SPACE-TIME-DOPPLER CODING OVER TIME-SELECTIVE FADING CHANNELS WITH MAXIMUM DIVERSITY AND CODING GAINS*

Georgios B. Giannakis[1], *Xiaoli Ma*[1], *Geert Leus*[2†], and *Shengli Zhou*[1]

[1] Dept. of ECE, Univ. of Minnesota, 200 Union St. SE, Minneapolis, MN 55455, USA
[2] Dept. of EE (ESAT), K.U.Leuven, Kasteelpark Arenberg 10, 3001 Leuven, Belgium

ABSTRACT

We rely on a Basis Expansion Model (BEM) for the channel, to design three Space-Time-Doppler (STD) codecs that enable maximum diversity gains, when block transmissions undergo time-selective fading effects. Within the constraints of each maximum-diversity design, it is also possible to achieve maximum coding gain, at least when the BEM parameters are i.i.d. The theoretical results are corroborated by simulation results and the BEM is validated.

1. INTRODUCTION

Modeling temporal channel variations and coping with time-selective fading are important and challenging tasks in mobile communications. Time-selectivity arises due to oscillator drifts, phase noise, multipath propagation, and relative motion between the transmitter and the receiver. It has been proved that time-selective channels can provide Doppler diversity [3]. It has also been pointed out that the diversity order of multi-antenna systems increases when the channel fading is faster [1, 6]. However, there is no detailed calculation of the maximum diversity gain in [1, 6].

In this paper, the time-selectivity is modeled by a Basis Expansion Model (BEM) (see [3] and references therein), which allows us to quantify the maximum diversity order. Using this BEM, we then develop three Space-Time-Doppler (STD) codecs that achieve this maximum diversity gain. One is related to the phase-sweeping idea [1], and is labeled as the Digital Phase Sweeping (DPS) scheme. The other two are based on the Space-Time-Multipath (STM) schemes presented in [9] for frequency-selective channels. They are labeled as the orthogonal STD schemes, and include a Cyclic Prefix (CP) based, and a Zero Padding (ZP) based orthogonal STD scheme.

Notation: Upper (lower) bold face letters will be used for matrices (column vectors). Superscript $^{\mathcal{H}}$ will denote Hermitian, * conjugate, ^{T} transpose, and † pseudo-inverse. We will reserve $\otimes$ for the Kronecker product and E[·] for expectation. $\mathbf{I}_N$ will denote the $N \times N$ identity matrix, and $\mathbf{F}_N$ the $N \times N$ normalized FFT matrix; diag[x] will stand for a diagonal matrix with $\mathbf{x}$ on its main diagonal.

2. BLOCK TRANSMISSION MODEL

We consider a wireless link with N_t transmit-antennas, N_r receive-antennas, and time-selective fading channels. Figure 1 depicts the discrete-time equivalent baseband model under consideration.

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† Postdoctoral Fellow of the FWO-Vlaanderen, Belgium.

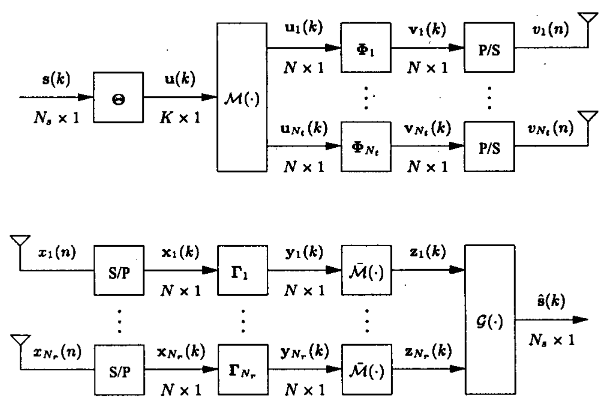

Fig. 1. Discrete-time model of transmitter and receiver.

The information bearing symbols $s(n)$ are drawn from a finite alphabet, and parsed into blocks of size $N_s \times 1$: $\mathbf{s}(k) := [s(kN_s), \ldots, s((k+1)N_s-1)]^T$. Each block $\mathbf{s}(k)$ is then linearly precoded by the $K \times N_s$ matrix $\mathbf{\Theta}$, resulting into $\mathbf{u}(k) := \mathbf{\Theta}\mathbf{s}(k)$. This operation will be termed the outer Space-Time-Doppler (STD) coding. Each block $\mathbf{u}(k)$ is further transformed into N_t blocks $\{\mathbf{u}_\mu(k)\}_{\mu=1}^{N_t}$ of size $N \times 1$ by $\mathcal{M}(\cdot)$: $\{\mathbf{u}_\mu(k)\}_{\mu=1}^{N_t} := \mathcal{M}(\mathbf{u}(k))$. This operation will be termed the middle STD coding. Each block $\mathbf{u}_\mu(k)$ is finally linearly precoded by the $N \times N$ matrix $\mathbf{\Phi}_\mu$, resulting into $\mathbf{v}_\mu(k) := \mathbf{\Phi}_\mu \mathbf{u}_\mu(k)$. This operation will be termed the inner STD coding. The sequence $v_\mu(n)$ obtained by serial-to-parallel converting $\mathbf{v}_\mu(k)$ is then transmitted from the μth transmit-antenna. The nth sample at the νth receive-antenna then is:

$$x_\nu(n) = \sum_{\mu=1}^{N_t} h_{\nu,\mu}(n) v_\mu(n) + \eta_\nu(n), \quad \nu \in [1, N_r], \qquad (1)$$

where $h_{\nu,\mu}(n)$ is the time-selective channel response from the μth transmit-antenna to the νth receive-antenna (notice the channel dependence on n), and $\eta_\nu(n)$ is additive noise at the νth receive-antenna. We model $h_{\nu,\mu}(n)$ as [3]:

$$h_{\nu,\mu}(n) := \sum_{q=0}^{Q} h_q^{(\nu,\mu)}(\lfloor n/N \rfloor) e^{j\omega_q \mathrm{mod}(n,N)},$$

where $\omega_q := 2\pi(q - Q/2)/N$, and $Q := 2\lceil f_{\max} N T_s \rceil$, with the parameter $f_{\max}$ denoting the channel's Doppler spread. Because it can be measured experimentally in practice, we assume that $f_{\max}$, and thus Q, is known and bounded.

At each receive-antenna, the received samples $x_\nu(i)$ are serial-to-parallel converted to form the $N \times 1$ blocks $\mathbf{x}_\nu(k) := [x_\nu(kN), x_\nu(kN+1), \ldots, x_\nu(kN+N-1)]^T$. The matrix-vector counterpart of (1) can then be expressed as:

$$\mathbf{x}_\nu(k) = \sum_{\mu=1}^{N_t} \mathbf{D}_H^{(\nu,\mu)}(k)\mathbf{v}_\mu(k) + \boldsymbol{\eta}_\nu(k), \quad \nu \in [1, N_r], \qquad (2)$$

where $\mathbf{D}_H^{(\nu,\mu)}(k) := \sum_{q=0}^{Q} h_q^{(\nu,\mu)}(k)\mathbf{D}_q$ is an $N \times N$ diagonal matrix, with $\mathbf{D}_q := \text{diag}[1, \exp(j\omega_q), \ldots, \exp(j\omega_q(N-1))]$, and $\boldsymbol{\eta}_\nu(k)$ is defined similar to $\mathbf{x}_\nu(k)$. Each block $\mathbf{x}_\nu(k)$ is then linearly processed by the $N \times N$ matrix $\boldsymbol{\Gamma}_\nu$, resulting into $\mathbf{y}_\nu(k) := \boldsymbol{\Gamma}_\nu \mathbf{x}_\nu(k)$. This operation is termed the inner STD decoding. Each block $\mathbf{y}_\nu(k)$ is further transformed into the block $\mathbf{z}_\nu(k)$, by $\bar{\mathcal{M}}(\cdot)$: $\mathbf{z}_\nu(k) := \bar{\mathcal{M}}(\mathbf{y}_\nu(k))$. This operation is termed the middle STD decoding. The blocks $\{\mathbf{z}_\nu(k)\}_{\nu=1}^{N_r}$ are finally decoded by $\mathcal{G}(\cdot)$ to obtain an estimate of $\mathbf{s}(k)$: $\hat{\mathbf{s}}(k) := \mathcal{G}(\{\mathbf{z}_\nu(k)\}_{\nu=1}^{N_r})$. This operation is termed the outer STD decoding.

From [3], we know that the maximum diversity gain that can be achieved is the rank of $\mathbf{R}_h(k) = \text{E}[\mathbf{h}(k)\mathbf{h}^{\mathcal{H}}(k)]$, where $\mathbf{h}(k) = [\mathbf{h}_{1,1}^T(k), \ldots, \mathbf{h}_{1,N_t}^T(k), \mathbf{h}_{2,1}^T(k), \ldots, \mathbf{h}_{N_r,N_t}^T(k)]^T$, with $\mathbf{h}_{\nu,\mu}(k) = [h_0^{(\nu,\mu)}(k), \ldots, h_Q^{(\nu,\mu)}(k)]^T$. We will denote this rank by r_h. We have shown[1] that the maximum possible r_h equals the dimensionality of $\mathbf{R}_h(k)$, namely $r_h^{max} = N_t N_r(Q+1)$. This can be reached when e.g., the BEM parameters $h_q^{(\nu,\mu)}(k)$ are i.i.d. In this paper, we show how to design the inner, middle, and outer STD code in order to achieve this maximum diversity gain. Since, in the following, we will only work on a block-by-block basis, we will drop the block index k. Finally, we only consider $N_r = 1$, and thus drop the receiver index ν. It is straightforward to extend the proposed methods for $N_r > 1$.

3. DIGITAL PHASE SWEEPING (DPS)

The first design that we study can be viewed as the dual of delay-diversity [5], which was originally developed for converting ST frequency-flat channels into a single frequency-selective channel, but can of course be extended for converting ST frequency-selective channels into a single longer frequency-selective channel. Digital Phase Sweeping (DPS) will convert ST time-selective channels into a single faster time-selective channel. For the DPS method, the middle STD codec, which is determined by $\mathcal{M}(\cdot)$ and $\bar{\mathcal{M}}(\cdot)$, is not in use, i.e., $\mathbf{u}_\mu = \mathbf{u}, \forall \mu \in [1, N_t]$ and $\mathbf{z} = \mathbf{y}$. Hence, we know that $K = N$. Using (2), $\mathbf{z}$ and $\mathbf{u}$ can then be related via:

$$\begin{aligned} \mathbf{z} &= \boldsymbol{\Gamma} \sum_{\mu=1}^{N_t} \mathbf{D}_H^{(\mu)} \boldsymbol{\Phi}_\mu \mathbf{u} + \boldsymbol{\Gamma}\boldsymbol{\eta} \\ &= \boldsymbol{\Gamma} \sum_{\mu=1}^{N_t} \sum_{q=0}^{Q} h_q^{(\mu)} \mathbf{D}_q \boldsymbol{\Phi}_\mu \mathbf{u} + \boldsymbol{\zeta}. \end{aligned} \qquad (3)$$

The matrices $\{\boldsymbol{\Phi}_\mu\}_{\mu=1}^{N_t}$ and $\boldsymbol{\Gamma}$, which determine the inner STD codec, are then designed as $\boldsymbol{\Phi}_\mu = \text{diag}[1, e^{j\phi_\mu}, \ldots, e^{j\phi_\mu(N-1)}]$ and $\boldsymbol{\Gamma} = \mathbf{I}_N$, where $\phi_\mu = 2\pi(\mu-1)(Q+1)/N$. Considering this design, (3) can be rewritten as

$$\mathbf{z} = \mathbf{D}_H \mathbf{u} + \boldsymbol{\zeta} = \sum_{q=0}^{N_t(Q+1)-1} h_q \mathbf{D}_q \mathbf{u} + \boldsymbol{\zeta}, \qquad (4)$$

where $h_q := h_{\text{mod}(q,Q+1)}^{(\lfloor q/(Q+1) \rfloor + 1)}$. Hence, the relationship between $\hat{\mathbf{s}}$ and $\mathbf{s}$ becomes $\hat{\mathbf{s}} = \mathcal{G}(\mathbf{D}_H \boldsymbol{\Theta}\mathbf{s} + \boldsymbol{\zeta})$. Comparing (4) with the model of [3], we observe that this DPS design transforms the N_t transmit-antenna scenario, where each channel can be expressed by $Q+1$ exponential bases, into a single transmit-antenna scenario, where the equivalent channel can be expressed by $N_t(Q+1)$ exponential bases. Hence, as outer STD codec, which is determined by $\boldsymbol{\Theta}$ and $\mathcal{G}(\cdot)$, we can now apply any SISO method for time-selective channels that achieves the maximum diversity gain. From Proposition 2 in [3], we know that ML decoding by means of $\mathcal{G}(\cdot)$ achieves r_h if the linear precoder $\boldsymbol{\Theta}$ is designed in such a way that $\boldsymbol{\Theta}\mathbf{e}$ has at least $N_t(Q+1)$ non-zero entries, for all possible error vectors $\mathbf{e} = \mathbf{s} - \mathbf{s}' \neq \mathbf{0}$.

One possibility is based on the Grouped Linear Constellation Precoding (GLCP) OFDM method proposed in [2] for frequency-selective channels. Using this method, we design a square $N \times N$ matrix $\boldsymbol{\Theta}$ that can be written as $\boldsymbol{\Theta} = \mathbf{I}_{N/N_{sub}} \otimes \boldsymbol{\Theta}_{sub}$, where N_{sub} is the group size chosen to satisfy $N_{sub} \geq N_t(Q+1)$, and $\boldsymbol{\Theta}_{sub}$ is a square $N_{sub} \times N_{sub}$ matrix chosen according to [2]. Note that we then have $N_s = N$. ML decoding by means of $\mathcal{G}(\cdot)$ can then be implemented by Sphere Decoding (SD) [7] on sub-blocks of size $N_{sub} \geq N_t(Q+1)$. We have shown that for i.i.d. BEM parameters $h_q^{(\mu)}$, the GLCP-based method achieves the maximum coding gain within the class of all square $N \times N$ LCP matrices $\boldsymbol{\Theta}$.

Another possibility is based on the (constellation-irrespective) Linearly Precoded (LP) OFDM method proposed in [8] for frequency-selective channels. Using this method, we design a tall $N \times [N - N_t(Q+1) + 1]$ matrix $\boldsymbol{\Theta}$ that can be written as $\boldsymbol{\Theta} = \mathbf{F}_N \mathbf{T}$, where $\mathbf{T} := [\mathbf{0}_{N_s \times (N_t(Q+1)-Q/2-1)}, \mathbf{I}_{N_s}, \mathbf{0}_{N_s \times Q/2}]^T$. Note that we then have $N_s = N - N_t(Q+1) + 1$. ML decoding by means of $\mathcal{G}(\cdot)$ can be implemented by SD on a block of size N_s. Since N_s is usually much bigger than $N_t(Q+1)$, this SD is more complex than the GLCP-based method described earlier. However, it is easy to show that $\mathbf{D}_H$ can be rewritten as

$$\mathbf{D}_H = \sum_{q=0}^{N_t(Q+1)-1} h_q \mathbf{D}_q = \mathbf{F}_N \mathbf{H} \mathbf{F}_N^{\mathcal{H}}, \qquad (5)$$

where $\mathbf{H}$ is a circulant $N \times N$ matrix with first column $[h_{Q/2}, \ldots, h_0, 0, \ldots, 0, h_{N_t(Q+1)-1}, \ldots, h_{Q/2+1}]^T$. Note that (5) shows that the BEM allows us to view a time-selective channel as a frequency-selective channel having the BEM parameters as taps. Hence, rewriting $\mathcal{G}(\cdot)$ as $\mathcal{G}(\mathbf{z}) = \mathcal{F}(\mathbf{F}_N^{\mathcal{H}}\mathbf{z})$, which can be done without loss of optimality, we obtain

$$\hat{\mathbf{s}} = \mathcal{F}(\mathbf{F}_N^{\mathcal{H}}\mathbf{z}) = \mathcal{F}(\mathbf{HTs} + \mathbf{F}_N^{\mathcal{H}}\boldsymbol{\zeta}).$$

It turns out that, due to the lower triangular Toeplitz structure of $\mathbf{HT}$, ML decoding by means of $\mathcal{F}(\cdot)$ can be implemented by the Viterbi Algorithm (VA) with a complexity that is exponential in $N_t(Q+1)$. Depending on N_t and Q, the complexity of the VA can be higher or lower than the one of the GLCP-based method described earlier. We have shown that for i.i.d. BEM parameters $h_q^{(\mu)}$, the LP-based method (using SD or VA) achieves the maximum coding gain within the class of all tall $N \times [N - N_t(Q+1)]$ LP matrices $\boldsymbol{\Theta}$.

Comparing both possibilities, the GLCP-based method is non-redundant, since $N_s = N$, whereas the LP-based method is redundant, since $N_s = N - N_t(Q+1) + 1$. However, because the LP-based design uses a tall precoder $\boldsymbol{\Theta}$, whereas the GLCP design uses a square precoder $\boldsymbol{\Theta}$, it is intuitively clear that the LP-based design will reach a higher coding gain than the GUP-based design. Moreover, when $N_t(Q+1)$ is very large, linear Zero-Forcing (ZF) decoding is ML for the LP-based design, whereas it is not ML for the GLCP-based design.

The matrices $\{\boldsymbol{\Phi}_\mu\}_{\mu=1}^{N_t}$ in (3) cause digital phase sweeping of our block transmissions, reminiscent of that used in [1], to in-

[1] Due to lack of space, proofs can only be found in the journal version of this paper.

crease the variation (and thus the potential for diversity) of time-selective channels at the price of reducing the number of channels (see also [4]). The differences between our design and [1] are: i) using GLCP or LP, we collect not only transmit-diversity, but also Doppler diversity; ii) we generalize to multiple transmit-antennas; iii) DPS can be used not only for coded, but also for uncoded systems.

4. ORTHOGONAL STD CODES

In this section, we follow a different approach. Here, the inner STD codec is used to transform the time-selective channels into frequency-selective channels by means of FFT and IFFT operations. As middle and outer STD codec, we can then use any of the existing orthogonal Space-Time-Multipath (STM) codecs to achieve the maximum diversity gain. For simplicity, we will only focus on $N_t = 2$ in this section.

The relationship between $\mathbf{x}$ and $\{\mathbf{u}_\mu\}_{\mu=1}^2$ can be written as:

$$\mathbf{x} = \sum_{\mu=1}^2 \mathbf{D}_H^{(\mu)} \mathbf{\Phi}_\mu \mathbf{u}_\mu + \boldsymbol{\eta} = \sum_{\mu=1}^2 \sum_{q=0}^Q h_q^{(\mu)} \mathbf{D}_q \mathbf{\Phi}_\mu \mathbf{u}_\mu + \boldsymbol{\eta}. \quad (6)$$

Similar to (5), $\mathbf{D}_H^{(\mu)}$ can be rewritten as

$$\mathbf{D}_H^{(\mu)} = \sum_{q=0}^Q h_q^\mu \mathbf{D}_q = \mathbf{F}_N \mathbf{H}_\mu \mathbf{F}_N^{\mathcal{H}}, \quad (7)$$

where $\mathbf{H}_\mu$ is a circulant $N \times N$ matrix with first column $[h_{Q/2}^{(\mu)}, \ldots, h_0^{(\mu)}, 0, \ldots, 0, h_Q^{(\mu)}, \ldots, h_{Q/2+1}^{(\mu)}]^T$. If we now design the inner STD coder $\mathbf{\Phi}_\mu$ as $\mathbf{\Phi}_\mu = \mathbf{F}_N, \forall \mu \in [1, 2]$, and the inner STD decoder $\mathbf{\Gamma}$ as $\mathbf{\Gamma} = \mathbf{F}_N^{\mathcal{H}}$, we obtain from (6) and (7):

$$\mathbf{y} = \sum_{\mu=1}^2 \mathbf{F}_N^{\mathcal{H}} \mathbf{D}_H^{(\mu)} \mathbf{F}_N \mathbf{u}_\mu + \mathbf{F}^{\mathcal{H}} \boldsymbol{\eta} = \sum_{\mu=1}^2 \mathbf{H}_\mu \mathbf{u}_\mu + \boldsymbol{\xi}. \quad (8)$$

Hence, the inner STD codec has transformed the ST time-selective channels into ST frequency-selective channels. In order to achieve the maximum diversity gain r_h, we can use any of the existing Space-Time-Multipath (STM) codecs as middle and outer STD codec. We discuss a Cyclic Prefix (CP) based approach, and a Zero Padding (ZP) based approach. For both approaches we define $\mathbf{u}_a$ ($\mathbf{u}_b$) as the first (last) $K/2$ entries of $\mathbf{u}$, and $\mathbf{y}_a$ ($\mathbf{y}_b$) as the first (last) $N/2$ entries of $\mathbf{y}$.

4.1. CP-Based Approach

The CP-based approach uses the STM codec presented in [9] as middle and outer STD codec. We design the middle STD coder $\mathcal{M}(\cdot)$ as:

$$\mathbf{u}_1 = (\mathbf{I}_2 \otimes \mathbf{T}_{cp})(\mathbf{I}_2 \otimes \mathbf{F}_{K/2}^{\mathcal{H}}) \begin{bmatrix} \mathbf{u}_a \\ -\mathbf{u}_b^* \end{bmatrix}, \quad (9)$$

$$\mathbf{u}_2 = (\mathbf{I}_2 \otimes \mathbf{T}_{cp})(\mathbf{I}_2 \otimes \mathbf{F}_{K/2}^{\mathcal{H}}) \begin{bmatrix} \mathbf{u}_b \\ \mathbf{u}_a^* \end{bmatrix}, \quad (10)$$

where $\mathbf{T}_{cp} := [\mathbf{T}_1, \mathbf{I}_{K/2}, \mathbf{T}_2]^T$, with

$$\mathbf{T}_1 := [\mathbf{I}_{Q/2}, \mathbf{0}_{Q/2 \times (K-Q)/2}]^T, \quad \mathbf{T}_2 := [\mathbf{0}_{Q/2 \times (K-Q)/2}, \mathbf{I}_{Q/2}]^T.$$

The middle STD decoder $\bar{\mathcal{M}}(\cdot)$ is designed as:

$$\mathbf{z} = (\mathbf{I}_2 \otimes \mathbf{F}_{K/2})(\mathbf{I}_2 \otimes \mathbf{R}_{cp}) \begin{bmatrix} \mathbf{y}_a \\ \mathbf{y}_b^* \end{bmatrix}, \quad (11)$$

where $\mathbf{R}_{cp} := [\mathbf{0}_{K/2 \times Q/2}, \mathbf{I}_{K/2}, \mathbf{0}_{K/2 \times Q/2}]$. Note that $\mathcal{M}(\cdot)$ increases the block size with $N_t Q = 2Q$, i.e., $N = K + 2Q$.

Plugging (9) and (10) into (8), we can rewrite (11) as

$$\mathbf{z} = \begin{bmatrix} \mathbf{F}_{K/2} \bar{\mathbf{H}}_1 \mathbf{F}_{K/2}^{\mathcal{H}} & \mathbf{F}_{K/2} \bar{\mathbf{H}}_2 \mathbf{F}_{K/2}^{\mathcal{H}} \\ \mathbf{F}_{K/2}^* \bar{\mathbf{H}}_2^* \mathbf{F}_{K/2}^T & -\mathbf{F}_{K/2}^* \bar{\mathbf{H}}_1^* \mathbf{F}_{K/2}^T \end{bmatrix} \begin{bmatrix} \mathbf{u}_a \\ \mathbf{u}_b \end{bmatrix} + \boldsymbol{\zeta} = \begin{bmatrix} \bar{\mathbf{D}}_H^{(1)} & \bar{\mathbf{D}}_H^{(2)} \\ \bar{\mathbf{D}}_H^{(2)*} & -\bar{\mathbf{D}}_H^{(1)*} \end{bmatrix} \begin{bmatrix} \mathbf{u}_a \\ \mathbf{u}_b \end{bmatrix} + \boldsymbol{\zeta} := \bar{\mathbf{D}}_H \mathbf{u} + \boldsymbol{\zeta}, \quad (12)$$

where $\bar{\mathbf{H}}_\mu$ is a $K/2 \times K/2$ circulant matrix with first column $[h_{Q/2}^{(\mu)}, \ldots, h_0^{(\mu)}, 0, \ldots, 0, h_Q^{(\mu)}, \ldots, h_{Q/2+1}^{(\mu)}]^T$, and $\bar{\mathbf{D}}_H^{(\mu)}$ is a $K/2 \times K/2$ diagonal matrix that can be written as $\bar{\mathbf{D}}_H^{(\mu)} = \mathbf{F}_{K/2} \bar{\mathbf{H}}_\mu \mathbf{F}_{K/2}^{\mathcal{H}}$. The outer STD decoder $\mathcal{G}(\cdot)$ will now multiply $\mathbf{z}$ with a specific unitary matrix, wich results into two decoupled problems: one in $\mathbf{u}_a$, and one in $\mathbf{u}_b$. This operation preserves the ML optimality. Designing the middle STD coder $\mathbf{\Theta}$ as $\mathbf{\Theta} = \mathbf{I}_2 \otimes \bar{\mathbf{\Theta}}$, we can then again make use of the GLCP-based approach or the LP-based approach in order to achieve the maximum diversity gain. The GLCP-based approach results into $N_s = K = N - 2Q$, whereas the LP-based approach results into $N_s = K - 2Q = N - 4Q$. For the GLCP-based approach the group size N_{sub} now has to satisfy $N_{sub} \geq Q + 1$. Hence, ML decoding can be implemented by SD on sub-blocks of size $N_{sub} \geq Q+1$, instead of on sub-blocks of size $N_{sub} \geq N_t(Q+1)$ as for the DPS scheme. For the LP-based approach, ML decoding has to be implemented by SD on a block of size $N_s/2$. Trading off performance with complexity non-linear (block DFE or BLAST), or linear (ZF or MMSE) alternatives are possible.

The LP-based method introduces redundancy not only by means of $\mathbf{T}_{cp}$, but also by means of the tall precoder $\bar{\mathbf{\Theta}}$. However, when considering a ZP instead of a CP, we can reduce this redundancy, without loosing performance. This is discussed in the next section.

4.2. ZP-Based Approach

The ZP-based approach uses the STM codec presented in [9] as middle and outer STD codec. We design the middle STD coder $\mathcal{M}(\cdot)$ as:

$$\mathbf{u}_1 = (\mathbf{I}_2 \otimes \mathbf{T}_{zp}) \begin{bmatrix} \mathbf{u}_a \\ -\mathbf{P}_{K/2} \mathbf{u}_b^* \end{bmatrix}, \quad (13)$$

$$\mathbf{u}_2 = (\mathbf{I}_2 \otimes \mathbf{T}_{zp}) \begin{bmatrix} \mathbf{u}_b \\ \mathbf{P}_{K/2} \mathbf{u}_a^* \end{bmatrix}, \quad (14)$$

where $\mathbf{T}_{zp} := [\mathbf{0}_{K/2 \times Q/2} \mathbf{I}_{K/2} \mathbf{0}_{K/2 \times (Q/2+1)}]^T$, and $\mathbf{P}_{K/2}$ is the $K/2 \times K/2$ matrix that time-reverses the entries of each block. The middle STD decoder $\bar{\mathcal{M}}(\cdot)$ is designed as:

$$\mathbf{z} = \begin{bmatrix} \mathbf{y}_a \\ \mathbf{P}_{N/2} \mathbf{y}_b^* \end{bmatrix}. \quad (15)$$

Plugging (13) and (14) into (8), we can rewrite (15) as

$$\mathbf{z} = \begin{bmatrix} \tilde{\mathbf{H}}_1 & \tilde{\mathbf{H}}_2 \\ \tilde{\mathbf{H}}_2^{\mathcal{H}} & -\tilde{\mathbf{H}}_1^{\mathcal{H}} \end{bmatrix} \begin{bmatrix} \mathbf{T}_{zp} \mathbf{u}_a \\ \mathbf{T}_{zp} \mathbf{u}_b \end{bmatrix} + \boldsymbol{\xi} := \tilde{\mathbf{H}}(\mathbf{I}_2 \otimes \mathbf{T}_{zp}) \mathbf{u} + \boldsymbol{\xi}, \quad (16)$$

where $\tilde{\mathbf{H}}_\mu$ is an $N/2 \times N/2$ circulant matrix with first column $[h_{Q/2}^{(\mu)}, \ldots, h_0^{(\mu)}, 0, \ldots, 0, h_Q^{(\mu)}, \ldots, h_{Q/2+1}^{(\mu)}]^T$. Note that $\mathcal{M}(\cdot)$ increases the block size with $N_t Q = 2Q$, i.e., $N = K + 2Q$. The outer STD decoder $\mathcal{G}(\cdot)$ will now multiply $\mathbf{z}$ with a specific unitary matrix, which results into two decoupled problems: one in $\mathbf{u}_a$ and one in $\mathbf{u}_b$. This operation again preserves the ML optimality. Designing the middle STD coder $\mathbf{\Theta}$ as $\mathbf{\Theta} = \mathbf{I}_{N_s}$ (i.e., $N_s = K$), we can use SD on blocks of size $N/2$ to implement ML decoding. As before, non-linear or linear alternatives are possible.

5. SIMULATED PERFORMANCE

We present simulations to test how well the BEM fits realistic time-varying channels, and confirm the performance of our maximum diversity schemes. QPSK modulation is adopted for both test cases.

Test Case 1: We compare DPS, CP-Based, and ZP-Based schemes with $(N_t, N_r) = (2, 1)$ antennas, $Q + 1 = 3$ bases per channel, and BEM parameters that are i.i.d., Gaussian, with mean zero, and variance $1/(Q + 1)$. We simulate DPS employing the LP-method with $N = K = 25$, and $N_s = 20$. Furthermore, we simulate the CP-based scheme employing the GLCP-method, and the ZP-based scheme. To keep the same transmission rate, we choose block sizes $N = 20$, and $K = N_s = 16$ for these schemes. Figure 2 depicts the bit-error rate (BER) performance of these three schemes. SD has been employed for all schemes. We observe that: i) from the slope of the BER curves, all three schemes guarantee maximum diversity order $r_h^{max} = N_t(Q + 1) = 6$; ii) CP-based and ZP-based schemes have quite similar performance, while DPS has worse performance since it has no orthogonal ST coding.

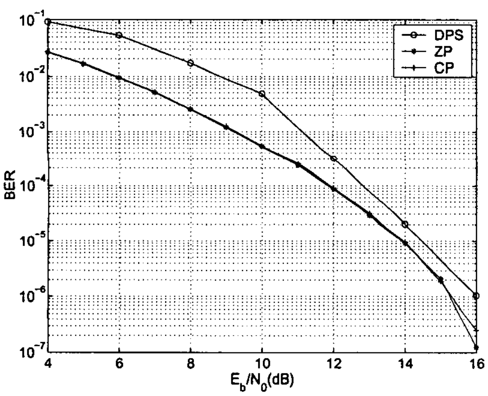

Fig. 2. Performance comparison of the three STD coding schemes.

Test Case 2: We now validate the BEM using Jakes' model. We consider a carrier frequency $f_0 = 900$ MHz, and a mobile speed $v_{\max} = 96$ km/hr. The transmitted block length is $N = 25$. Since we want to test the diversity for different Q's, we select the symbol period $T_s = Q/(2f_{\max}N)$. We generate the channels by Jakes' model. At the transmitter, we adopt our DPS design without any precoder. At the receiver, we consider MMSE equalization for two different scenarios: one uses the parameters generated by Jakes' model (corresponding curves are marked by "Jakes" in Fig. 3); and the other one uses the parameters of the BEM that approximates Jakes' model (see [3] for details). Figure 3 depicts the performance results. We observe that: i) our DPS design guarantees spatial diversity gains even for systems adhering to Jake's model; ii) when Q is small ($Q = 2$), model mismatch between the BEM and Jakes' model causes an error floor; iii) when Q is large, BEM matches Jakes' model well, and the BER performance improves considerably.

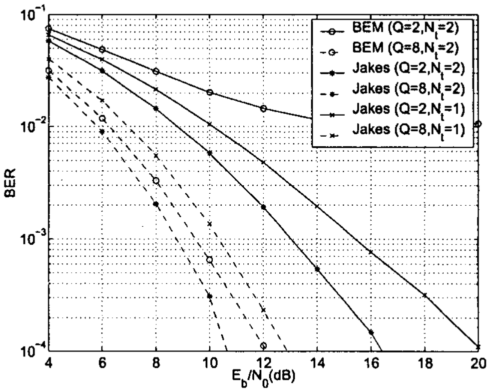

Fig. 3. Validation of the BEM based on Jakes' model

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