexing (OFDM) - the basic multicarrier modulation that has been adopted by many standards. CP requires the following design condition:

A1. *We select our redundant block length P to satisfy: $P \geq L + M$.*

Let N be defined as $N := P - L$ and suppose we wish to place a cyclic replica of the last L entries of an $N \times 1$ vector v at the top, to create a $P \times 1$ vector $\bar{v} = T_{cp}v$, The $P \times N$ CP-inducing precoder T_{cp} accomplishing this replicated augmentation is defined as

$$T_{cp} := \begin{bmatrix} 0_{L\times(N-L)} & I_L \\ \multicolumn{2}{c}{I_N} \end{bmatrix}_{P\times N}, \tag{3}$$

where I_L denotes the identity matrix of size L. With T_{cp} as in (3), we consider

$$\bar{A} = T_{cp}A, \quad \bar{b} = T_{cp}b, \tag{4}$$

where A is an $N \times M$ matrix and $b := [b(0), \ldots, b(N-1)]^T$ is an $N \times 1$ vector.

Substituting (4) into (1), we obtain the transmitted block

$$u(i) = T_{cp}[As(i) + b]. \tag{5}$$

At the receiver, we form $x(i) := R_{cp}\bar{x}(i)$, where $R_{cp} := [0_{N\times L}, I_N]_{N\times P}$ and substitute from (2) and (5) to obtain

$$x(i) = HAs(i) + Bh + w(i), \tag{6}$$

where: $h := [h(0), h(1), \ldots, h(L)]^T$; $w(i) := R_{cp}\eta(i)$; H is an $N \times N$ circulant matrix with first column $[h^T, 0, \ldots, 0]^T$; and B is an $N \times (L+1)$ column-wise circulant matrix with first column b; and in deriving (6) we used that $R_{cp}H_1 = \mathbf{0}$, and the commutativity of circular convolution to obtain $T_{cp}b = Bh$.

The model (6) can be considered as a *virtual* two-user model where one user transmits $s(i)$ and the other one h. They interfere with each other and thus it is desirable to eliminate the interference between them for channel estimation and symbol recovery. In this paper, we study and design affine precoders that enable such a separation using only linear operations *regardless* of the underlying FIR channels.

III. Decoupling Symbol from Channel Estimation

If B were full column rank, the LS estimate for h based on $x(i)$ would be

$$\hat{h} = B^{\dagger}x(i) = h + B^{\dagger}[HAs(i) + w(i)], \tag{7}$$

where $B^{\dagger} := (B^{\mathcal{H}}B)^{-1}B^{\mathcal{H}}$ is the pseudo-inverse of B and $^{\mathcal{H}}$ denotes conjugated transposition.

To minimize the estimation error and to guarantee symbol recovery, in the absence of noise, the following design conditions are necessary:

C1. *The training matrix B is chosen to be full column rank.*

C2. *Matrix $B^{\dagger}HA = \mathbf{0}$ for any FIR channel of order L,*

C3. *The precoding matrix A is chosen to be tall and full column rank.*

The following lemma can be readily obtained from (7):

Lemma 1. *Let A_m is a column-wise circulant matrix with first column equal to the mth column of A. Then, the necessary and sufficient condition for C2 under C1 is*

$$B^{\mathcal{H}}A_m = \mathbf{0}, \quad \text{for } m \in [0, M-1]. \tag{8}$$

Let F be the $N \times N$ FFT matrix with (m,n)th entry $[F]_{m,n} = N^{-\frac{1}{2}}W^{-mn}$, where $W := \exp(j2\pi/N)$. Let $B(z)$ be the $\mathcal{Z}$-transform of the training sequence $\{b(i)\}_{i=0}^{N-1}$. Since B is column-wise circulant, it can be expressed as

$$B = F^{\mathcal{H}}D_B F_{0:L} \tag{9}$$

where $F_{0:L}$ is a submatrix of F, corresponding to the first $L+1$ columns of F, and D_B is a diagonal matrix defined as $D_B := \mathrm{diag}[B(1), B(W), \ldots, B(W^{N-1})]$. Expressions for A_m similar to (9) can be readily obtained. Substituting these expressions into (8), one finds that (8) holds if and only if there exists an $(N-L-1) \times M$ matrix Φ such that

$$D_B F A = F_{L+1:N-L-1}\Phi, \tag{10}$$

where $\boldsymbol{F}_{L+1:N-L-1}$ is formed by the $(L+1)$st to the $(N-L-1)$st columns of $\boldsymbol{F}$.

It follows from (10) that the conditions on $\boldsymbol{A}$ and $\boldsymbol{b}$ for C2 depend on $\boldsymbol{D}_B$, and more specifically, on the number of non-zero $B(W^i)$'s. Let K be the number of non-zero $B(W^i)$'s, we define two sets of integer indices as following:

$$\mathcal{L} := \{l_k | B(W^l) \neq 0, l_k < l_{k+1}, k \in [0, K-1]\} \quad (11)$$

and

$$\mathcal{I} := \{i_k | B(W^i) = 0, i_k < i_{k+1}, k \in [0, N-K-1]\}. \quad (12)$$

Corresponding to $\mathcal{I}$, we also define the FIR filter $Q(z)$ of order $N-K-1$ as

$$Q(z) := \Pi_{i \in \mathcal{I}}(1 - W^i z^{-1}). \quad (13)$$

Based on these notational conventions we can state our first result as follows: (proofs are omitted due to lack of space)

Theorem 1. *Let the $\mathcal{Z}$-transform of the pilot block $\boldsymbol{b}$ be $B(z)$ and let K be the number of non-zero $B(W^i)$'s for $i \in [0, N-1]$, where $W := \exp(j2\pi/N)$. With the sets of integer indices as in (11) and (12), and for given M and L, the conditions C1–C3 are satisfied if and only if:*

1) $N \geq M + K$, $L + 1 \leq K \leq 2L + 1$, and the precoding matrix $\boldsymbol{A}$ is expressed through an $(N-K) \times M$ full column rank matrix $\boldsymbol{\Theta}$ as

$$\boldsymbol{A} = \boldsymbol{F}^{\mathcal{H}} \boldsymbol{P} \begin{bmatrix} \boldsymbol{\Theta} \\ \boldsymbol{0}_{K \times M} \end{bmatrix}, \quad (14)$$

where $\boldsymbol{P}$ is an $N \times N$ permutation matrix which permutes the i_kth row with the kth row for $k = 0, \ldots, N-K-1$ and $\boldsymbol{F}$ is an $N \times N$ FFT matrix.

2)$N \geq M + 2L + 1$, $2L + 1 < K \leq N$, and the precoding matrix $\boldsymbol{A}$ is expressed through an $(N-2L-1) \times M$ full column rank matrix $\boldsymbol{\Theta}$ as

$$\boldsymbol{A} = \boldsymbol{F}^{\mathcal{H}} \boldsymbol{P} \begin{bmatrix} \boldsymbol{I}_{N-K} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Lambda V} \end{bmatrix}_{N \times (N-2L-1)} \boldsymbol{\Theta}, \quad (15)$$

where $\boldsymbol{V}$ is a $K \times (K-2L-1)$ Vandermonde matrix with generators $\{W^{l_0}, W^{l_1}, \ldots, W^{l_{K-1}}\}$, and $\boldsymbol{\Lambda}$ is a $K \times K$ diagonal matrix given by

$$\boldsymbol{\Lambda} = \text{diag}\left[\frac{Q(W^{l_0})}{B(W^{l_0})} W^{(L+1)l_0}, \ldots, \frac{Q(W^{l_{K-1}})}{B(W^{l_{K-1}})} W^{(L+1)l_{K-1}}\right]. \quad (16)$$

Theorem 1 specifies the classes of redundant precoders $\boldsymbol{A}$ that satisfy C2 (and thus decouple channel estimation from symbol recovery) as a functions of: the channel order L, the redundant block size N and the modes (K) of the pilot block $\boldsymbol{b}$. Theorem 1 also implies that there are degrees of freedom in designing $\boldsymbol{A}$ and $\boldsymbol{b}$ to e.g., maximize bandwidth/power efficiency and/or minimize mean-squared channel estimation error.

The bandwidth efficiency can be defined as

$$\mathcal{E} := \frac{M}{P} = \frac{M}{N+L}. \quad (17)$$

It follows from Theorem 1 that if the number of non-zero FFT coefficients of $\boldsymbol{b}$ is $K = L+1$, that is $\boldsymbol{b}$ is composed of $L+1$ pilot tones, then for a fixed M the bandwidth efficiency is optimal among all cyclic prefixed affine precoders as per Theorem 1 and is given by $\mathcal{E} = M/(M + 2L + 1)$.

The mean-squared channel estimation error in (7) is given by

$$\sigma_{\hat{h}}^2 := E\{||\hat{\boldsymbol{h}} - \boldsymbol{h}||^2\} = \sigma_w^2 \operatorname{tr}[(\boldsymbol{B}^{\mathcal{H}}\boldsymbol{B})^{-1}] \quad (18)$$

where $E\{\cdot\}$ stands for the expectation operator, $||\cdot||$ is the norm of a vector, and σ_w^2 is the AWGN variance. If the power of the pilot block $\boldsymbol{b}$ is $||\boldsymbol{b}||^2 = \mathcal{P}_b$, then, using the fact that $\operatorname{tr}(\boldsymbol{B}^{\mathcal{H}}\boldsymbol{B}) = (L+1)\mathcal{P}_b$ and $\operatorname{tr}(\boldsymbol{B}^{\mathcal{H}}\boldsymbol{B}) \operatorname{tr}[(\boldsymbol{B}^{\mathcal{H}}\boldsymbol{B})^{-1}] \geq (L+1)^2$, we deduce that $\operatorname{tr}[(\boldsymbol{B}^{\mathcal{H}}\boldsymbol{B})^{-1}]$ is minimized if and only if $\boldsymbol{B}^{\mathcal{H}}\boldsymbol{B} = \mathcal{P}_b \boldsymbol{I}_{L+1}$. The minimum mean-squared channel estimation error will thus be

$$\sigma_{\hat{h}}^2 = \frac{L+1}{\mathcal{P}_b} \sigma_w^2. \quad (19)$$

As per Theorem 1, it is possible to design precoders with optimum bandwidth efficiency and superimposed pilots that optimize channel estimation performance if we: select (i) $K = L+1$ and $N = (L+1)J$ for some non-zero integer J; and, ii) choose the pilot tones with spacing and values satisfying:

$$l_i = l_0 + (L+1)i, \qquad |B(W^{l_i})|^2 = \frac{\mathcal{P}_b}{L+1}, \quad (20)$$

for some $l_0 \in [0, J-1]$ and $i \in [1, J]$. Eq. (20) implies that equally spaced and powered pilot tones optimize MMSE channel estimation performance.

IV. Designing Optimal Affine Precoders

Our ultimate goal is to design affine precoder that minimizes BER for a given transmit-power budget. However, it is not easy to obtain a closed form expression even for the upper bound of BER. Instead of BER, we will rely here on an approximate expression for the symbol mean-squared symbol (MSE) of the ZF (or LS) equalizer constructed from the LS channel estimate given by (7), assuming that the information block is white with zero mean and variance $\sigma_s^2 \boldsymbol{I}$. Because MSE (or BER) performance depends critically on the accuracy of the channel estimate, we will confine ourselves to equally spaced and equally powered pilot tones.

For the moment, let us consider the ZF equalizer based on the exact CSI which generates

$$\hat{\boldsymbol{s}}_{zf}(i) = (\boldsymbol{HA})^{\dagger} \boldsymbol{x}(i). \quad (21)$$

The symbol MSE can be bounded as follows:

$$\begin{aligned} E\{||\hat{\boldsymbol{s}}_{zf}(i) - \boldsymbol{s}(i)||^2\} &= ||(\boldsymbol{HA})^{\dagger}||_F^2 \sigma_w^2 \\ &\leq ||\boldsymbol{H}^{\dagger}||_F^2 ||\boldsymbol{A}^{\dagger}||_F^2 \sigma_w^2, \end{aligned} \quad (22)$$

where $||\boldsymbol{A}||_F := [\mathrm{tr}(\boldsymbol{A}^{\mathcal{H}}\boldsymbol{A})]^{\frac{1}{2}}$.

If CSI is available at the transmitter, then minimizing the symbol MSE amounts to minimizing $||(\boldsymbol{H}\boldsymbol{A})^{\dagger}||_F^2$ over $\boldsymbol{A}$. However, targeting a channel-independent precoder, it is reasonable to look for a $\boldsymbol{A}$ that minimizes $||\boldsymbol{A}^{\dagger}||_F^2$ in (22). Under the power constraint $\mathcal{P}_s := E\{||\boldsymbol{A}\boldsymbol{s}(i)||^2\} = \sigma_s^2||\boldsymbol{A}||_F^2$ and using the fact that $||\boldsymbol{A}||_F^2||\boldsymbol{A}^{\dagger}||_F^2 \geq M^2$, we find that $||\boldsymbol{A}^{\dagger}||_F^2$ is minimized if and only if $\boldsymbol{A}$ is orthogonal such that $\boldsymbol{A}^{\mathcal{H}}\boldsymbol{A} = c\boldsymbol{I}_M$, where $c := \mathcal{P}_s/(M\sigma_s^2)$. If the channel estimation error is sufficiently small, the same argument applies asymptotically to the symbol MSE of the ZF equalizer output that is based on the channel estimate. Thus, we henceforce restrict $\boldsymbol{A}$ to be orthogonal.

Let us define the normalized symbol MSE as

$$\sigma_{\hat{s}}^2 := \frac{E\{||\hat{\boldsymbol{s}}(i) - \boldsymbol{s}(i)||^2\}}{E\{||\boldsymbol{s}(i)||^2\}} = \frac{E\{||\hat{\boldsymbol{s}}(i) - \boldsymbol{s}(i)||^2\}}{M\sigma_s^2}. \quad (23)$$

Since $N \gg L$ and $\boldsymbol{A}$ is orthogonal, we may approximate $\boldsymbol{A}\boldsymbol{A}^{\mathcal{H}}$ as $\boldsymbol{A}\boldsymbol{A}^{\mathcal{H}} \approx c\boldsymbol{I}_N$. Under this approximation, we can show that if the channel matrix $\boldsymbol{H}$ is invertible and well-conditioned, then $\sigma_{\hat{s}}^2$ can be approximated as

$$\sigma_{\hat{s}}^2 \approx \frac{1}{Mc\sigma_s^2}||\boldsymbol{H}^{-1}||_F^2\left(\sigma_w^2 + c\sigma_s^2\sigma_{\hat{h}}^2\right). \quad (24)$$

We will load power on information and pilot symbols so that the symbol MSE given by (24) is minimized. For simplicity, and w.l.o.g, we can set $E\{||\boldsymbol{A}\boldsymbol{s}(i)||^2\} + ||\boldsymbol{b}||^2 = 1$ and define the power ratio between information symbols and pilot symbols as

$$\frac{E\{||\boldsymbol{A}\boldsymbol{s}(i)||^2\}}{||\boldsymbol{b}||^2} = \frac{\alpha}{1-\alpha}. \quad (25)$$

It follows from (19) and (24) that

$$\sigma_{\hat{s}}^2 \approx ||\boldsymbol{H}^{-1}||_F^2\,\sigma_w^2\left[\frac{1}{\alpha} + \frac{L+1}{M(1-\alpha)}\right]. \quad (26)$$

By differentiating the RHS of (26) with respect to α, the minimum is found to be

$$\sigma_{\hat{s},opt}^2 := ||\boldsymbol{H}^{-1}||_F^2\,\sigma_w^2\left(1 + \sqrt{\frac{L+1}{M}}\right)^2 \quad (27)$$

at $\alpha_{opt} := 1/(1 + \sqrt{L + 1/M})$. The optimum α_{opt} is a function of the ratio between the number of pilot symbols and that of information symbols. As the number M of information symbol increases, α_{opt} increases and more power is loaded on information symbols. Conversely, as the channel order L increases, α_{opt} decreases and more power is allocated to pilot symbols to obtain reliable channel estimates.

So far we assumed that the circulant channel matrix $\boldsymbol{H}$ is invertible in (7). It is well-known that if the channel has nulls on the FFT grids, then $\boldsymbol{H}$ becomes singular and hence symbol recovery is not guaranteed [1], [2]. To assure the invertibility of $\boldsymbol{H}\boldsymbol{A}$, we utilize the precoder developed for generalized multi-carrier (GMC) CDMA in [11].

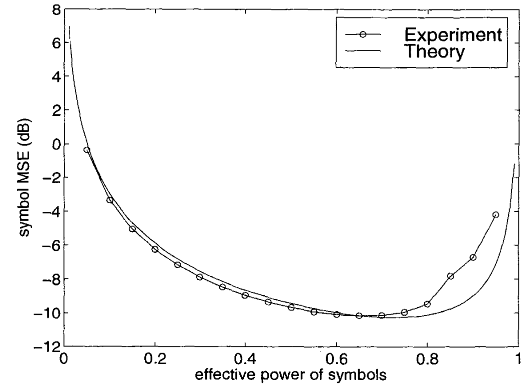

Fig. 2. Approximate and empirical symbol MSE.

We set the block size N to be $N = M + 2L + 1$ and the matrix $\boldsymbol{\Theta}$ for the precoding matrix given in (14) to be $\boldsymbol{F}_{M+L,0:M-1}$ that has the first M columns of the $(M + L) \times (M + L)$ FFT matrix; e.g., we select

$$\boldsymbol{A} = \boldsymbol{F}^{\mathcal{H}}\boldsymbol{P}\begin{bmatrix} \boldsymbol{F}_{M+L,0:M-1} \\ \boldsymbol{0}_{(L+1)\times M} \end{bmatrix}. \quad (28)$$

The channel matrix $\boldsymbol{H}$ can be diagonalized by $\boldsymbol{F}$ to obtain

$$\boldsymbol{H}\boldsymbol{A} = \tilde{\boldsymbol{F}}^{\mathcal{H}}\mathrm{diag}[H(W^{i_0}), \ldots, H(W^{i_{M+L-1}})]\boldsymbol{F}_{M+L,0:M-1}, \quad (29)$$

where $H(z)$ denotes the $\mathcal{Z}$-transform of $\{h(i)\}$ and $\tilde{\boldsymbol{F}}$ is an $(M + L) \times N$ matrix with nth column equal to the i_nth column of $\boldsymbol{F}$ for $n \in [0, M + L - 1]$. Because the channel has order L, at most L values of $H(W^{i_n})$ are zero. Since any M rows of $\boldsymbol{F}_{M+L,0:M-1}$ are linearly independent, it follows that $\boldsymbol{H}\boldsymbol{A}$ has full column rank regardless of the channel nulls. Therefore, $\boldsymbol{H}\boldsymbol{A}$ becomes invertible at the price of a slight decrease in bandwidth efficiency.

V. Numerical Results

In all experiments, we considered affine precoders with blocks of size $N = 64$. The guard interval length was set to be $L = 7$. The number of pilot tones was $L + 1 = 8$. Pilot tones were equally spaced and equally power-loaded. The channels were normalized to have unit norm, and the information symbols were drawn from a BPSK constellation.

A. Example 1

To validate our approximate expression (26) for the symbol MSE, we consider an FIR channel of order 7. The value of $||\boldsymbol{H}^{-1}||_F^2$ in (26) was 94.41, M and $\boldsymbol{\Theta}$ in (14) were set to be $M = N - (L + 1) = 56$ and $\boldsymbol{\Theta} = \boldsymbol{I}_M$, respectively. This

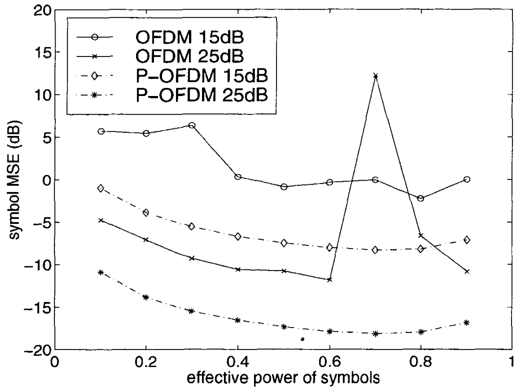

Fig. 3. Symbol MSE vs. effective symbol power α.

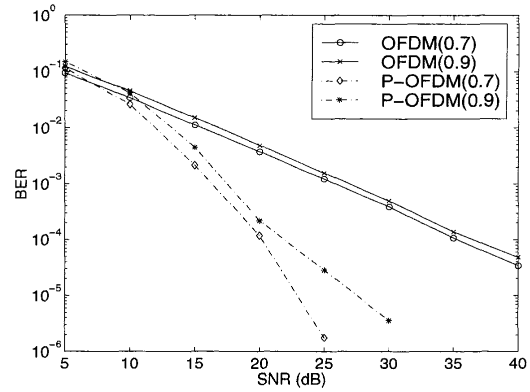

Fig. 4. Averaged BER vs. SNR.

affine precoder corresponds with uncoded OFDM with pilot tones. The symbol MSE was averaged over 100 Monte-Carlo experiments for different power ratio α. The optimum power ratio was found to be $\alpha_{opt} \approx 0.73$.

Fig. 2 depicts good agreement between the approximate symbol MSE and the empirical symbol MSE. It should be remarked that the symbol MSE is flat around its minimum.

B. Example 2

We compared the OFDM used in Example 1 with the affine precoder that guarantees the invertibility of $\boldsymbol{HA}$. We refer to the latter as Precoded OFDM. For P-OFDM, we set $M = 56-7 = 49$ and $\boldsymbol{\Theta} = \boldsymbol{F}_{M+L,0:M-1}$. For each SNR and power ratio, we averaged the symbol MSE and BER over 10^4 Monte-Carlo realizations of 7th order Rayleigh fading channels.

Fig. 3 compares the symbol MSE versus the power ratio α at fixed SNR. The fluctuation of the symbol MSE with OFDM at SNR$=$ 25dB is due to ill-conditioned channel matrices. Except for the fluctuation, we infer that both affine precoders have minimum MSE around α_{opt} and that the symbol MSE is flat around α_{opt}, as expected. P-OFDM enjoys 5dB gain over OFDM.

Fig. 4 shows BER versus SNR for $\alpha = 0.7$ and $\alpha = 0.9$. For both precoders, no major differences in the BER were observed between the two power ratio. As SNR increases, the BER of P-OFDM decreases fast, while that of the OFDM decreases linearly. This corroborates the importance of guaranteed symbol recovery.

VI. Conclusions

To enable low-complexity block-by-block processing, redundant precoders with cyclic prefix and superimposed pilots have been designed to decouple symbol recovery from least-squares based channel estimation. An approximate expression for the symbol MSE of the ZF equalizer based on the channel estimate has been derived, from which the optimum power ratio between information symbols and pilot symbols was designed and validated by simulations. An affine precoder that assures symbol recovery regardless of FIR channel nulls was adopted and verified by simulations to outperform the trivial one.

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