REDUNDANT FILTERBANK PRECODERS AND EQUALIZERS: UNIFICATION AND OPTIMAL DESIGNS†

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Abstract: Transmitter redundancy introduced using filterbank precoders generalizes existing modulations including OFDM, DMT, TDMA, and CDMA schemes encountered with single- and multi-user communications. Sufficient conditions are derived to guarantee that, with FIR filterbank precoders, FIR channels are equalized perfectly in the absence of noise by FIR zero-forcing equalizer filterbanks, *irrespective* of the channel zero locations. Multicarrier transmissions through frequency-selective channels can thus be recovered even when deep fades are present. Jointly optimal transmitter-receiver filterbank designs are also developed based on maximum output SNR and minimum mean-square error criteria under zero-forcing and fixed transmitted power constraints. Analytical performance results are presented for the zero-forcing filterbanks and are compared with mean-square error and ideal designs using simulations.

1. Introduction and Problem Statement

Orthogonal frequency division multiplexing (OFDM) has been successfully employed in digital audio and video broadcasting as an effective way to combat impulsive noise and intersymbol interference (ISI) due to frequency-selective fading [4]. Coded-OFDM (COFDM) has been later introduced to increase the robustness of OFDM against deep channel frequency nulls, at the expense of reduced efficiency. Both OFDM and COFDM are particular cases of modulations introducing redundancy at the transmitter via multirate filterbank precoders. In [2] and [7], transmitter induced cyclostationarity via filterbank precoding has been proved to allow for blind channel equalization using simpler receiver structures than solutions based on fractional sampling, or, space diversity [8]. In applications where the channel conditions are known at the transmitter, through feedback channels for example, the transmitter can optimize the precoding structure to combat channel fading. For example, discrete multitone (DMT) is a multi-carrier transmission scheme distributing the capacity on each subchannel as a function of the corresponding SNR [5]. In this paper, building on the general multirate filterbank precoding framework, we provide sufficient conditions guaranteeing *perfect zero-forcing equalization of FIR channels using FIR filterbanks* by introducing the *minimal amount of redundancy* and propose a *joint optimization of the transmit-receive filterbank pairs.*

Consider the discrete-time multirate equivalent scheme of the baseband communication system in Fig. 1. Downsamplers and upsamplers perform blocking (i.e., multiplexing) and un-blocking (de-multiplexing) operations. For every M symbols $s(n)$, we transmit P symbols $u(n)$, and with $P > M$ the rate $(P-M)/P$ represents the amount of redundancy introduced. Transmit filters $\{f_m(n)\}_{m=0}^{M-1}$ and receive-filters $\{g_p(n)\}_{p=0}^{P-1}$ are FIR of maximum order $P-1$ and K respectively, while the Lth order channel $\{h(l)\}_{l=0}^{L}$ includes multipath effects and timing ambiguities as delay factors. The input to the upsampler of the mth branch is $s_m(n) := s(nM+m)$. It represents the m-th symbol in the n-th block of M symbols, while in the multi-user case it stands for the mth user's symbols. With the insertion of $P-1$ zeros, the corresponding upsampler's output is: $u_m(n) = \sum_i s_m(i)\delta(n-iP)$, where $\delta(n)$ denotes Kronecker's delta. We will assume Nyquist signaling pulses; hence, the transmitted sequence is: $u(n) = \sum_{m=0}^{M-1} u_m(n) = \sum_i \sum_{m=0}^{M-1} s_m(i) f_m(n-iP)$. The received samples, $y(n) = x(n) + v(n)$, consist of the noise-free data:

$$x(n) = \sum_{i=-\infty}^{+\infty} \sum_{m=0}^{M-1} s_m(i) \sum_{l=0}^{L} h(l) f_m(n-l-iP), \qquad (1)$$

plus additive zero-mean stationary noise with covariance matrix $\boldsymbol{R}_{vv}$ $(= \sigma_{vv}^2 \boldsymbol{I}$ when the noise is white). Thanks to the FIR nature of $h(n)$ and $f_m(n)$, it can be shown [1] that for $M > L$ the p-th polyphase component (delayed and downsampled version) of $x(n)$, for $p = 0, \cdots, P-1$, is

$$\begin{aligned} x_p(n) &:= x(nP+p) = \sum_{m=0}^{M-1} s_m(n) \sum_{l=0}^{L} h(l) f_m(p-l) \\ &+ \sum_{m=0}^{M-1} s_m(n-1) \sum_{l=0}^{L} h(l) f_m(P+p-l), \qquad (2) \end{aligned}$$

where the terms $s_m(n)$ and $s_m(n-1)$ imply that the Lth order ISI entails at most two consecutive M data blocks. Let us define $M \times 1$ vector $\boldsymbol{s}(n) := (s_0(nM) \ldots s_{M-1}(nM))^T$,

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[1] Detailed proofs of claims and theorems in this paper can be found in the journal version [6];

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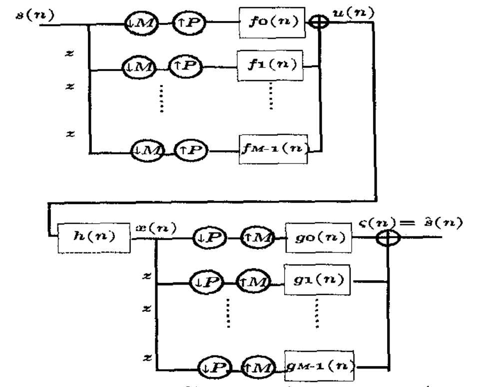

Figure 1: - Multirate discrete-time transmitter-channel-receiver model

$P \times 1$ vectors $\boldsymbol{x}(n) := (x(nP)x(nP+1)\ldots x(nP+P-1))^T$, $\boldsymbol{f}_m := (f_m(0)\ldots f_m(P-1))^T$, $P \times M$ precoder matrix $\boldsymbol{F} := (\boldsymbol{f}_0\ldots\boldsymbol{f}_{M-1})$, and $P \times P$ Toeplitz lower (upper) triangular matrix $\boldsymbol{H}_0$ ($\boldsymbol{H}_1$) with first (last) column $(h(0)\cdots h(L)0\cdots 0)^T$ $((h(1)\cdots h(L)\ 0\cdots 0)^T)$. Based on these definitions, we can cast (2) in matrix form:

$$\boldsymbol{x}(n) = \boldsymbol{H}_0\boldsymbol{F}\boldsymbol{s}(n) + \boldsymbol{H}_1\boldsymbol{F}\boldsymbol{s}(n-1), \qquad (3)$$

and state this paper's operational assumptions as follows:
(a0) Channel $h(l)$ is Lth order FIR with $h(0), h(L) \neq 0$.
(a1) (P, M) are chosen such that the triplet (P, M, L) satisfies: $P > M > L$
(a2) Filters $\{f_m(n)\}_{m=0}^{M-1}$ are linearly independent ($rank(\boldsymbol{F}) = M$), which guarantees one-to-one mapping and thus recovery of $s(n)$ from the coded symbols $u(n)$.

Under **(a0)**-**(a2)**, our objective is twofold: (i) to develop sufficient conditions for the existence of FIR zero-forcing (ZF) filterbanks which in the absence of noise equalize perfectly FIR channels (Section 2); and (ii) to derive jointly optimal FIR transmitter-receiver filterbank pairs according to the following criteria: 1) maximum SNR, subject to the zero-forcing (ZF) condition, i.e., no ISI; 2) minimum mean square error, considering both ISI and noise, subject to the transmitter average power constraint (Section 3). Simulation results are shown in Section 4.

Remark 1: A number of single and multiuser modulation schemes fall under the filterbank precoder of Fig. 1: OFDM, DMT, Fractional Sampling, Periodic input modulation, Spread-Spectrum, Interleaving, but also the multiuser schemes CDMA, TDMA, FDMA, where the blocking of data is created by multiplexing the data of M users [6].

2. FIR-ZF Equalizing Filterbanks

With moderate or large number of filters M in the precoder, the maximum likehood receiver implemented with Viterbi's algorithm has prohibitively large complexity which motivates looking for linear (and preferably low order FIR) equalizing filterbanks. In this section we will focus on ZF solutions because they offer (almost) perfect symbol recovery in (high SNR) noise free environments and their performance in terms of error probability is easily computable. Designs in the presence of noise are pursued in Section 3.

Our first result is summarized in the next theorem:
Theorem 1(leading equalizer zeros): *Suppose that* **(a0)**-**(a2)** *hold true, and in addition assume that:*
(a3)*Each column of the $QP \times QP$ matrix $(\boldsymbol{I}_{Q\times Q} \otimes \boldsymbol{F})$ ($\otimes$ stands for Kronecker product) cannot be expressed as a linear combination of less than $L+1$ Vandermonde vectors like $(1, \rho, \rho^2, .., \rho^{P-1})^T$.*
(a4) *For a given L, the triplet (P, M, Q) is selected to satisfy* **(a1)** *and also $P \geq M + \lceil L/Q \rceil$.*
Then, for a given precoder $\boldsymbol{F}$ and channel matrices $\boldsymbol{H}_0$, $\boldsymbol{H}_1$ in (3), there exists an FIR-ZF equalizing filterbank $\{\boldsymbol{G}_q\}_{q=0}^{Q-1}$ so that:

$$\boldsymbol{s}(n) = \sum_{q=0}^{Q-1} \boldsymbol{G}_q \boldsymbol{x}(n-q), \qquad (4)$$

where the $M \times P$ matrix $\boldsymbol{G}_q$ is formed with columns $\{\boldsymbol{g}_{q,p} = (g_p(qM), .., g_p(qM+M-1))^T\}_{p=0}^{P-1}$. If in addition to **(a0)**-**(a4)** *matrix $\boldsymbol{G}_{Q-1} = (\boldsymbol{0}_{M\times L}|\tilde{\boldsymbol{G}}_{Q-1})$, then the ZF equalizing filterbank $\mathcal{G} := (\tilde{\boldsymbol{G}}_{Q-1}|\boldsymbol{G}_{Q-2}|\cdots|\boldsymbol{G}_0)$ is unique and is given by:*

$$\mathcal{G} = (\boldsymbol{0}_{M\times(Q-1)M}|\boldsymbol{I}_{M\times M})(\mathcal{H}(\boldsymbol{I}_{Q\times Q} \otimes \boldsymbol{F}))^{\dagger}, \qquad (5)$$

where $\dagger$ denotes pseudo-inverse, and $\mathcal{H}$ is a $(QP-L)\times QP$ Sylvester matrix with first row $(h(L)\cdots h(0)0\cdots 0)$.

It is noteworthy that the existence and uniqueness conditions of Thm. 1 do not pose any constraints on the channel zeros. In contrast, FIR-ZF equalizers in [8] and [7] do not exist for certain configurations of channel zeros on the unit circle, and more important, performance degrades even when channels have zeros close to those non-invertible configurations.

Given the transmission block size P and the channel order L, **(a4)** is met easily by selecting appropriately M and/or the matrix equalizer length Q. Minimum block size P requires $P = M + 1$, and **(a4)** is then satisfied with equalizer length $Q \geq L$. On the other hand, simple zero-order ($Q = 1$) receiver filterbanks satisfy **(a4)** at the expense of extra redundancy: $P = M + L$ when M is fixed, or, with extra latency if both P and M increase in order to maintain fixed information rate. Assumption **(a3)** seems more technical but is satisfied by a number of special cases including those mentioned in Remark 1 with appropriate modifications. For spread-spectrum and CDMA systems, the $f_m(n)$ filters are often selected as pseudo-random codes and satisfy **(a3)** almost surely. TDMA systems have $\boldsymbol{F} = (\boldsymbol{I}_{M\times M}|\boldsymbol{0}_{M\times(P-M)})^T$ and will obey **(a3)**, if $M \geq L$, because the M canonical vectors in $\boldsymbol{F}$ cannot be generated by less than M linearly independent vectors, and $M > L$. This corroborates the result in [2], where a TDMA precoder accepts a ZF equalizer without channel zero restrictions.

A more general class of precoders fulfilling **(a3)** results if one chooses the $P \times 1$ columns $\{f_m(n)\}_{m=0}^{M-1}$ of $\boldsymbol{F}$ from the M-dimensional subspace spanned by the canonical basis of $\mathcal{R}^P$. Because $P > L$, each of the canonical basis vectors is given as a linear combination of exactly P Vandermonde vectors, and hence **(a3)** is satisfied. Trailing precoder zeros are a special case but **(a3)** is also satisfied if the L zeros are inserted in arbitrary positions of the filters $f_m(n)$.

Even when (**a0**)-(**a3**) hold and (**a4**) is satisfied as an equality, only existence (but not uniqueness) is guaranteed. Note that in Thm. 1 we also forced the equalizer filterbank to have leading zeros in order to assume uniqueness. An alternative approach is to force the transmitter filterbank to have trailing zeros which also satisfies (**a3**) and leads to ZF equalization as we summarize next:

Theorem 2: (trailing precoder zeros): *Suppose that* (**a0**) *holds,*(**a1**) *is satisfied with* $P = M + L$, *and transmitter matrix* $\boldsymbol{F}$ *has trailing zeros (and thus* (**a3**) *holds), and also obeys* (**a2**); *i.e.,* $\boldsymbol{F} = (\tilde{\boldsymbol{F}}_{M\times M}\boldsymbol{0}_{M\times L})^T$ *and* $rank(\tilde{\boldsymbol{F}}) = M$. *Then for a given* $\tilde{\boldsymbol{F}}$ *and channel matrix* $\boldsymbol{H}_0$, *there exists a zero-order* ($Q = 1$) *ZF equalizer filterbank so that* $\boldsymbol{G}\boldsymbol{x}(n) = \boldsymbol{s}(n)$. *With* $\tilde{\boldsymbol{H}}_0$ *denoting the first* M *columns of* $\boldsymbol{H}_0$, *the minimum norm ZF filterbank is unique and is given by:*

$$\boldsymbol{G} = (\boldsymbol{H}_0\boldsymbol{F}) = \tilde{\boldsymbol{F}}^{-1}\tilde{\boldsymbol{H}}_0^{\dagger} \tag{6}$$

Absence of the matrix $\boldsymbol{H}_1$ is reasonable, since the guard time of L trailing zeros avoids interblock interference which causes the second sum in (2) to disappear and simplifies (3) to $\boldsymbol{x}(n) = \boldsymbol{H}_0\boldsymbol{F}\boldsymbol{s}(n)$.

Remark 2: Conventional OFDM does not satisfy (**a3**), because the columns of $\boldsymbol{F}$ are themselves Vandermonde vectors [6]. However, Theorem 2 and the class of precoders with trailing zeros suggests a practical modification to the OFDM system: instead of the cyclic prefix one pads L trailing zeros. Such a modification leads to ZF equalizers even for channels with zeros uniformly spaced on the unit circle.

3. Transmitter-Receiver Optimal Designs

Assuming now that the channel matrices $\boldsymbol{H}_0$ and $\boldsymbol{H}_1$, along with the signal and noise covariance matrices $\boldsymbol{R}_{ss}$ and $\boldsymbol{R}_{vv}$, are known at the trasmitter side, we propose two methods for optimizing the precoder matrix $\boldsymbol{F}$ together with a zero order ($Q = 1$) equalizer $\boldsymbol{G}$. As per Thm. 1, a zero-order equalizer (**a4**) requires $P = M + L$, which we assume herein.

3.I Maximum SNR subject to ZF constraint

Our goal here is to select $\boldsymbol{G}$ and $\boldsymbol{F}$ that maximize SNR under the ZF constraint $\boldsymbol{G}\boldsymbol{H}_0\boldsymbol{F} = \mathcal{K}\boldsymbol{I}_{M\times M}$,where $\mathcal{K}$ is in general a complex constant whose magnitude $|\mathcal{K}| > 0$ depends on the transmitted power and controls the SNR at the equalizer output. The vector equalizer output is: $\hat{\boldsymbol{s}}(n) = \boldsymbol{s}(n) + \boldsymbol{G}\boldsymbol{v}(n)$, and, since the output SNR is only magnitude dependent, we take without loss of generality $\mathcal{K}$ real and positive. Clearly, unconstrained minimization of the output noise power (given by $tr(\boldsymbol{G}\boldsymbol{R}_{vv}\boldsymbol{G}^H)$), leads to the trivial solution $\boldsymbol{G} \equiv \boldsymbol{0}$ which contradicts $\boldsymbol{G}\boldsymbol{H}_0\boldsymbol{F} = \mathcal{K}\boldsymbol{I}_{M\times M}$ unless infinite power is transmitted. To avoid this solution, we formulate our max-SNR/ZF constrained optimization problem as follows:

$$\max_{\mathbf{G},\mathbf{F}} \mathcal{K}^2\frac{|tr(\boldsymbol{I}_{M\times M})|^2}{tr(\boldsymbol{G}\boldsymbol{R}_{vv}\boldsymbol{G}^H)} \text{ subjet to } \boldsymbol{G}\boldsymbol{H}_0\boldsymbol{F} = \mathcal{K}\boldsymbol{I}_{M\times M}\ . \tag{7}$$

Note that when $\boldsymbol{v}(n)$ is AGN, (7) is equivalent to minimizing the probability of error in a block-by-block detection scheme. The solution of (7) is summarized in the next theorem and is divided in two parts depending on whether the leading or trailing zeros are forced in the equalizer or in the precoder filterbank.

Theorem 3: (Max-SNR/ZF Equalizers) (**a**) *Let assumptions* (**a0**)-(**a4**) *be in force with* $P = M + L$ ($Q = 1$) *and* $\boldsymbol{G} = (\boldsymbol{0}_{M\times L}\bar{\boldsymbol{G}}_{M\times M})$ *corresponding to an equalizer with leading zeros (LZ). Let also* $\bar{\boldsymbol{H}}_0$ ($\bar{\boldsymbol{v}}(n)$) *denote the last* M *rows of* $\boldsymbol{H}_0$ ($\boldsymbol{v}(n)$), *and* $\bar{\boldsymbol{R}}_{vv}$ *stand for the covariance matrix of* $\bar{\boldsymbol{v}}(n)$. *Define* $M\times M$ *diagonal matrix* $\bar{\boldsymbol{\Lambda}}$ *from the eigendecomposition* $\bar{\boldsymbol{H}}_0\bar{\boldsymbol{R}}_{vv}^{-1}\bar{\boldsymbol{H}}_0^H = \bar{\boldsymbol{V}}\bar{\boldsymbol{\Lambda}}\bar{\boldsymbol{V}}^H$. *The output SNR in* (7) *is then maximized by the optimum ZF pair of filterbanks:*

$$\boldsymbol{F}_{opt} = \sqrt{\mathcal{K}}/\sigma_v\bar{\boldsymbol{V}}\bar{\boldsymbol{\Lambda}}^{-\frac{1}{2}},\ \ \bar{\boldsymbol{G}}_{opt} = \sigma_v\sqrt{\mathcal{K}}\bar{\boldsymbol{\Lambda}}^{-\frac{1}{2}}\bar{\boldsymbol{V}}^H\bar{\boldsymbol{H}}_0^H\bar{\boldsymbol{R}}_{vv}^{-1}. \tag{8}$$

(**b**) *Let assumptions* (**a0**)-(**a4**) *be in force with* $P = M + L$ ($Q = 1$) *and* $\boldsymbol{F} = (\tilde{\boldsymbol{F}}_{M\times M}\boldsymbol{0}_{M\times L})^T$ *corresponding to a precoder with trailing zeros* (TZ). *Define* $M\times M$ *diagonal matrix* $\tilde{\boldsymbol{\Lambda}}$ *from the eigendecomposition* $\tilde{\boldsymbol{H}}_0\tilde{\boldsymbol{R}}_{vv}^{-1}\tilde{\boldsymbol{H}}_0^H = \tilde{\boldsymbol{V}}\tilde{\boldsymbol{\Lambda}}\tilde{\boldsymbol{V}}^H$ *and let* $\tilde{\boldsymbol{H}}_0$ *denote the first* M *columns of* $\boldsymbol{H}_0$. *The output SNR in* (7) *is then maximized by the optimum* TZ-ZF *pair of filterbanks:*

$$\boldsymbol{F}_{opt} = \sqrt{\mathcal{K}}/\sigma_v\tilde{\boldsymbol{V}}\tilde{\boldsymbol{\Lambda}}^{-\frac{1}{2}},\ \ \tilde{\boldsymbol{G}}_{opt} = \sqrt{\mathcal{K}}\sigma_v\tilde{\boldsymbol{\Lambda}}^{-\frac{1}{2}}\tilde{\boldsymbol{V}}^H\tilde{\boldsymbol{H}}_0^H\tilde{\boldsymbol{R}}_{vv}^{-1}. \tag{9}$$

Interesting special cases appear when $\boldsymbol{v}(n)$ is white with variance σ_v^2. In this case, it suffices to replace $\bar{\boldsymbol{\Lambda}}$ with $\sigma_v^2\bar{\boldsymbol{\Lambda}}$ in (8), to obtain

$$\boldsymbol{F}_{opt} = \sqrt{\mathcal{K}}\,\bar{\boldsymbol{V}}\bar{\boldsymbol{\Lambda}}^{-\frac{1}{2}},\ \ \boldsymbol{G}_{opt} = (\boldsymbol{0}_{M\times L}\,\sqrt{\mathcal{K}}\,\bar{\boldsymbol{U}}^H)\ , \tag{10}$$

where $\bar{\boldsymbol{U}}$ is found from the SVD: $\bar{\boldsymbol{H}}_0 = \bar{\boldsymbol{U}}\bar{\boldsymbol{\Lambda}}^{\frac{1}{2}}\bar{\boldsymbol{V}}^H$. Analogous result can be obtained for Eq. (9)

$$\boldsymbol{F}_{opt} = \begin{pmatrix} \sqrt{\mathcal{K}}\,\tilde{\boldsymbol{V}}\tilde{\boldsymbol{\Lambda}}^{-\frac{1}{2}} \\ \boldsymbol{0}_{L\times M} \end{pmatrix}\ ,\ \ \boldsymbol{G}_{opt} = \sqrt{\mathcal{K}}\,\tilde{\boldsymbol{U}}^H\ , \tag{11}$$

where $\tilde{\boldsymbol{U}}$ is obtained from the SVD: $\tilde{\boldsymbol{H}}_0 = \tilde{\boldsymbol{U}}\tilde{\boldsymbol{\Lambda}}^{\frac{1}{2}}\tilde{\boldsymbol{V}}^H$. In both (10) and (11) the optimum precoder (equalizer) filterbank matrix is proportional to the left (right) singular vectors of the channel matrix. Transmitter filterbanks are also weighted by the inverse of the square root of the corresponding singular values.

Remark 3: With flat fading (the channel matrix is diagonal) and white noise, (10) and (11) yield precoder and equalizer filterbanks with diagonal structure. Since TDMA corresponds to $f_m(n) = \delta(n-m)$, we infer that TDMA possesses optimality (in the ZF maximum output SNR sense) when it comes to channels involving flat fading and white noise.

Remark 4:A similar solution was obtained in [3], under whiteness assumptions on symbols and noise. Our precoding scheme, however, is more general because it allows for ISI removal using the minimal amount of extra redundancy, i.e. $P = M+1$. Furthermore, the scheme of Fig. 1 leads to *deterministic blind* channel identification and direct equalization methods [6].

3.II MMSE subject to average transmit power

The ultimate criterion is to minimize error probability and although maximizing output SNR under the ZF constraint leads to simple closed form solutions, alternative criteria allowing residual ISI may come closer to the desired goal. One such candidate is the minimum mean-square error (MMSE) criterion which minimizes $\mathcal{E} := E\{tr(e(n)e^H(n))\}$, where $e(n) := \hat{s}(n) - s(n)$ is the error in the n-th data block. With $P = M + L$ and L trailing zeros, $y(n) = H_0 F s(n) + v(n) = \tilde{H}_0 \tilde{F} s(n) + v(n)$, and hence:

$$e(n) = Gy(n) - s(n) = G\tilde{H}_0\tilde{F}s(n) + Gv(n) - s(n). \quad (12)$$

Using (12), the MSE objective function becomes:

$$\mathcal{E} = tr((G\tilde{H}_0\tilde{F} - I)R_{ss}(G\tilde{H}_0\tilde{F} - I)^H)) + tr(GR_{vv}G^H). \quad (13)$$

But without any constraint, minimizing $\mathcal{E}$ leads to the trivial solution corresponding to $||G|| = 0$ and requiring infinite power to be transmitted $||F|| = \infty$. Imposing the ZF constraint not only leads to a nonlinear set of equations for F and G, but also gives rise to infinite solutions indicating the need for extra restrictions in the available degrees of freedom. A reasonable alternative is to constrain the transmitted power expressed as $P_0 := tr(\tilde{F}R_{ss}\tilde{F}^H)$. Our criterion thus becomes:

$$\min_{\tilde{F},G} \mathcal{E} \quad \text{subject to} \quad tr(\tilde{F}R_{ss}\tilde{F}^H) = P_0. \quad (14)$$

Analogous criteria formulated in the frequency domain for transmitter-receiver filter optimization can be found in the scalar case (e.g., [1, p. 333]), and in the more challenging multi-input-multi-output case [9]. Optimizing our criterion in (14) follows the steps in [9], but our time-domain matrix formulation will lead to closed form selection of the FIR filterbank matrices (in [9], IIR frequency-domain designs are optimized via iterative minimization of Lagrange multipliers). Our result is summarized in the following theorem:

Theorem 4: Constrained Power-MMSE Equalizer *Let* **(a0)**-**(a4)** *hold true with* $P = M + L$ *and* $F = (\tilde{F} \;\; 0)^T$ *corresponding to a precoder with trailing zeros. Let also the channel matrix* H_0 *be given and the diagonal matrices* Δ (Λ) *be determined by the eigen-decompositions:*

$$R_{ss} = U\Delta U^H, \quad \tilde{H}_0^H R_{vv}^{-1}\tilde{H}_0 = V\Lambda V^H, \quad (15)$$

where R_{ss} (R_{vv}) *denotes the information signal (noise) covariance matrix, which are also assumed to be available. Define the diagonal matrix* Φ *with* (i,i) *entry:*

$$|\Phi_{ii}|^2 = \frac{P_0 + tr(\Lambda^{-1})}{tr(\Lambda^{-\frac{1}{2}}\Delta^{\frac{1}{2}})}\frac{1}{\sqrt{\lambda_i\delta_i}} - \frac{1}{\lambda_i\delta_i} \quad (16)$$

where λ_i (δ_i) *is the* (i,i) *entry of* Λ (Δ). *The optimum* (F, G) *filterbank pair in the sense of (14) is given by:*

$$\tilde{F}_{opt} = V\Phi U^H, \quad (17)$$

$$G_{opt} = R_{ss}F_{opt}^H H_0^H(R_{vv} + H_0F_{opt}R_{ss}F_{opt}^H H_0^H)^{-1} \quad (18)$$

and $F_{opt} = (\tilde{F}_{opt} \;\; 0)^T$.

Assumption **(a2)** in Thm. 4 requires $rank(\tilde{F}) = M$ which under the fixed power constraint in (14) imposes the

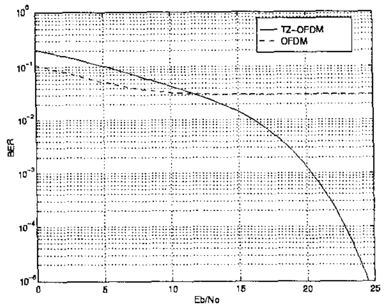

Figure 2: BER for OFDM (dashed); TZ-OFDM/ZF (solid)

(a) (b)

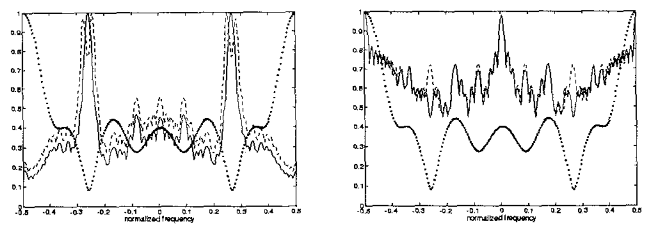

Figure 3: - $|H(\omega)|$ (dotted) vs. (a) $\max_m |F_m(\omega)|$ and (b) $\max_m |G_m(\omega)|$; TZ (solid), LZ (dashed).

following lower bound on P_0:

$$P_0 > \sqrt{\frac{tr(\Lambda^{-\frac{1}{2}}\Delta^{\frac{1}{2}})}{\min_i(\lambda_i\delta_i)}} - tr(\Lambda^{-1}). \quad (19)$$

Lower MMSE values may be reached only if one relaxes **(a2)** thereby avoiding the transmission of non recoverable symbols within the block.

4. Performance and Simulations

Example 1: (TZ-OFDM modification) We implemented the system in Fig. 1 with QPSK or BPSK inputs, $M = 8$ and $P = 11$ for an FIR channel of order $L = 3$ with zeros at -1, $0.9\exp(j9\pi/20)$, $1.1\exp(-j9\pi/20)$ and assumed knowledge of the channel impulse response at the receiver. Fig. 2 shows the average bit-error-rate (BER), theoretically computed, with BPSK $s(n)$ for OFDM (dashed) and TZ-OFDM/ZF (solid). Presence of the channel zero on the unit circle degrades performance of OFDM when compared to TZ-OFDM which according to Theorem 2 guarantees the ZF property irrespective of the channel zeros. As E_b/N_0 increases, TZ-OFDM improves its performance while conventional OFDM incurs a consistent number of symbol errors due to the channel fades and thus the corresponding curve in Fig. 2 levels off.

Example 2: (Optimum max-SNR/ZF Designs) Here we generated the optimal transmitter-receiver filterbank pairs of Theorem 3 with AWGN when the equalizer has leading zeros (LZ) as in (10), and when the precoder has trailing zeros (TZ) as in (11). Our system parameters were: $M = 32$, $P = 39$, $L = 7$, and the channel impulse response is $h^T = [1, -0.3, 0.5, -0.4, 0.1, -0.02, 0.3, -0.1]$. Figs. 3a and 3b depict for both the LZ and TZ solutions the envelopes of the transmit and receive-filters transfer functions $|\mathcal{F}(\omega)| := \max_{m\in[0,31]} |F_m(\omega)|$ and $|\mathcal{G}(\omega)| := \max_{m\in[0,31]} |G_m(\omega)|$, versus the channel transfer function magnitude $|H(\omega)|$. Note

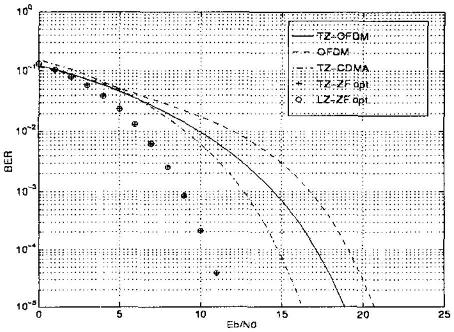

Figure 4: - BER vs. SNR

(a) (b)

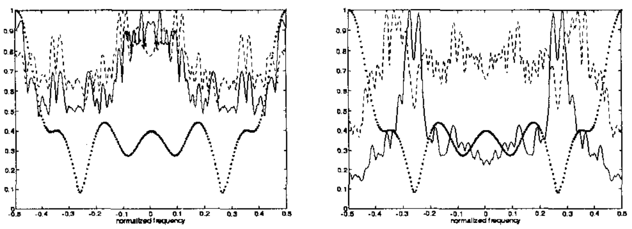

Figure 5: - $|H(\omega)|$ (dotted) vs. (a) $\max_m |F_m(\omega)|$ and (b) $\max_m |G_m(\omega)|$; $\sigma_v^2 = -6dB$ (dashed), $\sigma_v^2 = 40dB$ (solid).

that both optimal designs attempt to put more power (in at least one of their filters) at frequencies where the channel magnitude exhibits deep fades. To study the role of the TZ precoder, the choice of transmit filters with existing modulations and the effects of channel zero locations, we computed the theoretical BER with BPSK symbols considering a channel with zeros at: -0.9, $0.9\exp(j\pi 2/7)$, $0.7\exp(-j\pi/7)$. In Fig. 4 are compared the average BER vs. E_b/N_0 for the standard OFDM receiver where the channel is ideally compensated after the FFT, our proposed TZ-OFDM receiver (6), our optimized LZ-ZF and TZ-ZF transmitter-receiver pairs in (10) and (11) respectively, and a TZ-CDMA precoder using as filters the Hadamard basis with trailing zeros and the corresponding receiver in (6). All curves in Fig. 4 are generated with $M = 16$ and $P = 19$. It is evident that the LZ-ZF and TZ-ZF optimal designs of (10) and (11) perform best and have basically identical BER. Standard OFDM exhibits worst performance due to the zeros with magnitude 0.9. TZ-OFDM improves considerably OFDM's performance. Because Hadamard codes have wider spectrum, they are known to fight better against frequency-selective fading and this is verified in Fig. 4 where TZ-CDMA outperforms TZ-OFDM.

Example 3: Optimum MMSE/CP Designs Figs. 5 are the counterparts of Figs. 3 for the optimal transmitter-receiver filterbank pairs of Theorem 4 when the precoder has TZs. The same channel and parameters were used ($M = 32$, $P = 39$, $L = 7$) and the optimal MMSE designs under constrained power (CP). In this case the optimal filters concentrate most of the power at frequencies where the channel attenuation is low and are slightly sensitive to the SNR. Since the noise is white, this corresponds to send more power at those frequencies where the SNR is expected to be higher. Conversely, at the receiver side, the SNR has a clear im-

(a) (b)

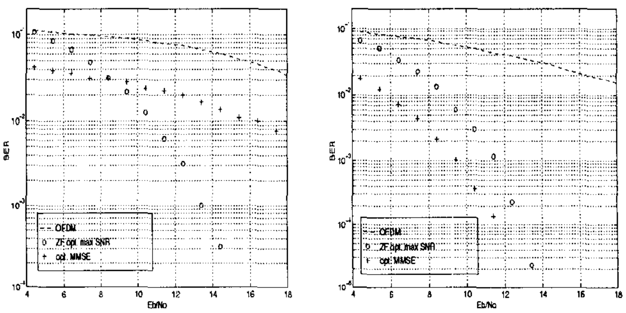

Figure 6: BER vs. SNR ($L = 3, M = 8, P = 1, P_0 = 64$)

pact on the filters depending on what is the main source of error, i.e., ISI or noise. To compare the BER performance under the two design criteria, we generated BPSK symbols and used $M = 8$ and $P = 11$. We then simulated both optimal designs with TZ precoders according to (16) and (22) and plotted their performance in Figs. 6a and 6b. Channel (a) had zeros at $-1.1, 0.9j, 0.7\exp(j3\pi/4)$ and channel (b) had zeros at $0.9, 0.7\exp(j2\pi 0.256)$, $0.4\exp(j2\pi 0.141)$. Transmitted power was fixed throughout this example at $P_0 = 64$. Both designs outperformed OFDM especially around the operational SNR range of $10 - 20$dB. Although the BER for the MMSE/CP design is lower (especially at low SNR) than the max-SNR/ZF design, its slope is lower and as the SNR increases the optimum ZF design becomes better than the MMSE/CP (Fig. 6a), or, at best the MMSE/CP approaches the max-SNR/ZF performance (Fig. 6b).

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