OPTIMAL TRAINING AND REDUNDANT PRECODING FOR BLOCK TRANSMISSIONS WITH APPLICATION TO WIRELESS OFDM

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ABSTRACT

The adoption of orthogonal frequency-division multiplexing (OFDM) by wireless local area networks and audio/video broadcasting standards testifies to the importance of recovering block precoded transmissions propagating through frequency-selective FIR channels. Existing block transmission standards invoke bandwidth-consuming error control codes to mitigate channel fades and training sequences to identify the FIR channels. To enable low-complexity block-by-block receiver processing, we design redundant precoders with cyclic prefix (CP) and superimposed training sequences for optimal channel estimation and guaranteed symbol recovery regardless of the underlying FIR frequency-selective channels. Numerical results are presented to access the performance of the designed training and precoding schemes.

1. INTRODUCTION

Block transmissions relying on linear redundant filterbank precoding with cyclic prefixed or zero padded blocks have gained increasing interest recently for mitigating frequency-selective multipath effects (see e.g., [2, 9, 10] and references therein). Redundancy removes inter block interference (IBI) and also facilitates (even blind) acquisition of channel state information (CSI) at the receiver. It leads to data efficient low-complexity linear equalization (of the zero-forcing (ZF) or minimum mean-squared error (MMSE) type) with guaranteed symbol recovery regardless of the zero locations of the underlying FIR channel [9].

When CSI is available at the transmitter (through a feedback channel) optimal precoders and decoders become available under various criteria [9]. However, rapid variations of the wireless channel render the CSI feedback to transmitter outdated, and motivate channel-independent precoders. On the other hand, because CSI is indispensable at the receiver, training sequences are needed to acquire it.

Instead of long training sequences, inserting training symbols is known as pilot symbol aided modulation (PSAM), and has been used for mitigating frequency-flat Rayleigh fading channels [1]. The inserted pilot symbols in PSAM are separated from the information symbols in the time domain [3, 4, 5], while the so-termed pilot tones (complex exponentials) are separated from the information symbols in the frequency domain [7].

This paper deals with linearly precoded symbol blocks with a superimposed pilot-block that can be jointly modeled as an *affine* precoder. Affine precoding was discussed in [6] but the type of affine precoders suitable for optimal channel estimation and symbol recovery were not specified. Unlike [6], we here specify the design constraints for IBI cancellation and low-complexity block-by-block reception enabling linear channel estimation that is decoupled from symbol recovery.

We then focus on channel estimation and symbol recovery through relatively fast varying FIR channels (Section 2). To enable low-complexity block-by-block processing, we first eliminate IBI by utilizing redundancy in the form of a cyclic prefix (CP). Decoupling channel estimation from symbol recovery naturally leads to a precoded (P) OFDM system with pilot tones (Section 3). Then, we derive the optimal pilot tones for channel estimation in the presence of white or colored noise (Section 4), and design P-OFDM to ensure symbol recovery regardless of channel nulls (Section 5). P-OFDM is shown to mitigate the channel effects and the noise color *deterministically*. Simulations are presented to corroborate P-OFDM's improved performance over conventional OFDM, thanks to its enhanced frequency diversity.

2. BLOCK MODELING AND PRELIMINARIES

We consider a discrete-time baseband equivalent model for block transmissions: The information bearing sequence $s(n)$ is parsed into blocks of size M. Each $M \times 1$ block $\boldsymbol{s}(i)$ is precoded by a tall $\bar{N} \times M$ precoding matrix $\bar{\boldsymbol{A}}$ with generally complex-valued entries. Selecting $\bar{N} > M$ introduces redundancy to mitigate the effects of frequency-selective propagation channels. An $\bar{N} \times 1$ block of training symbols $\bar{\boldsymbol{b}}$, which are also known to the receiver, is added to the precoded block to obtain

$$\bar{\boldsymbol{u}}(i) = \bar{\boldsymbol{A}}\boldsymbol{s}(i) + \bar{\boldsymbol{b}}. \qquad (1)$$

For the mapping in (1) to be invertible, we will choose the redundant precoder so that:

C1. *The $\bar{N} \times M$ matrix $\bar{\boldsymbol{A}}$ is tall and full column rank M.*

The channel is considered to be linear time-invariant over one received block but is allowed to vary from block-to-block. We omit time dependence and express the finite impulse response (FIR) of the discrete-time baseband equivalent channel as $\{h(n)\}$.

At the receiver, we assume perfect timing and carrier synchronization, and sample the output of the front-end

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filter (that is matched to the transmit-pulse) at the symbol rate $1/T_s$. With $\tau_{max,d}$ denoting the channel's maximum delay spread, our FIR channel order $L = \lceil \tau_{max,d}/T_s \rceil$, where $\lceil \ \rceil$ stands for integer-ceiling. We collect $\bar{N}$ noisy samples in an $\bar{N} \times 1$ received vector $\bar{\boldsymbol{x}}(i)$:

$$\bar{\boldsymbol{x}}(i) = \boldsymbol{H}_0 \bar{\boldsymbol{u}}(i) + \boldsymbol{H}_1 \bar{\boldsymbol{u}}(i-1) + \boldsymbol{\eta}(i), \tag{2}$$

where $\boldsymbol{H}_0$ and $\boldsymbol{H}_1$ are square Toeplitz channel convolution matrices with first column $[h(0), h(1), \ldots, h(L), 0, \ldots, 0]^T$ and with first row $[0, \ldots, 0, h(L), h(L-1), \ldots, h(1)]$, respectively; and $\boldsymbol{\eta}(i)$ is a zero-mean additive noise.

We first eliminate IBI by utilizing the so-called cyclic prefix (CP). Discarding the CP at the receiver removes IBI provided that the following design condition holds at the transmitter:

A1. *The length $\bar{N}$ of the redundant transmitted block is chosen to satisfy: $\bar{N} \geq L + M$.*

Let N be defined as $N := \bar{N} - L$. The CP insertion in matrix form can be described by the $\bar{N} \times N$ CP-inducing precoder $\boldsymbol{T}_{cp}$ (see e.g., [10] for details)

$$\boldsymbol{T}_{cp} := \begin{bmatrix} \boldsymbol{0}_{L\times(N-L)} & \boldsymbol{I}_L \\ \multicolumn{2}{c}{\boldsymbol{I}_N} \end{bmatrix}_{\bar{N}\times N}, \tag{3}$$

where $\boldsymbol{I}_L$ denotes the identity matrix of size L and $\boldsymbol{0}_{L\times(N-L)}$ stands for the $L \times (N-L)$ zero matrix. To enable insertion and removal of the CP and thus cancellation of IBI, we select our general design in (1) to satisfy:

A2. *Matrix $\bar{\boldsymbol{A}}$ and vector $\bar{\boldsymbol{b}}$ are chosen to incorporate the CP; i.e., $\bar{\boldsymbol{A}} = \boldsymbol{T}_{cp}\boldsymbol{A}$, $\bar{\boldsymbol{b}} = \boldsymbol{T}_{cp}\boldsymbol{b}$, where $\boldsymbol{A}$ is an $N \times M$ matrix and $\boldsymbol{b}$ is an $N \times 1$ vector.*

Taking A2 into account, we can re-write the transmitted block in (1) as $\bar{\boldsymbol{u}}(i) = \boldsymbol{T}_{cp}\boldsymbol{u}(i)$, where $\boldsymbol{u}(i) = \boldsymbol{A}\boldsymbol{s}(i) + \boldsymbol{b}$. Discarding the CP from $\bar{\boldsymbol{x}}(i)$, we arrive at

$$\boldsymbol{x}(i) = \boldsymbol{H}\boldsymbol{u}(i) + \boldsymbol{w}(i) = \boldsymbol{H}\boldsymbol{A}\boldsymbol{s}(i) + \boldsymbol{B}\boldsymbol{h} + \boldsymbol{w}(i), \tag{4}$$

where: $\boldsymbol{w}(i) := \boldsymbol{R}_{cp}\boldsymbol{\eta}(i)$; $\boldsymbol{h} := [h(0), h(1), \ldots, h(L)]^T$; $\boldsymbol{H}$ is an $N \times N$ circulant matrix with first column $[\boldsymbol{h}^T, 0, \ldots, 0]^T$; $\boldsymbol{B}$ is an $N \times (L+1)$ column-wise circulant matrix with first column $\boldsymbol{b}$; and in deriving (4) we used the commutativity of circular convolution to obtain $\boldsymbol{H}\boldsymbol{b} = \boldsymbol{B}\boldsymbol{h}$.

3. DECOUPLING SYMBOL FROM CHANNEL ESTIMATION

To be able to estimate $\boldsymbol{h}$ from $\boldsymbol{x}(i)$ using linear least-squares (LS), we should select our training vector $\boldsymbol{b}$ such that:

C2. *The $N \times (L+1)$ tall training circulant matrix $\mathbf{B}$ has full column rank $L+1$.*

When $\boldsymbol{b}$ is selected so that $\boldsymbol{B}$ satisfies C2, the LS channel estimator is given by [c.f. (4)]

$$\hat{\boldsymbol{h}} = \boldsymbol{B}^{\dagger}\boldsymbol{x}(i) = \boldsymbol{h} + \boldsymbol{B}^{\dagger}\left[\boldsymbol{H}\boldsymbol{A}\boldsymbol{s}(i) + \boldsymbol{w}(i)\right], \tag{5}$$

where $\boldsymbol{B}^{\dagger} := (\boldsymbol{B}^{\mathcal{H}}\boldsymbol{B})^{-1}\boldsymbol{B}^{\mathcal{H}}$ is the (minimum norm) pseudo-inverse of $\boldsymbol{B}$. Eq. (5) contains a symbol-dependent noise term which must be eliminated for the LS channel estimation error to be minimized. Eliminating this term amounts to designing $(\boldsymbol{A}, \boldsymbol{B})$ such that:

C3. *Matrix $\boldsymbol{B}^{\dagger}\boldsymbol{H}\boldsymbol{A} = \boldsymbol{0}$ for any FIR channel of order L.*

The conditions on $\boldsymbol{A}$ and $\boldsymbol{b}$ for C2 to be satisfied depend on the non-zero entries of the FFT of $\boldsymbol{b}$ [8]. Let $\boldsymbol{F}$ be the $N \times N$ FFT matrix with (m,n)th entry $[\boldsymbol{F}]_{m,n} = N^{-\frac{1}{2}}W^{-mn}$, where $W := \exp(j2\pi/N)$ and define $\tilde{\boldsymbol{b}} = \boldsymbol{F}\boldsymbol{b}$. Letting $N-K$ be the number of zero entries of $\tilde{\boldsymbol{b}}$, we define the set of ordered integer indices:

$$\mathcal{I}_0 := \{i_k | \tilde{b}_{i_k} = 0, i_k < i_{k+1}, k \in [0, N-K-1]\}, \tag{6}$$

where $\tilde{b}_i$ denotes the ith entry of $\tilde{\boldsymbol{b}}$. We also define its complement $\mathcal{I}_0^{\perp}$ containing K indices i_k corresponding to the non-zero values of $\tilde{\boldsymbol{b}}$. If the $K \times 1$ vector $\check{\boldsymbol{b}}$ contains the K non-zero entries of the FFT pilot vector $\tilde{\boldsymbol{b}}$, then the time-domain training vector can be written as:

$$\boldsymbol{b} = \boldsymbol{F}^{\mathcal{H}}\tilde{\boldsymbol{b}} = \boldsymbol{F}^{\mathcal{H}}\boldsymbol{P}_{\mathcal{I}_0}\begin{bmatrix} \check{\boldsymbol{b}} \\ \boldsymbol{0} \end{bmatrix} = \boldsymbol{F}^{\mathcal{H}}\boldsymbol{P}_{\mathcal{I}_0^{\perp}}\begin{bmatrix} \boldsymbol{0} \\ \check{\boldsymbol{b}} \end{bmatrix}, \tag{7}$$

where $\boldsymbol{P}_{\mathcal{I}_0}(\boldsymbol{P}_{\mathcal{I}_0^{\perp}})$ is a permutation matrix collecting the K possibly dispersed non-zero entries $\check{\boldsymbol{b}}$ of $\tilde{\boldsymbol{b}}$ at the top (bottom). We can now state our first result as follows (proofs are omitted due to lack of space but also available [8]):

Theorem 1 *Consider transmissions of information blocks of length M through an FIR channel of order L using CP to avoid ISI as per A1 and A2. For $K \in [L+1, 2L+1]$, guaranteed symbol recovery from precoded symbols, (C1), can be decoupled from linear channel estimation based on training symbols, (C2), regardless of the FIR channel, (C3), if and only if the precoded blocks have length $N \geq M+K$ and the affine precoders $(\boldsymbol{A}, \boldsymbol{b})$ are selected from the class:*

$$\boldsymbol{A} = \boldsymbol{F}^{\mathcal{H}}\boldsymbol{P}_{\mathcal{I}_0^{\perp}}\begin{bmatrix} \boldsymbol{\Theta}_{(N-K)\times M} \\ \boldsymbol{0}_{K\times M} \end{bmatrix}, \quad \boldsymbol{b} = \boldsymbol{F}^{\mathcal{H}}\boldsymbol{P}_{\mathcal{I}_0}\begin{bmatrix} \check{\boldsymbol{b}} \\ \boldsymbol{0} \end{bmatrix}, \tag{8}$$

where $\boldsymbol{\Theta}$ is any full column rank matrix, $\boldsymbol{P}_{\mathcal{I}_0}$ and $\boldsymbol{P}_{\mathcal{I}_0^{\perp}}$ are the permutation matrices and $\check{\boldsymbol{b}}$ is the non-zero vector defined in (7). Clearly, the minimum redundancy choice corresponds to $N = M+L+1$, and after accounting for CP, the transmitted block length is $\bar{N} = M+2L+1$.

4. OPTIMAL PILOT TONES

Relying on Theorem 1, we want to optimize the location (i.e., select the set $\mathcal{I}_0$ in (6)) as well as the power (i.e., choose $\check{\boldsymbol{b}}$) of pilots in (8). Our criterion will be to minimize the channel mean-square error (MSE) for a given transmit power budget $||\boldsymbol{b}||^2 := \mathcal{P}_b = ||\check{\boldsymbol{b}}||^2$, where $||\cdot||$ denotes the Euclidean norm. Considering (5) with $\boldsymbol{B}^{\dagger}\boldsymbol{H}\boldsymbol{A} = \boldsymbol{0}$, the channel MSE is given by

$$\sigma_{\hat{h}}^2 := E\{||\hat{\boldsymbol{h}} - \boldsymbol{h}||^2\} = \mathrm{tr}(\boldsymbol{B}^{\dagger}\boldsymbol{R}_w\boldsymbol{B}^{\dagger\mathcal{H}}), \tag{9}$$

where $E\{\cdot\}$ denotes the expectation operator, $\mathrm{tr}(\cdot)$ stands for the trace operator, and $\boldsymbol{R}_w$ is the correlation matrix of $\boldsymbol{w}(i)$ in (4). In addition to AWN with $\boldsymbol{R}_w = \sigma_w^2\boldsymbol{I}_N$, we will allow $\boldsymbol{w}(i)$ to be correlated in order to account for structured (e.g., adjacent channel) interference.

Suppose that the number of pilot tones is minimum for block-by-block channel estimation; i.e., $K = L + 1$. One can show that $\sigma_{\tilde{h}}^2$ is bounded as follows [8]:

$$\sigma_{\tilde{h}}^2 \leq N \operatorname{tr}(\check{\boldsymbol{F}}_{0:L}^{\mathcal{H}} \check{\boldsymbol{F}}_{0:L})^{-1} \cdot \max_{l_i \in \mathcal{I}_0^{\perp}} \frac{S_w(W^{l_i})}{|\tilde{b}_{l_i}|^2}, \qquad (10)$$

where $\check{\boldsymbol{F}}_{0:L}$ is the $K \times (L+1)$ matrix formed by the K rows of the first $L+1$ columns of $\boldsymbol{F}$ corresponding to the non-zero entries of $\tilde{\boldsymbol{b}}$; and $S_w(W^{l_i})$ denotes the power spectral density (psd) of the noise at W^{l_i} for $l_i \in \mathcal{I}_0^{\perp}$.

Minimizing the upper bound in (10) with respect to $\mathcal{I}_0^{\perp}$ and $|\tilde{b}_{l_i}|^2$ for $l_i \in \mathcal{I}_0^{\perp}$, leads to the following theorem [8]:

Theorem 2 *Let the number of pilot tones be minimum for channel estimation, that is, $K = L + 1$. Suppose that the additive noise is stationary with zero mean and spectrum $S_w(z)$. For a given power $||\boldsymbol{b}||^2 = \mathcal{P}_b$ on the pilots, the upper bound (10) for the channel MSE is asymptotically (as $N \to \infty$) minimized if pilot tones are equi-spaced and equi-powered. Then, the channel MSE is given by*

$$\sigma_{\tilde{h}}^2 \cong \frac{1}{\mathcal{P}_b} \sum_{l_i \in \mathcal{I}_0^{\perp}} S_w(W^{l_i}). \qquad (11)$$

When the additive noise is white, Theorem 2 conditions on the pilots are necessary and sufficient; they holds true for any finite N; and, (11) becomes exact: $\sigma_{\tilde{h}}^2 = (L+1)\sigma_w^2/\mathcal{P}_b$, where σ_w^2 is the noise variance. The sufficiency (but not the necessity) of equi-spaced and equi-powered pilot was also shown in [7] for white noise.

Equi-spaced and equi-powered pilot tones are possible if and only if we select:

i) $N = (L+1)J$ for some non-zero integer J;

ii) the spacing among pilot tones to satisfy: $l_i = l_0 + Jl$ for some $l_0 \in [0, J-1]$ and $l \in [1, L]$;

iii) the same power loaded on each pilot tone.

Design rule ii) suggests choosing $\mathcal{I}_0$ in (9) so that the pilot tones (entries of $\tilde{\boldsymbol{b}}$) are equi-spaced non-zero entries of $\tilde{\boldsymbol{b}}$ (when $l_0 \neq 0$ equi-spaced is understood in a circular sense; i.e., after periodically repeating $\tilde{\boldsymbol{b}}$).

According to our pilot design rules i) and ii), there are J possible sets of indices for equi-spaced pilot tones, defined as $\mathcal{I}_{0,j}^{\perp} = \{j + Jl | l \in [1, L]\}$ for $j \in [0, J-1]$. These sets yield different channel MSE in general (c.f. (11)). If the noise statistics are available at the transmitter, one may select the set with the minimum channel MSE. Otherwise, shifting (or hopping) these sets from block-to-block is well motivated because the *average* channel MSE will be lowered. If we randomly choose one set with equal probability, then the average channel MSE is found after scaling (11) with $J = N/(L+1)$ as follows:

$$\sigma_{\tilde{h}} \cong \frac{1}{\mathcal{P}_b J} \sum_{j=0}^{J-1} \sum_{l_i \in \mathcal{I}_{0,j}^{\perp}} S_w(W^{l_i}) = \frac{L+1}{\mathcal{P}_b N} \sum_{n=0}^{N-1} S_w(W^n). \qquad (12)$$

For N sufficiently large, the average channel MSE is asymptotically given by $(L+1)\sigma_w^2/\mathcal{P}_b$. Recall that the precoder must also hop according to the set of pilot tones through $\boldsymbol{P}_{\mathcal{I}_0^{\perp}}$ in (8); hence the resulting BER will be also averaged.

Now suppose that the noise spectrum is available at the transmitter (through a feedback channel). In this case, equi-spaced pilot tones are not always optimal. However, adopting equi-spaced pilot tones enables a simple solution for the (sub)optimum power loading [8]

$$\mathcal{I}_{0,opt}^{\perp} := \arg\min_{\mathcal{I}_{0,j}^{\perp}} \Big(\sum_{l_i \in \mathcal{I}_{0,j}^{\perp}} S_w^{\frac{1}{2}}(W^{l_i}) \Big)^2, \qquad (13)$$

$$|\tilde{b}_{l_i}|^2 = \mathcal{P}_b \frac{S_w^{\frac{1}{2}}(W^{l_i})}{\sum_{l_i' \in \mathcal{I}_{0,opt}^{\perp}} S_w^{\frac{1}{2}}(W^{l_i'})}, \quad l_i \in \mathcal{I}_{0,opt}^{\perp}. \qquad (14)$$

This suggests a waterfilling-like power loading when only the noise color (but not CSI) is available at the transmitter.

5. PRECODED OFDM

It is well-known that, if the channel has nulls on the FFT grid, then $\boldsymbol{HA}$ becomes singular and hence symbol recovery is not guaranteed [9, 10]. To assure *channel-independent* invertibility of $\boldsymbol{HA}$, we rely on the judicious selection of Θ suggested by [10] for generalized multi-carrier (GMC) CDMA. With this $\Theta \neq \boldsymbol{I}$ selection, our block transmission in (8) can be viewed as a precoded (P) OFDM modulation. P-OFDM uses $2L+1$ redundant subcarriers (hence $N = M + 2L + 1$), and adopts an $(M+L) \times M$ orthogonal matrix Θ in (8) such that:

A3. *Any M rows of Θ are linearly independent.*

The channel matrix $\boldsymbol{H}$ can be diagonalized by $\boldsymbol{F}$ to obtain $\boldsymbol{HA} = \check{\boldsymbol{F}}^{\mathcal{H}} \check{\boldsymbol{D}}(\tilde{\boldsymbol{h}})\Theta$, where for $i_n \in \mathcal{I}_0$, $\check{\boldsymbol{D}}(\tilde{\boldsymbol{h}}) := \operatorname{diag}[H(W^{i_0}), \ldots, H(W^{i_{M+L-1}})]$; $H(z)$ is the channel transfer function defined as $H(z) := \sum_0^L h(n)z^{-n}$; and $\check{\boldsymbol{F}}$ is an $N \times (M+L)$ matrix with nth column equal to the i_nth column of $\boldsymbol{F}$. Because the channel has order L, at most L values of $H(W^{i_n})$ are zero. Since any M rows of Θ are linearly independent, $\check{\boldsymbol{D}}(\tilde{\boldsymbol{h}})\Theta$ and hence $\boldsymbol{HA}$ have full column rank regardless of the channel nulls. Therefore, symbol recovery is guaranteed regardless of the channel nulls.

One choice for P-OFDM is to select Θ as the submatrix formed by M columns of the $(M+L) \times (M+L)$ FFT matrix; e.g., $\Theta = \boldsymbol{F}_{M+L,0:M-1}$, where $\boldsymbol{F}_{M+L,0:M-1}$ has the first M columns of the $(M+L) \times (M+L)$ FFT matrix. Suppose for simplicity that the channel estimate is perfect and that the channel has no nulls on the FFT grid. Then, the symbol estimate by the zero-forcing (ZF) equalizer is found to be: $\hat{\boldsymbol{s}}_{zf}(i) := \boldsymbol{A}^{\dagger}\boldsymbol{H}^{-1}\boldsymbol{x}(i) = \boldsymbol{s}(i) + \check{\boldsymbol{w}}(i)$, where $\check{\boldsymbol{w}}(i) := \boldsymbol{F}_{M+L,0:M-1}^{\mathcal{H}} \check{\boldsymbol{D}}^{-1}(\tilde{\boldsymbol{h}})\check{\boldsymbol{F}}^{\mathcal{H}}\boldsymbol{w}(i)$.

For M sufficiently large, diagonal entries of the correlation matrix of $\check{\boldsymbol{w}}(i)$ are found to be equal. It follows that the effects of the channel and the noise on each entry of $\hat{\boldsymbol{s}}_{zf}(i)$ are *averaged over one block*. This implies that the affine precoder with an FFT-based matrix Θ is *robust to the channel frequency response and the noise color.* In contrast, the conventional (uncoded) OFDM with $\Theta = \boldsymbol{I}$ has the nth entry of $\hat{\boldsymbol{s}}_{zf}(i)$ contaminated by noise that has variance $S_w(W^{i_n})/|H(W^{i_n})|^2$. This means that the BER for the nth entry of the symbol block becomes poor when $S_w(W^{i_n})$ is large and/or when $|H(W^{i_n})|$ is small. Thus, error control coding and interleaving are usually adopted by the conventional OFDM systems.

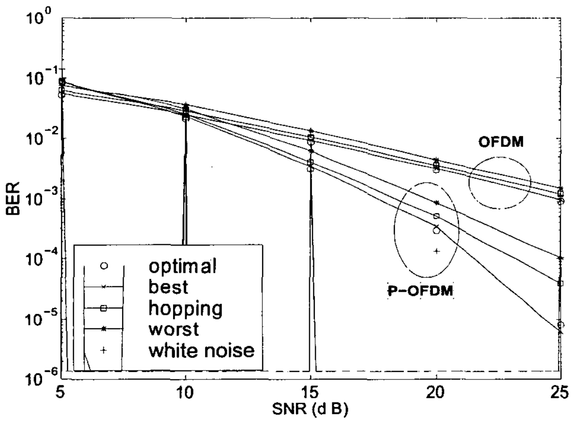

Figure 1: OFDM vs. P-OFDM ($\alpha = 0.8$, Rayleigh fading channels).

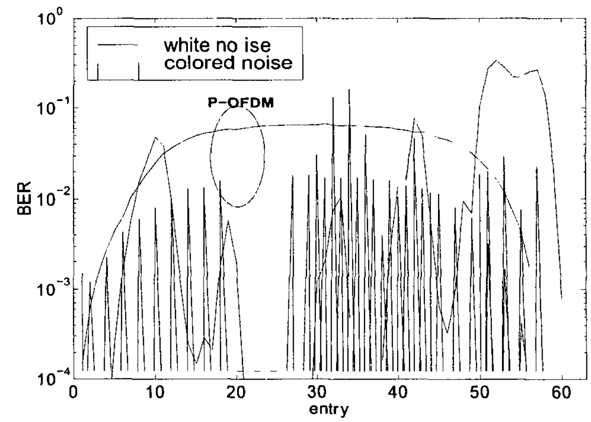

Figure 2: BER of OFDM and P-OFDM for each symbol in one block at SNR= 10dB (fixed channel).

6. SIMULATED PERFORMANCE

We set the block size to be $N = 64$ and the guard interval length (channel order) to be $L = 7$. We tested the conventional OFDM with $\boldsymbol{\Theta} = \boldsymbol{I}_M$ for $M = 56(= 64 - 8)$, and P-OFDM with $\boldsymbol{\Theta} = \boldsymbol{F}_{M+L,0:M-1}$ for $M = 49(= 56 - 7)$ where $L + 1 = 8$ equi-spaced pilot tones were utilized. We conducted 10^4 Monte-Carlo simulations and averaged the results. The information symbols were drawn from a BPSK constellation and the noise was either white or colored first-order Markov with coefficient -0.9. Channels were normalized to have unit norm. We loaded power on information-bearing symbols and pilot symbols such that $\alpha := E\{||\boldsymbol{As}(i)||^2\}/E\{||\boldsymbol{u}(i)||^2\} = 0.8$ $(||\boldsymbol{b}||^2/E\{||\boldsymbol{u}(i)||^2\} = 0.2)$. For channel estimation, we tested equi-spaced and equi-powered pilot tones (8 sets); hopping equi-powered pilots; and equi-spaced optimally power loaded pilots.

For Rayleigh channels of 7th order with i.i.d. complex zero-mean Gaussian taps, Fig. 1 shows the BER employing different types of pilot tones as a function of the signal-to-noise ratio, which is defined as $E\{||\boldsymbol{Hu}(i)||^2\}/E\{||\boldsymbol{w}(i)||^2\}$. P-OFDM outperforms OFDM above SNR= 15dB and the performance gain increases as SNR increases. We observed that hopping exhibits improved performance particularly for P-OFDM. No significant improvement by optimally loaded pilot tones could be seen in this example. Since hopping does not require any knowledge of the noise statistics at the transmitter and is easily implemented, it emerges as an attractive technique to improve BER.

For a fixed FIR channel of 7th order, Fig. 2 reports the BER of each symbol in the information block for OFDM and P-OFDM with hopping pilot tones, respectively. It can be observed that the BER of the nth symbol of OFDM depends on the channel and the noise spectra. On the other hand, the BER of each entry of the information symbol for P-OFDM is relatively flat regardless of the channel and the noise color except for both ends of the block, because the channel effects are averaged *deterministically* over one block by $\boldsymbol{\Theta}$.

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