ROBUST OFDM TRANSMISSIONS OVER FREQUENCY-SELECTIVE CHANNELS WITH MULTIPLICATIVE TIME-SELECTIVE EFFECTS

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ABSTRACT

OFDM systems enable simple and effective schemes to mitigate frequency selective fading channels. However, they are extremely sensitive to multiplicative fluctuations induced by time-varying multipath delays, carrier offsets and oscillators' phase noise. Although the duration of OFDM symbols is chosen smaller than the channel coherence time to avoid, or reduce, channel time variations, this choice limits system efficiency. In this paper, we propose a generalized OFDM scheme capable of estimating and then compensating channel frequency selectivity and multiplicative noise effects using a deterministic method that exploits the redundancy added to the transmitted sequence in the form of pilot tones and null guard intervals in the frequency domain. The proposed method applies to channels modeled as the cascade of dispersive filters whose output is corrupted by both additive and multiplicative noise.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) systems have received considerable attention in the recent past as they provide simple and effective means of combating frequency selective fading. However, OFDM systems exhibit sensitivity to fast channel variations arising from Doppler effects, carrier offsets and oscillators' phase noise [7], [13]. OFDM systems usually operate with a symbol duration T_s smaller than the channel coherence time T_c to limit the effects of channel variations. However, this choice puts an upper bound on the system efficiency. With the cyclic prefix introduced to simplify the equalization procedure, transmission efficiency cannot exceed $\epsilon = M/(M + L_t)$, where M is the information block length and L_t is the prefix length. Hence, for a given L_t, the efficiency approaches one by increasing the value of M. Conversely, setting an upper bound on M is equivalent to limiting the maximum value of ϵ. To overcome this limitation, it is thus necessary to devise methods able to compensate for the unknown multiplicative noise effects are well motivated.

Statistical methods were proposed in [4] to counteract rapid channel variability in OFDM systems, assuming that the correlation function of the time-varying channel transfer function is separable. More recently, an equalization technique for OFDM transmissions over time-varying channels was proposed, assuming that the channel varies linearly in time [3]. Time and carrier synchronization in OFDM systems has been also extensively studied [11], [5]. In fact, carrier offset can be viewed as a special case of multiplicative complex exponential noise with zero support in the frequency domain. Estimation and compensation of multiplicative noise having a finite non-zero spectral support was proposed in [10], assuming that the channel is purely multiplicative and that the noise is strictly band limited. Generalizations to time and frequency dispersive channels were proposed in [12].

In this paper we extend the method of [10] to transmissions through frequency selective channels and in the presence of additive and bandlimited multiplicative noise. Furthermore, we analyze performance in the presence of multiplicative noise with non strictly bandlimited spectrum. Such a multiplicative noise model captures oscillators' phase noise, which is one of the main performance limiting factors in high carrier frequency systems [13]. Specifically, we propose a precoding strategy and a *deterministic* method for estimating and mitigating both dispersive channels and multiplicative noise. The method exploits the redundancy added to the transmitted sequence in the form of pilot tones and null guard intervals in the frequency domain (also termed virtual carriers in [8]) and is able to provide error free estimates for both dispersive channel and multiplicative noise, in high (ideally infinite) SNR.

2. PRECODING AND CHANNEL MODELING

The discrete-time baseband equivalent model of the overall system is depicted in Figure 1. The information symbol stream $s(n)$ is parsed into consecutive blocks of $NM - L_t$ symbols. Each block contains L_t unity symbols ($s(n) = 1$) and N blocks of L_f zero symbols ($s(n) = 0$) which are inserted according to the following rule: The all-zero blocks are equidistant from each other while the unity symbols are positioned in order to maximize their distance subject to the constraint that they do not fall within any block of zeros; we select L_t to be an over-estimate of the time-invariant channel order Q_t, and L_f to be an overestimate of the frequency support Q_f of the multiplicative noise power spectral density.

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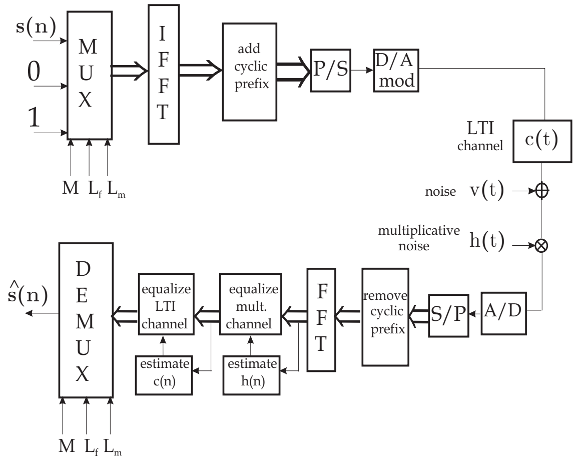

Figure 1: Proposed precoding and channel model.

The multiplexed data stream passes through a serial-to-parallel converter, whose output block feeds an inverse FFT (IFFT) module. The cyclic prefix of length L_t is then added to the IFFT output to facilitate channel equalization and avoid inter-block interference at the receiver, as in standard OFDM. In formulas, given a sequence of information symbols we insert the unity symbols to form $s(n)$ and express the p-th transmitted block as

$$u_p(n) = \sum_{i=0}^{N-1}\sum_{m=0}^{M-1} s_p(iM+m)e^{j2\pi\frac{(iP+m)n}{NP}}, \;\; n \in \; [-L_t, NP-1] \tag{1}$$

where $s_p(iM+m) := s(pMN+iM+m)$. Insertion of the unity symbols adds pilot tones whereas the zeros introduce null guard intervals in the frequency domain. Since in (1) n goes from $-L_t$ to $NP-1$, a cyclic prefix of length L_t is automatically included. Our precoding scheme differs from standard OFDM only in the insertion of null guard intervals in the *frequency* domain. As we will see later on, these frequency gaps are instrumental for estimating and compensating for the multiplicative noise.

Let $c(n)$ denote the discrete time impulse response of the LTI dispersive channel and $H(q)$ the spectrum of the multiplicative noise $h(n)$, evaluated on a grid of NP points, i.e., $H(q) := \sum_{n=0}^{NP-1} h(n) \exp{(j2\pi qn/NP)}$. We rely upon the following assumptions and design choices:

a1) $c(n) = 0$, for $n \notin [0, Q_t]$;
a2) $H(q) = 0$, for $q \notin [0, Q_f]$;
a3) $P = M + L_f$, with $L_f \geq Q_f$ and $L_t \geq Q_t$;
a4) The symbols' matrix $\boldsymbol{S}_p := [\boldsymbol{s}_{p,0}, \ldots, \boldsymbol{s}_{p,P-1}]$, with $\boldsymbol{s}_{p,k}{:=}(s_p(kM) \ldots s_p(kM+M-1))^T$, is full column rank.

Assumptions a1) and a2) are approximately valid for most communication channels. Nonetheless, we will also study the performance of the proposed method in situations where those assumptions are not exactly satisfied. Assumption a3) requires only the availability of an overestimate of the LTI channel time support and of the multiplicative noise frequency support. Both Q_t and Q_f are readily available from physical constraints such as for example maximum delay/Doppler and oscillators' bandwidth.

The precoded sequence $u_p(n)$ passes through $c(n)$ and is received in additive noise $w(n)$ and multiplicative noise $h(n)$. The baseband received sequence is thus

$$x_p(n) = y_p(n)h(n) + w(n), \quad n \; \in \; [0, NP-1], \tag{2}$$

where $y_p(n) = u_p(n){\star}c(n)$ and $\star$ denotes convolution. After removing the guard interval, $y_p(n)$ depends only on one block of symbols:

$$y_p(n) = \sum_{i=0}^{N-1}\sum_{m=0}^{M-1} s'_p(iM+m)e^{j2\pi(iP+m)n/NP} \;, \tag{3}$$

where $s'_p(iM+m) := s_p(iM+m)C((iP+m)/NP)$ and $C(f)$ is the dispersive channel transfer function: $C(f) := \sum_{l=0}^{Q_t} c(l)e^{j2\pi fl}$. Using (2) and (3), the NP-point FFT of the received sequence $x_p(n)$ is

$$X_p(l) = \frac{1}{NP}\sum_{i=0}^{N-1}\sum_{m=0}^{M-1} s'_p(iM{+}m)H(l{-}iP{-}m){+}W_p(l), \tag{4}$$

where $v = 0, \ldots, P-1$, $k = 0, \ldots, P-1$ and $W_p(l)$ is the FFT of the additive noise. Setting $l = kP+v$, with $v = 0, \ldots, P-1$, we also have

$$X_p(kP{+}v) = \frac{1}{NP}\sum_{i=0}^{N-1}\sum_{m=0}^{M-1} s'_p(iM+m)H((k-i)P+v{-}m) + W_p(kP+v). \tag{5}$$

Because of a2), the term $H((k-i)P+v-m) \neq 0$ for $i = k$, so that (5) simplifies into

$$X_p(kP+v) = \frac{1}{NP}\sum_{m=0}^{M-1} s'_p(kM{+}m)H(v-m) + W_p(kP+v). \tag{6}$$

For each k, the $P \times 1$ vector $\boldsymbol{X}_{p,k} := (X_p(kP), \ldots, X_p(kP+P-1))^T$ is given by the convolution of the $M \times 1$ vector $\boldsymbol{s}'_{(p,k)}{:=}(s'_p(kM), \ldots, s'_p(kM+M-1))^T$ with the vector $\boldsymbol{h} := (H(0), \ldots, H(Q_f))^T$. Because of a1)÷ a4), after removing the time guard interval, there is no superposition among different blocks. Hence, we can rewrite (6) in a more compact matrix form as

$$\boldsymbol{X}_p := [\boldsymbol{X}_{p,0}, \ldots, \boldsymbol{X}_{p,P-1}] = \boldsymbol{H}\boldsymbol{S}'_p + \boldsymbol{W}_p, \tag{7}$$

where $\boldsymbol{S}'_p := [\boldsymbol{s}'_{p,0}, \ldots, \boldsymbol{s}'_{p,P-1}]$, $\boldsymbol{W}_p := [\boldsymbol{W}_{p,0}, \ldots, \boldsymbol{W}_{p,P-1}]$ and $\boldsymbol{H}$ is the $P{\times}M$ Toeplitz matrix whose first row and column are $(H(0), 0, \ldots, 0)$ and $(H(0), \ldots, H(Q_f), 0, \ldots, 0)^T$, respectively.

From (7) we can spot the two main distortions arising to OFDM transmissions through the channel of Fig. 1: the convolutive channel introduces multiplication of the information symbols $s(n)$ by the values of the channel transfer function on the corresponding sub-carrier frequencies, whereas the multiplicative channel gives rise to convolution of each symbol block with the vector $\boldsymbol{h}$, which causes ISI. The extra non-zero pilot tones do not facilitate estimating or removing the ISI effect, for in both the time and the frequency domain the data are multiplied by an unknown modulating sequence and are convolved with an unknown channel. Both effects render maximum likelihood estimation a rather complicated non-linear least-squares task, despite the number of training bits one could use. The advantage of zero-guard precoding and the resulting estimation

method that we describe in Section 3 is twofold:
i) it requires a minimal persistence of excitation assumption on $\boldsymbol{S}'_p$, allowing to estimate the multiplicative channel first and the multipath channel next, after $\boldsymbol{H}$ is equalized, with a linear method; ii) it enables multiplicative noise equalization also in the presence of deep fading in time.

3. CHANNEL ESTIMATION/COMPENSATION

A deterministic method for blind estimation and zero-forcing equalization of FIR time-invariant channels was proposed in [9], based on linear block precoding using time domain null guard intervals. The channel estimation method was proved to be asymptotically efficient, in the sense that at high SNR the covariance of the channel estimator reaches the Cramér-Rao bound (CRB), in the presence of additive white Gaussian noise [1]. Since the mathematical formulation in [9] is equivalent to (7), we can use the method of [9] here to estimate the matrix $\boldsymbol{H}$. We recall that the method of [9] is capable of providing ideally error-free estimates in the absence of noise, using a *finite* number of samples. Furthermore, since the FFT of the additive noise samples leaves the additive noise white and Gaussian, extending [1] we infer that if we treat $h(n)$ as deterministic, we can estimate it with accuracy tending to the CRB as the SNR increases.

For convenience, we review briefly the method of [9]. In the additive noise-free case, i.e., $\boldsymbol{W}_p = \boldsymbol{0}$, the $P \times M$ matrix $\boldsymbol{X}_p$ has rank M and thus nullity $L_f = P - M$, because $\boldsymbol{H}$ is full rank by construction and, by virtue of a4), $\boldsymbol{S}'_p$ is also full rank. Taking the SVD of the observed matrix $\boldsymbol{X}_p$, we compute the L_f vectors $\tilde{\boldsymbol{u}}_l$, $l = 1, \ldots, L_f$, as the left singular vectors associated to the L_f smallest singular values of $\boldsymbol{X}_p$. Since $\boldsymbol{S}'_p$ is full rank, the vectors $\tilde{\boldsymbol{u}}_l$ span also the null space of $\boldsymbol{H}$. Hence, we can write $\tilde{\boldsymbol{u}}_l \boldsymbol{H} = \boldsymbol{0}^H$, $l = 1, \ldots, L_f$. But since $\boldsymbol{H}$ is Toeplitz, we have equivalently that

$$\tilde{\boldsymbol{h}}^H \tilde{\mathcal{U}} := \boldsymbol{h}^H [\tilde{\mathcal{U}}_1, \ldots, \tilde{\mathcal{U}}_{L_f}] = \boldsymbol{0}^T, \tag{8}$$

where $\tilde{\mathcal{U}}_k$ is the $(L_f + 1) \times M$ Hankel matrix having first column $(\tilde{u}_k(1), \ldots, \tilde{u}_k(L_f + 1))$ and last row $(\tilde{u}_k(L_f + 1), \ldots, \tilde{u}_k(P))$. Under a1)$\div$ a4), the nullity of $\tilde{\mathcal{U}}$ is one [9], so that $\boldsymbol{h}$ is obtained as the unique solution (within a scale) of (8). The overall algorithm equalizing frequency and time-selective effects can be thus summarized in the following steps:

s1. Build matrix $\boldsymbol{X}_p$, as in (7);
s2. Compute the L_f left singular vectors $\tilde{\boldsymbol{u}}_k$, $k = 1, \ldots, L_f$ associated to the L_f smallest singular values of $\boldsymbol{X}_p$;
s3. Compute $\tilde{\boldsymbol{h}}$ by solving (8);
s4. Build the Toeplitz matrix $\tilde{\boldsymbol{H}} = \mathcal{T}(\tilde{\boldsymbol{h}})$ and compensate for the multiplicative noise by taking $\tilde{\boldsymbol{S}}'_p = \tilde{\boldsymbol{H}}^\dagger \boldsymbol{X}_p$, where $\dagger$ denotes pseudo-inverse;
s5. Estimate $c(n)$ using the pilot tones present in $\tilde{\boldsymbol{S}}'_p$;
s6. Compensate for $c(n)$ using a one-tap equalizer on each sub-carrier.

Various simplifications are possible. For example, instead of s4, we can divide the received sequence, before the FFT, by the channel estimate $\tilde{h}(n)$. Furthermore, the pilot tones and steps s5-s6 could be eliminated by resorting to differential

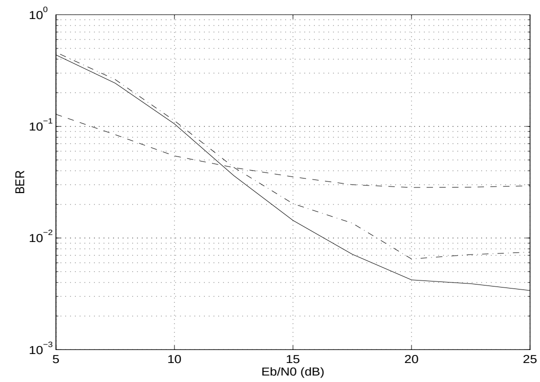

Figure 2: BER vs. E_b/N_0 corresponding to methods A (solid line), B (dashed and dotted line), and C (dashed line), using the same number of samples.

encoding, even though this would come at the price of extra SNR to achieve the same performance.

4. PERFORMANCE AND CONCLUSION

The method proposed in the previous section assumes that the multiplicative noise is strictly bandlimited. However, in some applications this assumption is only approximately valid. An important example is phase noise that arises due to the nonzero bandwidth of the oscillators' loop filters [2]. Since phase noise is one of the major performance limiting factors for OFDM systems [13], we have tested our method in the presence of Wiener phase noise generated as $h(t) = \exp(j\beta\phi(t))$, where $\phi(t) := \int_0^t \nu(\tau)d\tau$, β is the frequency modulation index and $\nu(\tau)$ is a zero mean Gaussian process with autocorrelation function $R_\nu(\tau)$ [13]. The noise power spectral density is then approximately Lorentzian at high frequencies [6]

$$S_{hh}(\omega) \simeq \frac{2\rho\tau_c\beta^2}{\omega^2 + \rho^2\tau_c^2\beta^4}, \tag{9}$$

where τ_c is the maximum correlation lag of $\nu(t)$ and $\rho = R_\nu(0)$. Indeed, this kind of noise is quite troublesome for our method because its spectrum has heavy tails. At low frequencies, model (9) is rather pessimistic because the phase noise spectrum of practical oscillators usually decays faster than $1/\omega^2$ at low ω [2]. In Fig. 2 we report the BER when transmitting over a multipath channel composed of 6 independent Rayleigh fading paths. The results shown in Fig. 2 correspond to the following methods: A) the phase noise is estimated blindly and equalized by multiplying the vector $\boldsymbol{X}_p$ by the pseudo-inverse of $\boldsymbol{H}$; B) after estimating of $\boldsymbol{H}$, the phase noise $\tilde{h}(n)$ is reconstructed and the received sequence, before FFT, is divided by $\tilde{h}(n)$; C) plain OFDM with pilot tones and guard time interval. After compensation of the phase noise, all the methods equalize the LTI channel, by estimating the channel frequency response based on pilot tones and then applying a simple zero-forcing one-tap equalizer. The parameters are chosen as follows: $M = 32$, $L_t = 6$, $L_f = 8$. The BER is estimated by averaging over 800 independent Rayleigh fading channels and

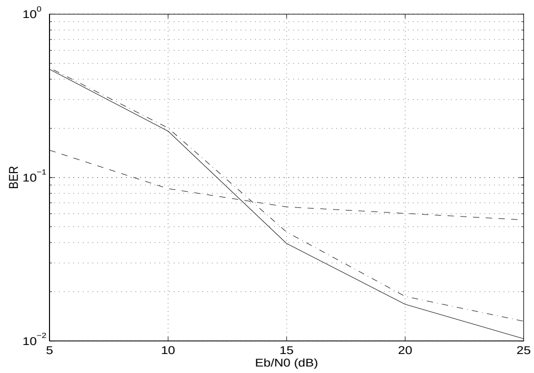

Figure 3: BER vs. E_b/N_0 corresponding to methods A (solid line), B (dashed and dotted line), and C (dashed line), under the same efficiency condition.

phase noise realizations (the number of BPSK symbols in each iteration is 10,240). We can clearly observe that our method is able to partially compensate for the phase noise, despite the non finite support of the noise spectrum. Better compensation requires larger gap intervals in the frequency domain, so that the finite support assumption of the multiplicative noise spectrum is better met; but this would come at the expense of extra redundancy.

In Fig. 2, both plain OFDM and our OFDM system operate with the same number of samples. Instead of fixing the number of samples, it is also interesting to compare the two systems at the same efficiency. Because of the null guard intervals in the frequency domain, the same efficiency can be achieved only by transmitting blocks of different size. Specifically, the efficiency of OFDM with pilot tones and time guard interval is

$$\epsilon_{OFDM} = \frac{M}{M + 2L_t}, \tag{10}$$

whereas the efficiency for our system is

$$\epsilon_{our} = \frac{PM}{P(M + L_f) + 2L_t}. \tag{11}$$

In Fig. 3 we compare the two systems, after setting $\epsilon_{OFDM} = \epsilon_{our}$ and choosing the parameter M of the standard OFDM system accordingly. It is important to see that also in this case there is an advantage in using the proposed approach.

In a nutshell, considering that null guard intervals in the frequency domain are sometimes introduced in OFDM systems for frequency synchronization (which is one of the most critical subjects for OFDM) in this paper we have shown that null guard intervals of appropriate dimension and number can also be used to compensate for multiplicative noise and time-selective effects whose spectrum has a non zero frequency support.

5. REFERENCES

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