

SPACE-TIME-FREQUENCY BLOCK CODED OFDM WITH SUBCARRIER GROUPING AND CONSTELLATION PRECODING*

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ABSTRACT

This paper proposes novel space-time-frequency (STF) block coding for multi-antenna OFDM transmissions over frequency-selective Rayleigh fading channels. Incorporating subcarrier grouping and choosing appropriate system parameters, we first convert our system into a set of group STF (GSTF) systems. This enables simplification of STF block coding within each GSTF system. We derive design criteria for STF block coding, and exploit existing ST coding techniques to construct STF block codes. The resulting codes are shown capable of achieving both maximum diversity and coding gains, while affording low-complexity decoding. The performance merits of our design is confirmed by corroborating simulations, and compared with existing alternatives.

1. INTRODUCTION

Space-time (ST) coding relies on simultaneous coding across space and time to achieve diversity gain without necessarily sacrificing precious bandwidth. In ST coding, the maximum achievable diversity advantage is equal to the product of the number of transmit- and receive-antennas; and therefore, it is constrained by the size and cost a system can afford. The latter motives exploitation of extra diversity dimensions, such as multipath (or frequency) diversity.

Multipath diversity becomes available when frequency selectivity is present. As proved in [2, 7, 11], multi-antenna transmissions over frequency selective fading channels can potentially provide a maximum diversity gain that is multiplicative in the number of transmit-antennas, receive-antennas, and the channel length. A number of coding schemes have been proposed recently to exploit frequency diversity. Because they offer low-complexity equalization-decoding, and facilitate the support of multirate services, multicarrier transmissions are typically adopted by those schemes [2, 4, 7, 11]. Among them, [4, 11] rely on combining ST codes with redundant or non-redundant linear precoders. Maximum diversity gain is achieved in [4, 11] at the expense of bandwidth efficiency [4], or, increased decoding complexity [4, 11]. On the other hand, [2, 7] are based on space-frequency (SF) coding, which amounts to simultaneously coding over space and frequency. However, due to the prohibitive complexity in constructing the codes, no-SF codes have been designed in [2, 7]. Instead, [2, 7] simply adopt existing codes without maximum diversity gain guarantees. Moreover, issues pertaining to maximizing the coding gain have not been addressed so far.

Focusing on multi-antenna OFDM transmissions through frequency selective Rayleigh fading channels, this paper proposes

a novel concept: joint space-time-frequency (STF) coding. Resorting to the subcarrier grouping we introduced in [6], and by choosing proper system parameters, we first divide the set of generally correlated OFDM subchannels into groups of independent subchannels. We thus convert our system into a set of what we term group STF (GSTF) sub-systems, within which STF coding is considered. By exploiting the fact that the subchannels within each GSTF system are statistically independent in space, time, and frequency, we derive design criteria for STF codes, which provide a link between STF codes, and existing ST codes. We prove that subcarrier grouping does preserve maximum diversity gains, while simplifying not only the code construction, but also the decoding algorithm significantly. Aiming at maximum diversity and coding gains, we construct STF block (STFB) codes, whose performance is investigated both by theoretical analyses, and by corroborating simulations.

2. PRELIMINARIES

2.1. System Model

We consider a wireless communication system with N_t transmit-antennas and N_r receive-antennas, where OFDM utilizing N_c subcarriers is employed per antenna transmission. The fading channel between the μ th transmit-antenna and the ν th receive-antenna is assumed to be frequency-selective, and is described by $\mathbf{h}_{\mu\nu} := [h_{\mu\nu}(0), \dots, h_{\mu\nu}(L)]^T$, where L is the channel order.

Let $x_n^\mu(p)$ be the data symbol transmitted on the p th subcarrier from the μ th transmit-antenna during the n th OFDM symbol interval. As defined, the symbols $\{x_n^\mu(p), \mu = 1, \dots, N_t, p = 0, 1, \dots, N_c - 1\}$ are transmitted in parallel on N_c subcarriers by N_t transmit-antennas. $x_n^\mu(p)$ can be viewed as a point in a 3-D STF parallelepiped. After FFT processing, the received data sample $y_n^\nu(p)$ at the ν th receive-antenna can be expressed as:

$$y_n^\nu(p) = \sum_{\mu=1}^{N_t} H_{\mu\nu}(p)x_n^\mu(p) + w_n^\nu(p), \quad \nu = 1, \dots, N_r, \quad (1)$$

where, $H_{\mu\nu}(p) := \sum_{l=0}^L h_{\mu\nu}(l)e^{-j\frac{2\pi}{P}lp}$; and the additive noise $w_n^\nu(p)$ is zero-mean, complex Gaussian with variance N_0 .

Eq. (1) represents a general model for multi-antenna OFDM systems, including those considered in [2, 7, 11]. The difference among those systems lies in how $x_n^\mu(p)$'s are generated from the information symbols s_n , which eventually leads to corresponding tradeoffs among performance, decoding complexity, and transmission rate. In our system, the generation of $x_n^\mu(p)$ is performed via what we term STF coding that we describe next.

2.2. STF Coding

Recalling that each $x_n^\mu(p)$ is a point in 3-D, we define each STF codeword as the collection of transmitted symbols within the par-

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allelepipied, spanned by N_t transmit-antennas, N_x OFDM symbol intervals, and N_c subcarriers. Mathematically, one STF codeword can be represented by a block matrix:

$$\mathbf{X} := [\mathbf{X}(0) \mathbf{X}(1) \cdots \mathbf{X}(N_c - 1)] \in \mathbb{C}^{N_t \times N_c N_x}, \quad (2)$$

where, $\mathbf{X}(p)$ is the $N_t \times N_x$ matrix with (μ, n) th entry $[\mathbf{X}(p)]_{\mu n} = x_{\mu n}^p(p)$. Let us define the MIMO channel matrix $\mathbf{H}(p) \in \mathbb{C}^{N_r \times N_t}$ with $[\mathbf{H}(p)]_{\nu \mu} = H_{\nu \mu}(p)$; and the received sample matrix $\mathbf{Y}(p) \in \mathbb{C}^{N_r \times N_x}$ with $[\mathbf{Y}(p)]_{\nu n} = y_{\nu n}^p(p)$. It follows from (1) that our 3-D STF system can be modeled as:

$$\mathbf{Y}(p) = \mathbf{H}(p)\mathbf{X}(p) + \mathbf{W}(p), \quad \forall p \in [0, N_c - 1]. \quad (3)$$

Suppose that \mathbf{X} has been generated by \bar{N}_I information symbols collected in the block $\mathbf{s} := [s_0, \dots, s_{\bar{N}_I}]^T$. STF coding is then defined as an one-to-one mapping $\Psi: \mathbf{s} \rightarrow \mathbf{X}$.

Because \mathbf{X} in (2) is described by three dimensions, STF coding simultaneously encodes information over space, time, and frequency, as its name reveals. Let $\mathcal{A}_s \ni s_n$ be the alphabet set to which the information symbol s_n belongs, and let $|\mathcal{A}_s|$ be the cardinality of \mathcal{A}_s . Since \mathbf{X} is uniquely mapped from \mathbf{s} , the number of possible STF codewords \mathbf{X} is $|\mathcal{A}_s|^{\bar{N}_I}$, which we collect into a finite set \mathcal{A}_x with $|\mathcal{A}_x| = |\mathcal{A}_s|^{\bar{N}_I}$. From a conceptual point of view, STF coding is equivalent to constructing the finite set \mathcal{A}_x , as well as specifying the mapping Ψ . Based on (1) or (3), our goal is to achieve maximum diversity and coding advantages, by carefully designing Ψ , and properly choosing system parameters.

3. SUBCARRIER GROUPING

Our design of STF coding Ψ involves designing the set \mathcal{A}_x with codewords \mathbf{X} of size $N_t \times N_x N_c$. However, N_c is typically large in practice. Thinking of the difficulties already encountered in designing ST codes of a much smaller size, it can be expected that this design will be far more challenging, without any effort to alleviate the ‘‘curse of dimensionality’’. The tool we will use to reduce the dimensionality, and thus facilitate design and decoding is subcarrier grouping.

Subcarrier grouping was originally suggested in [6] to reduce design and decoding complexity, while preserving both diversity and coding advantages, for *single-antenna* unitary precoded OFDM systems. For STF coding, the first step towards subcarrier grouping is to choose:

$$N_c = N_g(L + 1), \quad (4)$$

for a certain positive integer N_g denoting the number of groups. We assume that: **as1)** the channel taps $h_{\mu\nu}(l)$'s are i.i.d., zero-mean, complex Gaussian with variance $1/(2L+2)$ per dimension.

Under **as1)**, it can be verified that: $\forall \mu, \mu', \nu, \nu'$,

$$E[H_{\mu\nu}(p_1)H_{\mu'\nu'}^*(p_2)] = 0, \quad \text{if } \text{mod}(p_1 - p_2, N_g) = 0, \quad (5)$$

from which, it is deduced that $H_{\mu\nu}(p_1)$ and $H_{\mu'\nu'}(p_2)$ are statistically independent. The second step is to split the $N_t \times N_c N_x$ STF codeword \mathbf{X} into N_g group STF (GSTF) codewords \mathbf{X}_g :

$$\mathbf{X}_g = [\mathbf{X}_g(0), \mathbf{X}_g(1), \dots, \mathbf{X}_g(L)], \quad \forall g \in [0, N_g - 1], \quad (6)$$

where $\mathbf{X}_g(l) := \mathbf{X}(N_g l + g)$. Accordingly, we divide the STF system (3) into N_g GSTF subsystems, which we describe through the input-output relationships:

$$\mathbf{Y}_g(l) = \mathbf{H}_g(l)\mathbf{X}_g(l) + \mathbf{W}_g(l), \quad \forall l \in [0, L], \quad (7)$$

where $\mathbf{Y}_g(l) := \mathbf{Y}(N_g l + g)$, and $\mathbf{H}_g(l) := \mathbf{H}(N_g l + g)$.

It is important to recognize the following property that will be used to simplify our design criteria.

Property 1 Under **as1)** and with the choice of parameters in (4), all subchannels within $\{\mathbf{H}_g(l)\}_{l=0}^L$ of each GSTF subsystem are statistically independent.

Each GSTF subsystem is nothing but a simplified STF system with a much smaller size in the frequency dimension, as compared to the original STF system. To take advantage of subcarrier grouping, we will consider STF coding within each GSTF subsystem; i.e., we will perform STF coding to generate \mathbf{X}_g 's individually, rather than generating \mathbf{X} as a whole. As we will show in the next subsection, doing so will not incur any reduction in the diversity advantage, while it will reduce the design complexity considerably. To distinguish GSTF from the STF coding, we hereafter name the STF coding for each GSTF subsystem as GSTF coding, and denote it by the unique mapping $\Psi_g: \mathbf{s}_g \rightarrow \mathbf{X}_g$, where, $\mathbf{s}_g \in \mathbb{C}^{N_I \times 1}$ is the information symbol block used to generate \mathbf{X}_g . It is clear that we have $\bar{N}_I = N_g N_I$.

So far, we have converted the design of Ψ into the design of the set $\{\Psi_g\}_{g=0}^{N_g-1}$. Since all Ψ_g 's are basically uniform, we will only focus on one of them in the ensuing discussion.

4. DESIGN CRITERIA

We derive here design criteria for the GSTF codes \mathbf{X}_g . In addition to **as1)**, we further assume that: **as2)** maximum likelihood (ML) detection is performed with perfect channel state information at the receiver; and **as3)** high SNR is observed at the receiver. It is noted that assumptions **as1)**-**as3)** are also made in [2, 7]. Under **as1)**-**as3)**, our derivations rely on analyzing the diversity and coding advantages of GSTF transmissions modeled in (7). Due to the lack of space, we omit details, and state the main results only.

Let \mathcal{A}_{x_g} be the set of all possible \mathbf{X}_g 's. We are interested in designing \mathcal{A}_{x_g} such that both diversity and coding advantages are maximized. The design criteria are summarized as follows.

C1) (Sum-of-ranks criterion) Design \mathcal{A}_{x_g} such that: $\forall \mathbf{X}_g \neq \mathbf{X}'_g \in \mathcal{A}_{x_g}$, the matrices

$$\mathbf{\Lambda}_e(l) = [\mathbf{X}_g(l) - \mathbf{X}'_g(l)][\mathbf{X}_g(l) - \mathbf{X}'_g(l)]^H, \quad \forall l \in [0, L] \quad (8)$$

have full rank.

C2) (Product-of-determinants criterion) For the set of matrices satisfying **C1)**, design \mathcal{A}_{x_g} such that: $\forall \mathbf{X}_g \neq \mathbf{X}'_g \in \mathcal{A}_{x_g}$, the minimum of $\prod_{l=0}^L \det[\mathbf{\Lambda}_e(l)]$ is maximized.

Two remarks are due at this point:

Remark 1 Checking the dimensionality of $\mathbf{\Lambda}_e(l)$ reveals that the maximum diversity advantage is $G_d^{\max} = N_t N_r (L + 1)$, which coincides with that of STF systems without subcarrier grouping [11]. Thus, our subcarrier grouping does not sacrifice the diversity order. It is not difficult to show that this result holds true even with arbitrary subcarrier grouping (instead of (6)) as long as each GSTF subsystem contains $L + 1$ subcarriers. However, arbitrary subcarrier grouping generally involves correlated subchannels per GSTF system [c.f. Property 1], which will decrease the coding advantage. Thus, our subcarrier grouping scheme in (6) is optimal in the sense of maximizing coding advantage for a GSTF system of a given size.

Remark 2 Because codeword size affects directly the design complexity, our scheme enjoys lower design complexity relative to [2, 7], since $N_c > N_t(L + 1)$ in typical applications. More important, $\mathbf{\Lambda}_e(l)$ in both **C1)** and **C2)** is related to $\mathbf{X}_g(l)$ in a much

simpler way, as compared to that in [2, 7]. This will lead to a much simpler construction of our codes relative to those in [2, 7]¹.

Having obtained the design criteria, we proceed to design GSTF block codes. GSTF trellis codes are designed in [5].

5. GSTF BLOCK CODES

Based on the observation that the design of \mathbf{X}_g can be accomplished by the joint design of the ST codewords $\{\mathbf{X}_g(l)\}_{l=0}^L$ [c.f. C1) and C2)], the encoding of GSTF codes is carried out in two successive stages: constellation precoding, and ST component coding. Constellation precoding will enable frequency diversity, while ST component coding will collect spatial diversity. Although our design applies to arbitrary number of transmit and receive-antennas [5], for brevity, we just present the case of $N_t = 2$ and $N_r = 1$, starting from the encoding process.

5.1. Encoding

We first choose the parameter $N_l = 2(L + 1)$. Then, we demultiplex the information symbol block \mathbf{s}_g into two sub-blocks: $\{\mathbf{s}_{g,i} \in \mathbb{C}^{(L+1) \times 1}, i = 0, 1\}$ so that: $\mathbf{s}_g := [\mathbf{s}_{g,0}^T, \mathbf{s}_{g,1}^T]^T$. The stage of constellation precoding is first invoked to distribute information symbols over multiple subcarriers by precoding $\mathbf{s}_{g,i}$ to obtain $\tilde{\mathbf{s}}_{g,i} := \Theta \mathbf{s}_{g,i}$, where $\Theta \in \mathbb{C}^{(L+1) \times (L+1)}$ denotes our square constellation precoder. The precoded blocks $\tilde{\mathbf{s}}_{g,i} \in \mathbb{C}^{(L+1) \times (L+1)}$ are subsequently processed to form the GSTF codeword \mathbf{X}_g via the stage of ST component coding. Combining these two stages, we will be able to achieve G_d^{\max} , if constellation precoding and ST component coding are designed properly, as we describe next.

5.1.1. ST Component Coding

Let us define $\tilde{\mathbf{s}}_{g,i} := [\tilde{s}_{g,i,0}, \dots, \tilde{s}_{g,i,L}]^T$. To perform ST component coding, we construct $\mathbf{X}_g(l)$ as:

$$\mathbf{X}_g(l) = \begin{bmatrix} \tilde{s}_{g,0,l} & -\tilde{s}_{g,1,l} \\ \tilde{s}_{g,1,l} & \tilde{s}_{g,0,l} \end{bmatrix}, \quad (9)$$

which is nothing but Alamouti's ST coding [1]. We next move on to design the constellation precoder.

5.1.2. Constellation Precoding

With θ_l^T denoting the l th row of Θ , we can write $\tilde{s}_{g,i,l} = \theta_l^T \mathbf{s}_{g,i}$. It follows from (9), that (8) can be re-written as:

$$\Lambda_e(l) = \left[\sum_{i=0}^1 |\theta_l^T (\mathbf{s}_{g,i} - \mathbf{s}'_{g,i})|^2 \right] \mathbf{I}_2, \quad (10)$$

where the meaning of $\mathbf{s}'_{g,i} \neq \mathbf{s}_{g,i}$ is clear from the context.

Because C1) will be automatically satisfied if C2) is satisfied, we consider C2) only. Plugging (10) into C2), we have

$$\xi(\mathbf{s}_g, \mathbf{s}'_g) := \prod_{l=0}^L \det[\Lambda_e(l)] = \prod_{l=0}^L \left[\sum_{i=0}^1 |\theta_l^T (\mathbf{s}_{g,i} - \mathbf{s}'_{g,i})|^2 \right]^2. \quad (11)$$

Using the arithmetic-geometric mean inequality, (11) can be lower-bounded by

$$\xi(\mathbf{s}_g, \mathbf{s}'_g) \geq 4 \prod_{l=0}^L \prod_{i=0}^{N_s-1} |\theta_l^T (\mathbf{s}_{g,i} - \mathbf{s}'_{g,i})|, \quad (12)$$

¹Indeed, no code construction is offered by [2, 7] due to its difficulty.

where the equality is satisfied when $|\theta_l^T (\mathbf{s}_{g,i} - \mathbf{s}'_{g,i})|^2 = |\theta_l^T (\mathbf{s}_{g,i'} - \mathbf{s}'_{g,i'})|^2, \forall i \neq i'$. According to C2), we are dealing with a max-min problem for all possible $\mathbf{s}_g \neq \mathbf{s}'_g$. Thus, it is meaningful to maximize only the lower bound in (12). Because the lower bound is attained when $|\theta_l^T (\mathbf{s}_{g,i} - \mathbf{s}'_{g,i})|$'s are equal for different i 's, we use \bar{s} to denote a generic $\mathbf{s}_{g,i}$. From C2), the design criterion for Θ can be reduced to:

C3) (Product distance criterion) Design Θ to maximize

$$\min_{\bar{s} \neq \bar{s}'} \prod_{l=0}^L |\theta_l^T (\bar{s} - \bar{s}')|. \quad (13)$$

Interestingly, C3) is exactly the design criterion used in [10] to construct the so-called constellation precoder for flat fading channels. Therefore, we will not pursue the detailed construction of Θ in this paper. Instead, we will simply borrow the precoders found in [10]. For example, when $L = 1$, and QPSK modulation is employed, the precoder Θ is given by [10]:

$$\Theta = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & e^{j\frac{\pi}{4}} \\ 1 & e^{j\frac{3\pi}{4}} \end{bmatrix}. \quad (14)$$

Different from the redundant and constellation-irrespective linear precoders used in [9], it is worthwhile to underscore that Θ is square (thus non-redundant), and its constellation-specific design depends on the finiteness of \mathcal{A}_s .

Before we proceed to describe STFB decoding, we illustrate in Fig. 1 our STF block coded OFDM system with $N_g = 2$, where $\{\Phi_g\}_{g=0}^1$ represent our subcarrier selectors that are used to assign $L + 1$ subcarriers to each GSTF subsystem according to our subcarrier grouping scheme in Section 3.

5.2. Decoding of GSTF Block Codes

The decoding of GSTF block codes follows the reverse order of the encoding process. We first use low-complexity Alamouti's ST block decoding algorithm [1] to obtain decision statistics of $\tilde{\mathbf{s}}_{g,i}$, from which ML decoding of $\mathbf{s}_{g,i}$ can be performed by using the sphere decoding algorithm whose complexity is irrespective of the constellation size, and polynomial in $L + 1$. Because L is typically small, the decoding complexity of GSTFB codes is relatively low.

6. SIMULATIONS

Here, we present simulations to investigate the performance of our designs with $N_t = 2$ and $N_r = 1$. Our figure of merit is OFDM symbol error rate (OFDM-SER), which we average over 10,000 channel realizations. In our simulations, we will let L_{real} denote the physical channel order, and L the channel order assumed in designing STF block codes.

Example 1 We compare our GSTF block coding schemes to the SF coding schemes in [2, 7]. The 16-state TCM code with effective length 3, and the 16-state ST trellis code [8, Fig. 5] are used to generate the SF codes of [7] and [2], respectively. QPSK modulation and $N_c = 64$ are chosen for all schemes. The random channels ($L_{\text{real}} = 8$) are based on the HiperLan 2 channel model A [3]. The design of GSTF block codes is based on $L = 1$. Fig. 2 shows that GSTF block codes outperform SF codes considerably. Note that our GSTF codes are designed regardless of the real channel order $L_{\text{real}} = 8$, channel correlation, and power profile in HiperLan 2 channels, which speaks for the robustness of our design.

Example 2 In order to appreciate the importance of frequency diversity, we simulate the performance of GSTF block coding in multi-ray Rayleigh fading channels with $L_{\text{real}} = 1, 2, 3$. We design GSTF block codes with $L = L_{\text{real}}$. The number of subcarriers

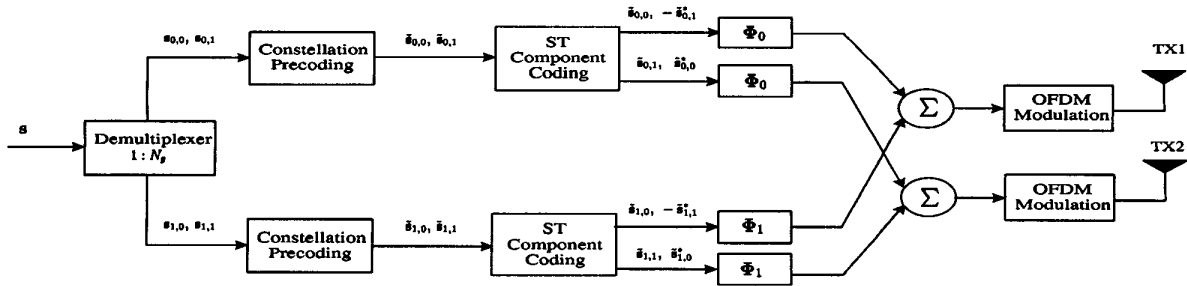


Fig. 1. STF block coded OFDM with $N_t = 2$, and $N_g = 2$

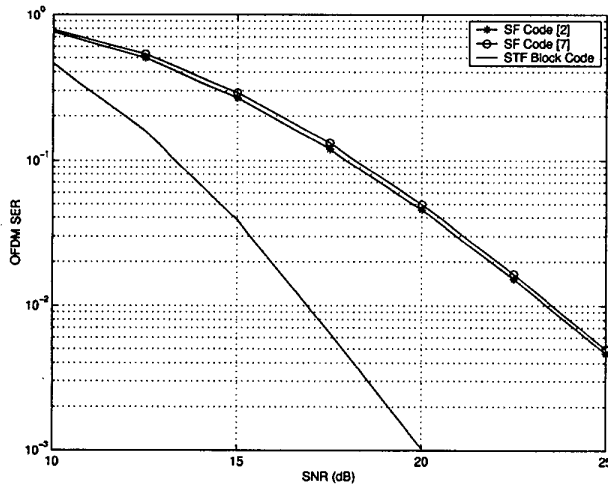


Fig. 2. Comparison with SF codes (HiperLan 2 channels)

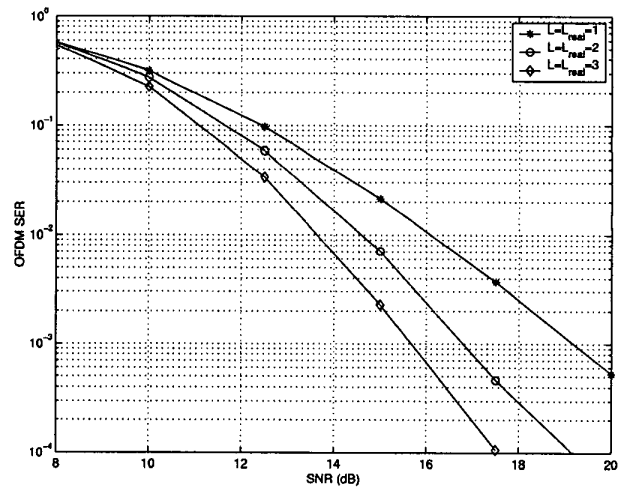


Fig. 3. Importance of frequency diversity (multi-ray channels)

is chosen as $N_c = 48$. Fig. 3 confirms that GSTF codes achieve higher diversity gain as the channel order increases, which justifies the importance of GSTF coding that accounts for frequency diversity.

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