OPTIMAL TRANSMITTER EIGEN-BEAMFORMING AND SPACE TIME BLOCK CODING BASED ON CHANNEL MEAN

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ABSTRACT

Optimal transmitter designs obeying the water-filling principle are well-documented, and widely applied when the propagation channel is deterministically known, and regularly updated at the transmitter. Because channel state information is impossible to be known perfectly at the transmitter in practical wireless systems, we develop in this paper optimal transmitter design based on the knowledge of mean values of the underlying channels. Applied to a multiple transmit-antenna paradigm, our optimal transmitter design turns out to be an eigen-beamformer with multiple beams pointing to orthogonal directions along the eigenvectors of the channel correlation matrix conditioned on channel mean, and with proper power loading across beams. The optimality pertains to minimizing a tight bound on the symbol error rate. Coupled with orthogonal space time block codes, two-directional eigen-beamforming emerges as a more attractive choice than conventional one-directional beamforming with uniformly improved performance, without rate reduction.

1. INTRODUCTION

Multi-antenna diversity is well motivated for wireless communications through fading channels. Although receive-antenna diversity has been widely applied in practice, in certain cases, e.g., cellular downlink, multiple receive antennas may be expensive or impractical to deploy, which endeavors transmit-diversity systems.

Multi-antenna systems can further enhance performance and capacity, when perfect or partial channel state information (CSI) is made available at the transmitter [4, 8]. One particular form of partial CSI is the noisy channel estimates. For example, the receiver can feed back to the transmitter the unquantized (or quantized) estimates of slowly time-varying wireless channels. Based on noisy channel estimates, the transmitter obtains the knowledge of the channel mean, while models the uncertainty about the channel around its mean value as an additive white Gaussian noise.

In this paper, we design optimal transmit-diversity precoders for widely used constellations based on channel mean. Our performance-oriented designs rely on the Chernoff bound on symbol error rate (SER). The optimal precoder turns out to be a generalized beamformer with multiple beams pointing to orthogonal directions along the eigenvectors of the channel correlation matrix conditioned on the channel mean; hence the name, optimal transmitter eigen-beamforming. The optimal eigen-beams are power loaded according to a spatial water-filling principle.

To increase the data rate, we also develop parallel transmissions equipped with orthogonal space time block coding (STBC) [1, 2, 7] across optimally loaded eigen-beams. Wedding optimal precoding with orthogonal STBC leads to a two-directional eigen-beamforming which turns out to enjoy uniformly better performance than the conventional one-directional beamforming, without rate reduction. The combination of orthogonal STBC with beamforming has also been studied in [3]. The major differences between our approach and [3] are as follows: i) [3] starts from a predetermined orthogonal space-time code and try to improve the performance by a linear transformation (beamforming). We start from general precoders, and the optimal transmitter leads itself naturally to the application of orthogonal STBC on optimally loaded eigen-beams; ii) with mean feedback, [3] only proposed a semi analytical solution, that relies on numerical search. We here provide simple closed-form solutions, which are appealing for online implementations. Notice also that in this paper, we only focus on single receive antenna (the extension to multiple receive antennas is provided in [9]), while [3] deals with multiple receive antennas.

Notation: Bold upper (lower) letters denote matrices (column vectors); $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^{\mathcal{H}}$ denote conjugate, transpose, and Hermitian transpose, respectively; $|\cdot|$ stands for the absolute value of a scalar, and $\|\cdot\|$ for the Euclidean norm of a vector; $\mathrm{E}\{\cdot\}$ stands for expectation, $\mathrm{tr}\{\cdot\}$ for the trace of a matrix; $\mathrm{Re}\{\cdot\}$ stands for the real part of a complex number; $\mathbf{I}_K$ denotes the identity matrix of size K; $\mathbf{0}_{K\times P}$ denotes an all-zero matrix with size $K\times P$; $\mathrm{diag}(\mathbf{x})$ stands for a diagonal matrix with $\mathbf{x}$ on its diagonal; $[\cdot]_p$ denotes the pth entry of a vector. The special notation $\mathbf{h} \sim \mathcal{CN}(\bar{\mathbf{h}}, \mathbf{\Sigma}_h)$ indicates that $\mathbf{h}$ is complex Gaussian distributed with mean $\bar{\mathbf{h}}$ and covariance $\mathbf{\Sigma}_h$.

2. SYSTEM MODEL

Fig. 1 depicts the block diagram of a transmit diversity system with a single receive- and N_t transmit- antennas. In the μth ($\mu \in [1, N_t]$) transmit-antenna, each information-bearing symbol $s(i)$ is first spread by the code $\mathbf{c}_\mu := [c_\mu(0), \ldots, c_\mu(P-1)]^T$ of length P to obtain the chip sequence: $u_\mu(n) = \sum_{i=-\infty}^{\infty} s(i)c_\mu(n-iP)$. The transmission channels are flat faded (frequency non-selective) with complex fading coefficients $\{h_\mu\}_{\mu=1}^{N_t}$. The received samples in the presence of additive white Gaussian noise $w(n)$ are thus given by:

$$x(n) = \sum_{i=-\infty}^{\infty} \sum_{\mu=1}^{N_t} h_\mu c_\mu(n-iP)s(i) + w(n). \quad (1)$$

To cast (1) into a convenient matrix-vector form, we define the $P \times 1$ vectors $\mathbf{x}(i) := [x(iP+0), \ldots, x(iP+P-1)]^T$, and $\mathbf{w}(i) := [w(iP+0), \ldots, w(iP+P-1)]^T$; the $N_t \times 1$ channel vector $\mathbf{h} := [h_1, \ldots, h_{N_t}]^T$, and the $P \times N_t$ spreading code matrix $\mathbf{C} := [\mathbf{c}_1, \ldots, \mathbf{c}_{N_t}]$. The block version of (1) can be re-written as: $\mathbf{x}(i) = \mathbf{C}\mathbf{h}s(i) + \mathbf{w}(i)$. Because we will focus on symbol by symbol detection, we omit the symbol index i, and subsequently deal with the input-output model

$$\mathbf{x} = \mathbf{C}\mathbf{h}s + \mathbf{w}. \quad (2)$$

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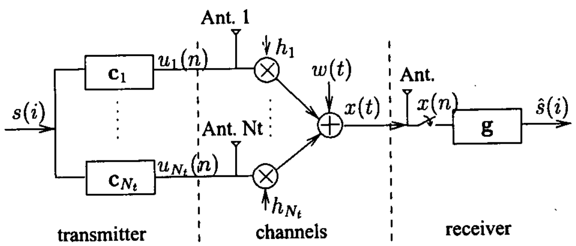

Fig. 1. Discrete-time baseband equivalent model

At the receiver, we first acquire the channel $\mathbf{h}$ to enable maximum ratio combining (MRC) using

$$\mathbf{g}_{opt}^{\mathcal{H}} := [g(0), \ldots, g(P-1)] = (\mathbf{Ch})^{\mathcal{H}}. \quad (3)$$

The MRC receiver is known to maximize the signal to noise ratio (SNR) at its output, that yields the symbol estimate $\hat{s} = \mathbf{g}_{opt}^{\mathcal{H}}\mathbf{x} = \mathbf{h}^{\mathcal{H}}\mathbf{C}^{\mathcal{H}}\mathbf{x}$.

For a given precoder $\mathbf{C}$, equation (3) specifies the optimal receiver $\mathbf{g}$ in the sense of maximizing the output SNR. The question that arises is how to select an optimal precoder $\mathbf{C}$ if perfect or imperfect channel state information is available at the transmitter. In this paper, we look for optimal $\mathbf{C}$ based on the channel mean.

3. OPTIMAL EIGEN-BEAMFORMING

Throughout this paper, we will adopt the following assumptions:
a0) the noise $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I}_P)$.
a1) based on partial CSI $f(\mathbf{h})$, the transmitter infers that the conditional channel $\check{\mathbf{h}} := \mathbf{h}|f(\mathbf{h})$ has distribution $\mathcal{CN}(\bar{\mathbf{h}}, \sigma_\epsilon^2\mathbf{I}_{N_t})$.

Assumption a1) corresponds to the mean feedback in [8]. Depending on partial CSI, the desired parameters $(\bar{\mathbf{h}}, \sigma_\epsilon^2)$ can be calculated differently [4,8]. We highlight the following simple example, and will use it in simulations.

Example 1 (delayed feedback) [4,8]: We assume that the transmit antennas are well separated and the true channel coefficients (Gaussian random variables with zero mean and variance σ_h^2) can be feedback to the transmitter after delay $D(PT_c)$, i.e., $f(\mathbf{h}(i)) = \mathbf{h}(i-D)$. The channel feedback quality is determined by the correlation coefficient ρ from $\mathrm{E}\{\mathbf{h}(i)\mathbf{h}^{\mathcal{H}}(i-D)\} = \rho\sigma_h^2\mathbf{I}_{N_t}$. It is shown in [4] that:

$$\bar{\mathbf{h}} = \rho f(\mathbf{h}), \qquad \sigma_\epsilon^2 = \sigma_h^2(1-|\rho|^2). \quad (4)$$

Our goal in this paper is to design optimal precoder $\mathbf{C}$ based on a0) and a1). In the following, we will first derive a closed-form SER expression, that will facilitate our optimal precoder design.

Under a1), we can express $\check{\mathbf{h}} = \bar{\mathbf{h}} + \Delta\mathbf{h}$, where $\Delta\mathbf{h} \sim \mathcal{CN}(\mathbf{0}, \sigma_\epsilon^2\mathbf{I}_{N_t})$. For each realization of $\Delta\mathbf{h}$, the signal to noise ratio at the MRC receiver output is

$$\begin{aligned}\gamma &= \mathrm{E}\{|\check{\mathbf{h}}^{\mathcal{H}}\mathbf{C}^{\mathcal{H}}\mathbf{C}\check{\mathbf{h}}s|^2\}/\mathrm{E}\{\check{\mathbf{h}}^{\mathcal{H}}\mathbf{C}^{\mathcal{H}}\mathbf{w}\mathbf{w}^{\mathcal{H}}\mathbf{C}\check{\mathbf{h}}\} \\ &= \check{\mathbf{h}}^{\mathcal{H}}\mathbf{C}^{\mathcal{H}}\mathbf{C}\check{\mathbf{h}}\, E_s/N_0,\end{aligned} \quad (5)$$

where $E_s := \mathrm{E}\{|s|^2\}$ is the average energy of the underlying signal constellation. Since the SER depends on the SNR differently for different constellations, we will consider two widely used constellations: M-ary phase shift keying (M-PSK), and square M-ary quadrature amplitude modulation (M-QAM).

To simplify (5), we first pursue the following eigen-decomposition:

$$\mathbf{C}^{\mathcal{H}}\mathbf{C} = \mathbf{U}_c\mathbf{D}_c\mathbf{U}_c^{\mathcal{H}}, \quad \mathbf{D}_c := \mathrm{diag}(\delta_1, \ldots, \delta_{N_t}), \quad (6)$$

where $\mathbf{U}_c$ is unitary, and δ_μ denotes the μth eigenvalue of $\mathbf{C}^{\mathcal{H}}\mathbf{C}$, that is non-negative: $\delta_\mu \geq 0$. Without loss of generality, we can arrange δ_μ in an non-increasing order: $\delta_1 \geq \cdots \geq \delta_{N_t}$, by re-ordering the eigenvectors in $\mathbf{U}_c$.

Define $\tilde{\mathbf{h}} := \mathbf{U}_c^{\mathcal{H}}\check{\mathbf{h}} = \mathbf{U}_c^{\mathcal{H}}\bar{\mathbf{h}} + \Delta\tilde{\mathbf{h}}$, where $\Delta\tilde{\mathbf{h}} := \mathbf{U}_c^{\mathcal{H}}\Delta\mathbf{h}$ has the same variance as $\Delta\mathbf{h}$. Denote the μth entry of $\tilde{\mathbf{h}}$ as $\tilde{h}_\mu$. The SNR of (5) then reduces to

$$\gamma = \tilde{\mathbf{h}}^{\mathcal{H}}\mathbf{D}_c\tilde{\mathbf{h}}E_s/N_0 = \sum_{\mu=1}^{N_t}\gamma_\mu, \quad \gamma_\mu := \delta_\mu|\tilde{h}_\mu|^2E_s/N_0. \quad (7)$$

Notice that the SNR expression (7) coincides with that of the MRC output for N_t independent channels [5], with γ_μ denoting the μth subchannel's SNR. Since the channel coefficient on each path $|\tilde{h}_\mu|$ is Ricean distributed, the quality of each path is determined by two important factors. The first is the Ricean factor:

$$\mathcal{K}_\mu := |[\mathbf{U}_c^{\mathcal{H}}\bar{\mathbf{h}}]_\mu|^2/\sigma_\epsilon^2. \quad (8)$$

The second is the average power for each subchannel

$$\bar{\gamma}_\mu = \mathrm{E}\{\gamma_\mu\} = \delta_\mu(1+\mathcal{K}_\mu)\sigma_\epsilon^2E_s/N_0. \quad (9)$$

Averaging over the Ricean distributed $|\tilde{h}_\mu|$, closed form SER expressions are then available [5,6]:

$$P_{s,PSK} = \frac{1}{\pi}\int_0^{(M-1)\pi/M}\prod_{\mu=1}^{N_t}I_\mu(\bar{\gamma}_\mu, g_{PSK}, \theta)d\theta, \quad (10)$$

$$\begin{aligned}P_{s,QAM} = {}&\frac{b_{QAM}}{\sqrt{M}}\int_0^{\pi/4}\prod_{\mu=1}^{N_t}I_\mu(\bar{\gamma}_\mu, g_{QAM}, \theta)d\theta \\ &+ b_{QAM}\int_{\pi/4}^{\pi/2}\prod_{\mu=1}^{N_t}I_\mu(\bar{\gamma}_\mu, g_{QAM}, \theta)d\theta,\end{aligned} \quad (11)$$

where $b_{QAM} := 4(1-1/\sqrt{M})/\pi$, and $I_\mu(x, g, \theta)$ is the moment generating function of the probability density function (p.d.f) of $|\tilde{h}_\mu|$ evaluated at $-gx/\sin^2\theta$ [5, eq. (24)]. The constellation-specific g is given by:

$$g_{PSK} = \sin^2\left(\frac{\pi}{M}\right), \qquad g_{QAM} = \frac{3}{2(M-1)}. \quad (12)$$

Because $|\tilde{h}_\mu|$ is Ricean distributed, the moment generating function $I_\mu(x, g, \theta)$ assumes the following form [5]:

$$I_\mu(x, g, \theta) = \frac{(1+\mathcal{K}_\mu)\sin^2\theta}{(1+\mathcal{K}_\mu)\sin^2\theta + gx}\exp\left(\frac{-\mathcal{K}_\mu\, gx}{(1+\mathcal{K}_\mu)\sin^2\theta + gx}\right).$$

Direct optimization based on the exact SER (10) and (11) turns out to be difficult because of the integration involved. Instead, we rely on a tight Chernoff bound on SER to design the optimal $\mathbf{C}$, that will enable simple closed-form precoder solutions.

Since $I_\mu(x, g, \theta)$ peaks at $\theta = \pi/2$, the Chernoff bound for the SER in (10), and (11) can be obtained in a unifying form [6]:

$$\begin{aligned}P_{s,bound} &= \alpha\prod_{\mu=1}^{N_t}I_\mu(\bar{\gamma}_\mu, g, \pi/2) \\ &= \alpha\prod_{\mu=1}^{N_t}\frac{1}{1+\delta_\mu\beta}\exp\left(\frac{-\mathcal{K}_\mu\delta_\mu\beta}{1+\delta_\mu\beta}\right),\end{aligned} \quad (13)$$

where for notational brevity we have defined

$$\alpha := (M-1)/M, \quad \beta := g\sigma_\epsilon^2 E_s/N_0, \tag{14}$$

with g taking constellation-specific values as in (12).

From (13), we next determine optimal $\mathbf{U}_c$ and $\mathbf{D}_c$. Consider the eigen decomposition of $\bar{\mathbf{h}}\bar{\mathbf{h}}^{\mathcal{H}}$ and the correlation matrix conditioned on channel mean $\mathbf{R}_{\check{h}\check{h}} := \mathrm{E}\{\check{\mathbf{h}}\check{\mathbf{h}}^{\mathcal{H}}\}$ as:

$$\bar{\mathbf{h}}\bar{\mathbf{h}}^{\mathcal{H}} = \mathbf{U}_h\mathbf{D}_h\mathbf{U}_h^{\mathcal{H}}, \quad \mathbf{R}_{\check{h}\check{h}} = \mathbf{U}_h(\mathbf{D}_h + \sigma_\epsilon^2\mathbf{I}_{N_t})\mathbf{U}_h^{\mathcal{H}}. \tag{15}$$

where $\mathbf{D}_h := \mathrm{diag}(\lambda, 0, \ldots, 0)$ with $\lambda := \|\bar{\mathbf{h}}\|^2$, and $\mathbf{U}_h$ is unitary. We prove in [9] the following result for optimal $\mathbf{U}_c$.

Corollary 1: *Under a0) and a1), the optimal* $\mathbf{U}_c$ *is* $\mathbf{U}_c = \mathbf{U}_h$.

We next proceed to optimize $\mathbf{D}_c$ with the optimal $\mathbf{U}_c$. We impose the following constraint: $\mathrm{tr}\{\mathbf{C}^{\mathcal{H}}\mathbf{C}\} = \mathrm{tr}\{\mathbf{D}_c\} = 1$, so that the average transmitted power is E_s per symbol. Adopting the special notation $[x]_+ := \max(x, 0)$, and defining

$$\begin{aligned} a &:= (1 + N_t/\beta)^2, \quad c := N_t(N_t - 1), \\ b &:= [\lambda/(\beta\sigma_\epsilon^2) + (1 + N_t/\beta)(2N_t - 1)], \end{aligned} \tag{16}$$

the optimal solution that minimizes $P_{s,bound}$ under the power constraint is provided in [9] as:

$$\begin{aligned} \delta_2 &= \ldots = \delta_{N_t} = \left[\frac{2a}{b + \sqrt{b^2 - 4ac}} - \frac{1}{\beta}\right]_+, \\ \delta_1 &= 1 - (N_t - 1)\delta_2. \end{aligned} \tag{17}$$

Although (17) provides the optimal solution that can be efficiently solved, we next derive a suboptimum but simpler solution. The importance of this alternative approach lies in the fact that it allows us to extend our results to multiple receive antennas [9].

It is well known that a Ricean distribution with Ricean factor $\mathcal{K}_\mu$ can be well approximated by a Nakagami distribution with parameter m_μ if $m_\mu = (1 + \mathcal{K}_\mu)^2/(1 + 2\mathcal{K}_\mu)$ [6, p. 23]. The moment generating function for the Nakagami distribution has a simpler form [5, 6]. We thus obtain another simple closed-form solution in [9] as:

$$\begin{aligned} \delta_2 &:= \cdots = \delta_{N_t} = \begin{cases} \delta_2^o & E_s/N_0 > \gamma_{th} \\ 0 & E_s/N_0 \le \gamma_{th} \end{cases}, \\ \delta_1 &= 1 - \delta_2(N_t - 1), \end{aligned} \tag{18}$$

where

$$\begin{aligned} \delta_2^o &= \frac{\sigma_\epsilon^2(\sigma_\epsilon^2 + 2\lambda)}{N_t\sigma_\epsilon^2(\sigma_\epsilon^2 + 2\lambda) + \lambda^2}\left[1 + \frac{1}{\beta}\left(N_t - \frac{\lambda}{\sigma_\epsilon^2 + 2\lambda}\right)\right] - \frac{1}{\beta}, \\ \gamma_{th} &:= \frac{\lambda}{g\sigma_\epsilon^4}\left(\frac{\sigma_\epsilon^2 + \lambda}{\sigma_\epsilon^2 + 2\lambda}\right). \end{aligned}$$

Both (17) and (18) guarantee that $\delta_1 \ge \cdots \ge \delta_{N_t}$ [9]. Thus the power loading obeys a water filling principle, by allocating more power to stronger subchannels [8].

Having decided optimal $\mathbf{U}_c$ and $\mathbf{D}_c$, the optimal precoder $\mathbf{C}$ can be expressed as

$$\mathbf{C} = \mathbf{\Phi}\mathbf{D}_c^{\frac{1}{2}}\mathbf{U}_h^{\mathcal{H}}, \tag{19}$$

where the columns of $\mathbf{\Phi}$ are orthonormal. The factorization of $\mathbf{C}$ in (19) suggests that the optimal precoder is a generalized beamformer with multiple beams pointing to orthogonal directions along the eigenvectors of the channel correlation matrix conditioned on the channel mean; thus, the name eigen-beamformer.

We summarize our results so far in the following.

Proposition 1: *Suppose a0) and a1) hold true. The optimum receive-filter* $\mathbf{g}_{opt}$ *is given by (3), and the optimum precoding matrix* $\mathbf{C}_{opt} = \mathbf{\Phi}\mathbf{D}_c^{\frac{1}{2}}\mathbf{U}_h^{\mathcal{H}}$ *has* $\mathbf{U}_h$ *and* $\mathbf{D}_c$ *formed as in (15), (6), (17) or (18), with* $\mathbf{\Phi}$ *an arbitrary orthonormal* $P \times N_t$ *matrix. Optimality in* $\mathbf{g}_{opt}$ *refers to maximum-SNR, while optimality in* $\mathbf{C}_{opt}$ *pertains to minimizing the Chernoff bound on the average symbol error rate.*

4. BEAMFORMING AND SPACE-TIME BLOCK CODING

In the system model (2), we transmit only one symbol over P time slots (chip-periods), which essentially amounts to repetition coding (or a spread-spectrum) scheme. To overcome the associated rate loss, it is possible to send K symbols, $s_1, \ldots, s_K$ simultaneously, with optimal precoder $\mathbf{C}_k = \mathbf{\Phi}_k\mathbf{D}_c\mathbf{U}_h^{\mathcal{H}}$ for s_k. Certainly. this would require symbol separation at the receiver. Fortunately, this degree of freedom can be afforded by our design in Section 3 because so far the $\mathbf{\Phi}_k$'s are only required to have orthonormal columns. The desired means of data multiplexing that enables symbol separability at the receiver is indeed possible through orthogonal space-time block coding (STBC) [1, 2, 7], as summarized in the following for complex symbols; real symbols can be treated similarly.

Let s_k^R and s_k^I denote the real and imaginary part of s_k, respectively. The following orthogonal STBC designs are available for complex symbols [2, 7]:

Definition: *For complex symbols* $\{s_k = s_k^R + js_k^I\}_{k=1}^K$, *and* $P \times N_t$ *matrices* $\{\mathbf{\Phi}_k, \mathbf{\Psi}_k\}_{k=1}^K$ *each having entries drawn from* $\{1, 0, -1\}$, *the space time coded matrix*

$$\mathbf{Z}_c = \sum_{k=1}^K \mathbf{\Phi}_k s_k^R + j\sum_{k=1}^K \mathbf{\Psi}_k s_k^I \tag{20}$$

is termed a generalized complex orthogonal design (GCOD) in variables $\{s_k\}_{k=1}^K$ *of size* $P \times N_t$ *and rate* K/P, *if* $\mathbf{Z}_c^{\mathcal{H}}\mathbf{Z}_c = (\sum_{k=1}^K |s_k|^2)\mathbf{I}_{N_t}$ *[7].*

For complex symbols $s_k = s_k^R + js_k^I$, we define two precoders corresponding to $\{\mathbf{\Phi}_k, \mathbf{\Psi}_k\}$ as: $\mathbf{C}_{k,1} = \mathbf{\Phi}_k\mathbf{D}_c^{\frac{1}{2}}\mathbf{U}_h^{\mathcal{H}}$, and $\mathbf{C}_{k,2} = \mathbf{\Psi}_k\mathbf{D}_c^{\frac{1}{2}}\mathbf{U}_h^{\mathcal{H}}$. The transmitted space-time coded matrix is now

$$\mathbf{Z}_c' = \sum_{k=1}^K \mathbf{C}_{k,1}s_k^R + j\sum_{k=1}^K \mathbf{C}_{k,2}s_k^I = \mathbf{Z}_c\mathbf{D}_c^{\frac{1}{2}}\mathbf{U}_h^{\mathcal{H}}. \tag{21}$$

For the kth detector, the decision variable is formed by

$$\begin{aligned} y_k &= \mathrm{Re}\{\check{\mathbf{h}}^{\mathcal{H}}\mathbf{C}_{k,1}^{\mathcal{H}}\mathbf{x}\} + j\mathrm{Re}\{-j\check{\mathbf{h}}^{\mathcal{H}}\mathbf{C}_{k,2}^{\mathcal{H}}\mathbf{x}\} \\ &= \check{\mathbf{h}}^{\mathcal{H}}\mathbf{U}_h\mathbf{D}_c\mathbf{U}_h^{\mathcal{H}}\check{\mathbf{h}}s_k + w_k, \quad \forall k \in [1, K], \end{aligned} \tag{22}$$

where w_k has variance $N_0\check{\mathbf{h}}^{\mathcal{H}}\mathbf{U}_h\mathbf{D}_c\mathbf{U}_h^{\mathcal{H}}\check{\mathbf{h}}$; and the second equality in (22) can be easily verified from the property of orthogonal codes [2, 9]. Notice that the SNR from (22) is the same as the MRC output for the single symbol transmission studied in Section 3; thus, the optimal loading enables space-time block coded transmissions to minimize the Chernoff bound on SER, but with symbol rate K/P.

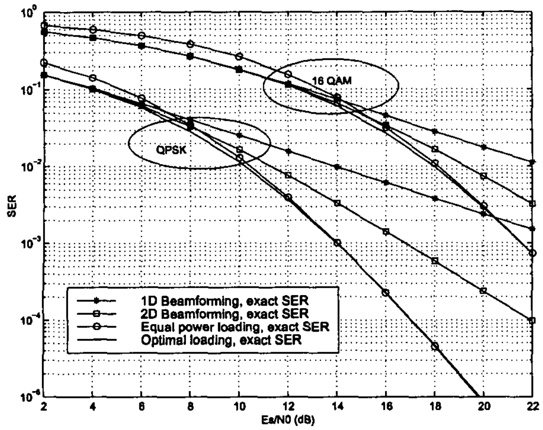

Fig. 2. SER versus E_s/N_0 ($\rho = 0.6$)

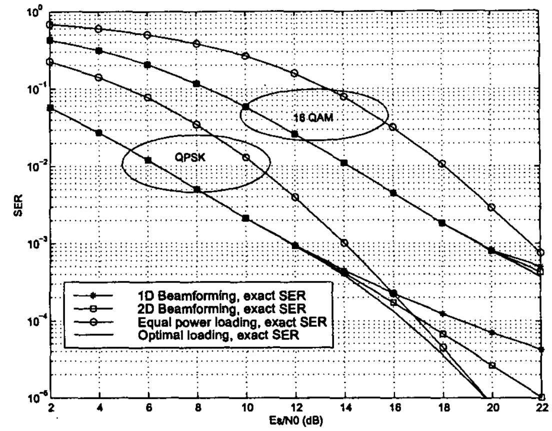

Fig. 3. SER versus E_s/N_0 ($\rho = 0.9$)

For complex symbols, a rate 1 GCOD only exists for $N_t = 2$. It corresponds to the well-known Alamouti code [1]:

$$\mathbf{Z}_{c,2\times 2} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \begin{matrix} \rightarrow \text{space} \\ \downarrow \text{time} \end{matrix} . \quad (23)$$

For $N_t = 3, 4$, rate $3/4$ orthogonal STBC exist, while for $N_t > 4$, only rate $1/2$ codes have been constructed [7], [2].

Therefore, for complex symbols, the N_t-directional eigen-beamformer of (21) achieves optimal performance with no rate loss only when $N_t = 2$, and pays a rate penalty when $N_t > 2$. To tradeoff the optimal performance for a constant rate 1 transmission, it is possible to construct a two-directional (2D) eigen-beamformer with the Alamouti code applied to the strongest two eigen-beams. Specifically, we can force $\mathbf{Z}'_c$ to be a $2 \times N_t$ matrix

$$\mathbf{Z}'_c := [\mathbf{Z}_{c,2\times 2}, \mathbf{0}_{2\times(N_t-2)}]\mathbf{D}_c^{\frac{1}{2}}\mathbf{U}_h^{\mathcal{H}}. \quad (24)$$

Similarly, we can construct a one-directional (1D) beamformer. Notice that if $\mathbf{D}_c$ has only one nonzero entry $\delta_1 = 1$, the 2D eigen-beamformer reduces to the 1D beamformer, with s_1 and $-s_2^*$ transmitted during consecutive time-slots, as confirmed by (23).

5. NUMERICAL RESULTS

We consider an uniform linear array with $N_t = 4$ antennas at the transmitter, and a single antenna at the receiver. We assume that the transmit antennas are well separated so that $\mathbf{h} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_t})$. We consider a delayed channel feedback scenario as described in Example 1. The quality of the channel feedback is thus determined by the correlation coefficient ρ. We will present simulation results for two constellations: QPSK (4-PSK), and 16-QAM. Simulation results are averaged over 10,000 channel realizations.

We compared optimal power loading based on the Ricean distribution (17) with that based on Nakagami distribution (18). It turns out that both approaches have almost identical performance [9]. For this reason, we omit the comparison result, and subsequently plot only the performance of optimal power loading based on (18).

Figs. 2 and 3 compare optimal power loading, equal power loading, 1D beamforming and 2D beamforming, for both QPSK and 16QAM. When the feedback quality is low with $\rho = 0.6$, Fig. 2 shows that optimal power loading performs close to equal power loading, while it considerably outperforms conventional 1D beamforming. On the other hand, when the feedback quality improves to $\rho = 0.9$, equal power loading is highly suboptimum. The conventional beamforming performs close to the optimal power loading at low SNR range, while it becomes inferior at sufficiently high SNR. Notice that the 2D beamformer outperforms 1D beamformer uniformly, although both are suboptimal when $E_s/N_0 > \gamma_{th}$, thus deviate from the optimal N_t-directional beamformer at the same time.

6. REFERENCES

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