REDUCED COMPLEXITY EQUALIZERS FOR ZERO-PADDED OFDM TRANSMISSIONS

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ABSTRACT

The widespread application of OFDM for local area mobile wireless broadband systems is strongly motivated by the simple equalization it affords. OFDM's bottleneck of channel-dependent performance was only recently addressed by replacing the cyclic prefix (CP) with trailing zeros (TZ). Such zero-padded OFDM transmissions enable FIR channel-irrespective symbol recovery which results in significant BER gains. The price paid is increased receiver complexity compared to the classical CP-OFDM. To reduce complexity, this paper proposes two novel equalizers for TZ-OFDM. One is based on an overlap and add approach and exhibits exactly the same performance and complexity as CP-OFDM. The second offers BER performance close to the TZ-OFDM MMSE equalizer and incurs a moderate complexity increase. An extension of an existing CP-OFDM pilot-based channel estimation method is also derived for the proposed equalization schemes. Simulations are conducted in a realistic HiperLAN/2 scenario comparing the various OFDM transceivers.

1. INTRODUCTION

Though unnoticed for some time, there has been an increasing interest towards multicarrier and in particular Orthogonal Frequency Division Multiplexing (OFDM), not only for digital audio- and video-broadcasting (DAB and DVB) but also for high-speed modems over Digital Subscriber Lines (xDSL), and more recently for small area mobile wireless broadband systems (ETSI BRAN HiperLAN/2 similar to IEEE802.11a; see e.g., [3, 1] and references therein).

OFDM entails block redundant transmissions and enables very simple (FFT-based) equalization of Lth-order frequency-selective FIR channels, thanks to the IFFT precoding and the insertion of the so called Cyclic Prefix (CP) at the transmitter. Present in each block of size P, the CP consists of L redundant symbols preceding (and circularly replicated from) the $P-L=M$ IFFT-precoded non-redundant symbols. At the receiver end, CP is discarded to avoid interblock interference (IBI) and each truncated block is FFT processed – an operation converting the frequency-selective channel output into M parallel flat-faded independent subchannel outputs, each corresponding to a different subcarrier. Unless zero, flat fades are removed by dividing each subchannel output with the channel transfer function at the corresponding subcarrier. At the expense of bandwidth over-expansion, coded-OFDM ameliorates (but does not eliminate) performance losses incurred by channels having nulls on (or close to) the transmitted subcarriers. Recently, it was proposed to replace the generally non-zero CP by Trailing Zeros (TZ) [4, 7]. Specifically, in each P-long block of the so termed TZ-OFDM transmission, L all-zero symbols are appended after the $M=P-L$ IFFT-precoded information symbols. Unlike CP-OFDM and without bandwidth consuming channel coding, TZ-OFDM guarantees symbol recovery and assures FIR (even zero-forcing) equalization of FIR channels *regardless* of channel zero locations. The price paid is somewhat increased receiver complexity (the single FFT required by CP-OFDM is replaced by FIR filtering) [4, 7].

In this paper we take a closer look at TZ-OFDM and propose two novel equalizers that enable trading off BER performance for extra savings in complexity (Section 2). The simplest one (TZ-OFDM-OLA) has computational complexity equivalent to CP-OFDM, but similar to CP-OFDM it also suffers from channel-dependent performance. The second equalizer (TZ-OFDM-FAST) is slightly more complex than CP-OFDM, but similar to TZ-OFDM it guarantees symbol recovery and offers BER performance close to TZ-OFDM-MMSE.

Because linear equalizers require channel status information (CSI), we also develop here a channel estimator for zero-padded OFDM transmissions (Section 3). It extends the pilot-based channel estimator developed in [6] for CP-OFDM to the TZ-OFDM transmission format.

2. SYMBOL RECOVERY AND COMPLEXITY

Figure 1 depicts the baseband discrete-time block equivalent model of a TZ-OFDM system. The $M \times 1$ input digital vector[1] $\mathbf{s}_M(i)$ is first modulated by the IFFT matrix $\mathbf{F}_M^{\mathcal{H}}$ with entries $M^{-1/2}\exp\{j2\pi mk/M\}$. Then L trailing zeroes are padded at the end of the resulting vector $\tilde{\mathbf{s}}_M(i)$. The corresponding $P \times 1$ transmitted vector $\tilde{\mathbf{s}}_{TZ}(i) = \mathbf{F}_{TZ}^{\mathcal{H}}\mathbf{s}_M(i)$,

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[1]Lower (upper) boldface symbols will be used for column vectors (matrices) sometimes with subscripts M or P emphasizing their sizes (for square matrices only); tilde will denote IFFT precoded quantities; subscripts m and k will stand for uncoded and precoded quantities, respectively; argument i will be used to index blocks of symbols; $^{\mathcal{H}}$ (^{T}) will denote Hermitian (transpose).

where $\mathbf{F}_{TZ}^{\mathcal{H}} = [\mathbf{F}_M \; \mathbf{0}]^{\mathcal{H}}$, is then serialized and transmitted through the Lth-order FIR channel with impulse response $h_l = 0, \forall l \notin [0, L]$. The all-zero $L \times M$ matrix $\mathbf{0}$ eliminates IBI. Let $\mathbf{H}{:=}[\mathbf{H}_o \, , \mathbf{H}_{TZ}]$ denote a partition of the $P \times P$ convolution matrix $(\mathbf{H})_{ij} = h_{i-j}$ between its first M and last L columns. The received noise-free $P \times 1$ vector is then:

$$\tilde{\mathbf{x}}_P(i) = \mathbf{H}\mathbf{F}_{TZ}^{\mathcal{H}}\mathbf{s}_M(i) = \mathbf{H}_o\mathbf{F}_M^{\mathcal{H}}\mathbf{s}_M(i).$$

Corresponding to the first M columns of $\mathbf{H}$, the $P \times M$ matrix $\mathbf{H}_o$ is Toeplitz and is always guaranteed to be invertible, which enables symbol recovery regardless of the channel zero locations. The corresponding ZF and MMSE equalizers are given by [7]: $\mathbf{G}_{zf} = \mathbf{F}_M\mathbf{H}_o^{\dagger}$ and $\mathbf{G}_{mmse} = \mathbf{F}_M\mathbf{H}_o^{\mathcal{H}}(\sigma_n^2\mathbf{I} + \mathbf{H}_o\mathbf{H}_o^{\mathcal{H}})^{-1}$, where σ_n^2 denotes the AWN variance and the symbols are assumed w.l.o.g. to have variance $\sigma_s^2 = 1$.

In addition, one can readily verify that $\mathbf{F}_P\mathbf{H}_o\mathbf{F}_M^{\mathcal{H}} = \mathbf{D}_P(H)\mathbf{V}$, where $\mathbf{F}_P$ is the $P \times P$ FFT matrix with entries $P^{-1/2}\exp\{-j2\pi mk/P\}$, $\mathbf{D}_P(H)$ is a $P \times P$ diagonal matrix with $(p+1, p+1)$st entry $H(2\pi p/P){:=}\sum_{l=0}^{L} h_l \exp(-j2\pi lp/P)$, and $\mathbf{V}$ is a known $P \times M$ structured matrix with (p, m)th entry[2]:

$$[\mathbf{V}]_{p,m} = \begin{cases} \dfrac{1}{\sqrt{PM}} \dfrac{1 - e^{j2\pi(m-pM/P)}}{1 - e^{j2\pi(m/M-p/P)}}, & \frac{m}{M} - \frac{p}{P} \neq 0, p \neq 0, \\ \sqrt{M/P}, & \frac{m}{M} - \frac{p}{P} = 0, \\ 0, & p \neq 0, m = 0. \end{cases} \quad (1)$$

Implementing the multiplication $\mathbf{F}_P\tilde{\mathbf{x}}_P(i) := \mathbf{x}_P(i)$ with a P-point FFT, the TZ-OFDM receiver output yields:

$$\begin{aligned} \mathbf{x}_P(i) &= \mathbf{F}_P\mathbf{H}_o\mathbf{F}_M^{\mathcal{H}}\mathbf{s}_M(i) = \mathbf{D}_P(H)\mathbf{V}\mathbf{s}_M(i) \\ &\Rightarrow \hat{\mathbf{s}}_M(i) = (\mathbf{D}_P(H)\mathbf{V})^{\dagger}\mathbf{x}_P(i), \end{aligned} \quad (2)$$

where the estimated $\hat{\mathbf{s}}_M(i)$ equals $\mathbf{s}_M(i)$ only in the absence of noise.

2.1. TZ-OFDM-FAST

Because $H(z)$ is of order L, $\mathbf{D}_P(H)$ can have at most L zero-diagonal entries. But unlike CP-OFDM, the remaining (at least $P - L = M$ non-zero) entries guarantee recovery of $\mathbf{s}_M(i)$ in TZ-OFDM regardless of the underlying Lth-order FIR channel nulls, as any M rows of matrix $\mathbf{V}$ form a full rank matrix. Equation (2) requires computing the pseudo-inverse of a $P \times M$ matrix in general. Targeting lower complexity equalizers, we pursue two options:

Option 1: (TZ-OFDM-FAST-ZF) Suppose none of the L channel roots is located on the P-point FFT grid $\{\exp(-j2\pi k/P)\}_{k=0}^{P-1}$. Matrix $\mathbf{D}_P(H)$ then has full rank P. In this case,

$$\hat{\mathbf{s}}_M(i) = \mathbf{V}^{\dagger}\mathbf{D}_P^{\dagger}(H)\mathbf{x}_P(i), \quad (3)$$

and we can form our equalizer in two steps after the P-point FFT $\mathbf{F}_P$ is applied to $\tilde{\mathbf{x}}_P(i)$: first, we obtain an estimate of $\mathbf{y}_P(i){:=}\mathbf{V}\mathbf{s}_M(i)$ as $\hat{\mathbf{y}}_P(i) = \mathbf{D}_P^{\dagger}(H)\mathbf{x}_P(i)$; and then we find $\hat{\mathbf{s}}_M(i) = \mathbf{V}^{\dagger}\mathbf{y}_P(i)$. Because $\mathbf{V}$ is not channel-dependent, its pseudo-inverse $\mathbf{V}^{\dagger}$ needs to be computed only once, while in the operational mode we simply need to invert the diagonal $\mathbf{D}_P(H)$ instead of the pseudo-inverse in (2). We term the ZF equalization based on (3) the TZ-OFDM-FAST-ZF algorithm. This scheme would fail if the channel has a zero at one of the P-point FFT frequencies because $(\mathbf{D}_P(H)\mathbf{V})^{\dagger} \neq \mathbf{V}^{\dagger}\mathbf{D}_P^{\dagger}(H)$ and (3) does not hold. Even when a zero is close to the P-point FFT grid, performance degrades because noise is enhanced in the first step.

Option 2: (TZ-OFDM-FAST-MMSE) To obtain a low cost equalizer that mitigates the noise enhancement problem, one could replace the first step in the TZ-OFDM-FAST-ZF by the MMSE estimator of $\mathbf{y}_P(i)$:

$$\begin{aligned} \hat{\mathbf{y}}_P(i) &= \mathrm{E}[\mathbf{y}_P(i)\mathbf{x}^{\mathcal{H}}(i)] \cdot \mathrm{E}^{-1}[\mathbf{x}_P(i)\mathbf{x}^{\mathcal{H}}(i)] \\ &= \mathbf{V}\mathbf{V}^{\mathcal{H}}\mathbf{D}_P^{\mathcal{H}}(\sigma_n^2\mathbf{I} + \mathbf{D}_P\mathbf{V}\mathbf{V}^{\mathcal{H}}\mathbf{D}_P^{\mathcal{H}})^{-1}. \end{aligned} \quad (4)$$

As an approximation, we take $\mathbf{V}\mathbf{V}^{\mathcal{H}} \approx \mathbf{I}$ and (4) simplifies to $\hat{\mathbf{y}}_P(i) = \mathbf{D}_P(\sigma_n^2\mathbf{I} + \mathbf{D}_P\mathbf{D}_P^{\mathcal{H}})^{-1}$, which involves inversion of a diagonal matrix only. We term this equalizer TZ-OFDM-FAST-MMSE. At this point, we wish to underscore that both options are computationally fast but in general they do not implement the minimum-norm solution in (2).

2.2. TZ-OFDM-OLA

At the expense of channel-irrespective invertibility, one may pursue an alternative option at the receiver, which we term TZ-OFDM-OLA because it originates from the overlap-add (OLA) method for block convolution (see also Figure 2). Specifically, we can split $\tilde{\mathbf{x}}_P(i)$ into its upper $M \times 1$ part, $\tilde{\mathbf{x}}_u(i) = \mathbf{H}_u\tilde{\mathbf{s}}_M(i)$, and its lower $L \times 1$ part $\tilde{\mathbf{s}}_l(i) = \mathbf{H}_l\tilde{\mathbf{s}}_M(i)$, where $\mathbf{H}_u$ ($\mathbf{H}_l$) denotes the corresponding $M \times M$ ($L \times M$) partition of $\mathbf{H}_o$. Padding $M - L$ zeros in $\tilde{\mathbf{x}}_l(i)$ and adding the resulting vector to $\tilde{\mathbf{x}}_u(i)$ we can form:

$$\begin{aligned} \overline{\mathbf{x}}_M(i) &{:=} \tilde{\mathbf{x}}_u(i) + \begin{bmatrix} \tilde{\mathbf{x}}_l(i) \\ \mathbf{0}_{(M-L)\times 1} \end{bmatrix} \\ &= \left(\mathbf{H}_u + \begin{bmatrix} \mathbf{H}_l \\ \mathbf{0}_{(M-L)\times M} \end{bmatrix} \right) \tilde{\mathbf{s}}_M(i) {:=} \mathbf{H}_c\tilde{\mathbf{s}}_M(i). \end{aligned} \quad (5)$$

Matrix $\mathbf{H}_u$ is lower-triangular with (i, j)th entry h_{i-j} and $\mathbf{H}_l$ is upper triangular Toeplitz with (i, j)th entry h_{i-j+M}. Hence, the $M \times M$ sub-matrix $\mathbf{H}_c$ in (5) has (i, j)th entry $h_{i-j} + h_{i-j+M}$, which verifies that it is circulant and can thus be diagonalized by $M \times M$ (I)FFT matrices; i.e.,

$$\begin{aligned} \mathbf{F}_M\overline{\mathbf{x}}_M(i) &= \mathbf{F}_M\mathbf{H}_c\mathbf{F}_M^{\mathcal{H}}\mathbf{s}_M(i) = \mathbf{D}_M(H)\mathbf{s}_M(i) \\ &\Rightarrow \mathbf{s}_M(i) = \mathbf{D}_M^{\dagger}(H)\mathbf{F}_M\overline{\mathbf{x}}_M(i), \end{aligned} \quad (6)$$

where $\mathbf{D}_M(H)$ is a $M \times M$ diagonal matrix with $(m+1, m+1)$st entry $H(2\pi p/M){:=}\sum_{l=0}^{L} h_l \exp(-j2\pi l/M)$.

¿From (6), we see that TZ-OFDM-OLA is not only equivalent to CP-OFDM in the overall $M \times M$ transceiver transfer function $\mathbf{D}_M(H)$, but has also identical complexity (two M-point FFTs are involved). However, the complexity penalty paid by TZ-OFDM is precisely what equips it with FIR channel-irrespective symbol recovery [4, 7].

[2] We use $[\cdot]_{p,m}$ to denote the (p, m)th entry of a matrix and $[\cdot]_p$ to denote the pth entry of a vector.

3. PILOT-BASED CSI FOR TZ-OFDM

Channel estimation in CP-OFDM is performed in the frequency domain using pilot-symbols [6]. The channel transfer function $H(f)$ at each subcarrier $f_m = 2\pi m/M$ can be estimated from the noisy CP-OFDM symbols:

$$\mathbf{r}_M(i) := \mathbf{x}_M(i) + \mathbf{n}_M(i) = \mathbf{D}_M(H)\mathbf{s}_M(i) + \mathbf{n}_M(i),$$

by simply dividing them by the pilot data:

$$\hat{H}^{(i)}(2\pi m/M) = [\mathbf{r}_M(i)]_m \,/\, [\mathbf{s}_M(i)]_m,\ m \in [0, M-1]. \quad (7)$$

Since TZ-OFDM-OLA is equivalent to the classical CP-OFDM, (7) applies directly to TZ-OFDM when one acquires CSI from the OLA receiver. Our simulations have confirmed that for a given SNR, the channel estimation accuracy with CP-OFDM and with TZ-OFDM-OLA are similar and their BER performance is thus comparable. However, when the channel's delay-spread is longer than the CP, TZ-OFDM-FAST-MMSE exhibits improved BER performance over TZ-OFDM-OLA.

Because the TZ-OFDM-FAST variants operate with the P-point FFT of the channel frequency response, they entail an extra M-point IFFT and a P-point FFT to retrieve $H(2\pi p/P)$ from $H(2\pi m/M)$. However, a more direct channel estimator for TZ-OFDM follows from (2). Indeed, with $\mathbf{y}_P(i) := \mathbf{V}\mathbf{s}_M(i)$, a channel estimate based on the ith-block can be found as [c.f. (2)]

$$\hat{H}^{(i)}(2\pi p/P) = [\mathbf{r}_P(i)]_p \,/\, [\mathbf{y}_P(i)]_p, \quad p \in [0, P-1]. \quad (8)$$

For noise robustness, the pilot symbols in $\mathbf{y}_P(i)$, and hence in $\mathbf{s}_M(i)$, need to be designed accurately [8]. A possible choice minimizing the MSE of $\hat{H}^{(i)}(2\pi p/P)$ in (8) is to send the same pilot symbol on all subcarriers. The resulting MMSE for a channel with $||\mathbf{h}||^2 = 1$ is then given by: $\epsilon_P^2 = E[(\mathbf{h} - \hat{\mathbf{h}})^{\mathcal{H}}(\mathbf{h} - \hat{\mathbf{h}})] = \sigma_n^2 P/M$, and is equal to the MMSE ϵ_M^2 of the channel estimated using the OLA structure. Furthermore, it is possible for both (7) and (8) to improve the $\hat{H}^{(i)}(2\pi m/M)$ and $\hat{H}^{(i)}(2\pi p/P)$ estimates by taking advantage of the fact that the channel is FIR of order $\approx L$ [9]. This can be achieved by applying an IFFT to $\hat{H}^{(i)}$ for removing the spurious taps located after the CP, before switching back to the frequency domain. The MMSE of the resulting M- and P-sampled channel estimates for the TZ-OFDM-OLA and -FAST, respectively, turns out to be: $\epsilon_M^2 = \sigma_n^2 LP/M^2$ and $\epsilon_P^2 = \sigma_n^2 L/M = \epsilon_M^2 M/P < \epsilon_M^2$. Thus, for $L = M/4$, the fast equalizers for TZ-OFDM gain $10\log_{10}(P/M) = 0.96$ dB compared to the pilot-based estimation method for CP-OFDM [6].

4. SIMULATIONS AND DISCUSSION

This section compares the equalizers of this paper and the corresponding TZ-OFDM performance with the classical CP-OFDM in the practical context of the HiperLAN/2 (HL2) broadband wireless communication standard. HL2 is a multicarrier systems operating over 20MHz in the 5GHz band. The number of carriers is $M = 64$ and the TZ/CP length is $L = 16$ resulting in transmitted blocks of $P = 80$ symbols. BER curves are based on Monte Carlo simulations, with each trial corresponding to a different realization of the typical 5GHz wireless Channel Models A and E specified by HL2 [2]. The channel is assumed to be unknown and is estimated at the beginning of each frame using either the improved channel estimation method of Section 3 for TZ-OFDM, or, the one in [6] for CP-OFDM. The frame duration is 100 OFDM symbol-blocks ($i \in [1, 100]$) and the channel is assumed to be constant over the frame. In order to account for clipping effects arising due to nonlinear power amplification, the input powers of CP-OFDM and TZ-OFDM are set identical, which implies the same clipping thresholds. This results in a smaller operating SNR at the receiver input for TZ-OFDM than for CP-OFDM which explains the BER difference between CP-OFDM and TZ-OFDM-OLA curves.

Figures 4 and 5 plot the uncoded BERs for channels A (fair channel) and E (difficult channel) as a function of the symbol SNR E_s/N_0 for a QPSK modulation. We infer that the guaranteed symbol recovery of the TZ precoder leads to significant performance gains of about 5dB for 10^{-3} BER when using the TZ-OFDM-MMSE equalizer. With our reduced complexity TZ-OFDM-FAST-MMSE equalizer, the guaranteed symbol recovery still affords a significant gain ($\approx$3dB for 10^{-3} BER). It can also be seen that the improvement is pronounced for a channel with long delay-spread (channel E) since the probability for a channel zero to be located on a subcarrier increases with the channel order.

In a nutshell, we have demonstrated that the TZ-OFDM-FAST-MMSE equalizer of this paper outperforms the classical CP-OFDM with complexity lower than the TZ-OFDM-MMSE equalizer of [7]. With the fast equalizers developed herein, we have further evinced the superiority of TZ-OFDM over CP-OFDM in the following facets:

i) channel-irrespective linear equalizability and guaranteed symbol recovery [4, 7];

ii) flexibility in pursuing complexity-scalable TZ-OFDM variants such as OLA/FAST/MMSE combinations;

iii) pilot-based channel estimation with improved tracking capability of channel variations (see also [7, 5]).

The subjects deserving further exploration are: i) how TZ-OFDM (and its fast variants proposed herein) compares with CP-OFDM when it comes to clipping effects induced by high-power nonlinear amplification, and ii) how to ensure efficient time and frequency synchronization for TZ-OFDM.

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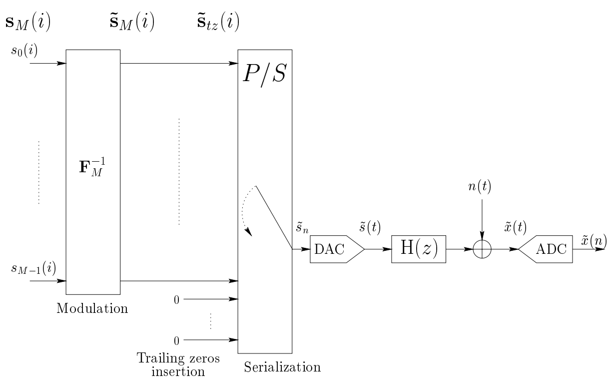

Figure 1: Discrete model of the TZ precoder

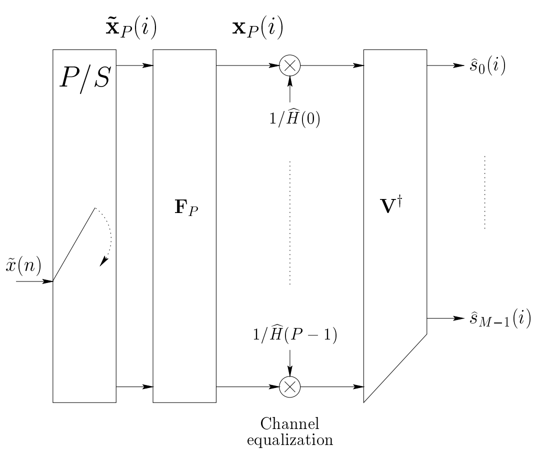

Figure 2: TZ-OFDM-FAST Equalizer

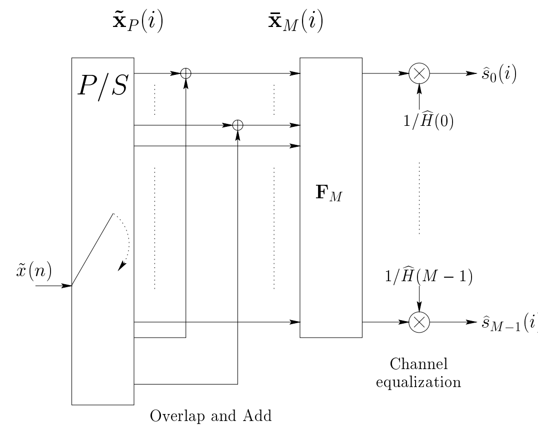

Figure 3: OLA equalization scheme

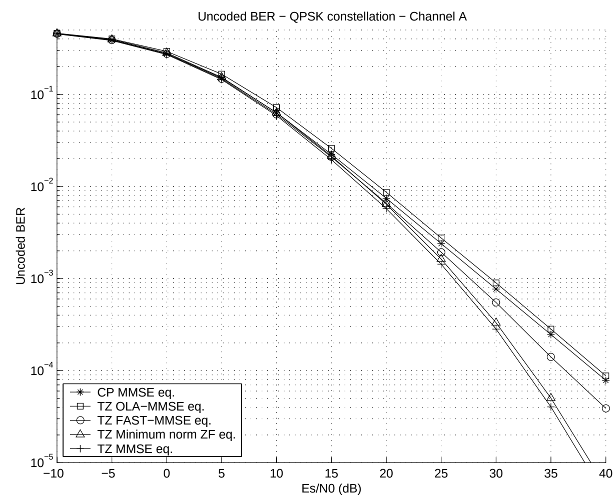

Figure 4: BER for the HiperLAN/2 channel model A

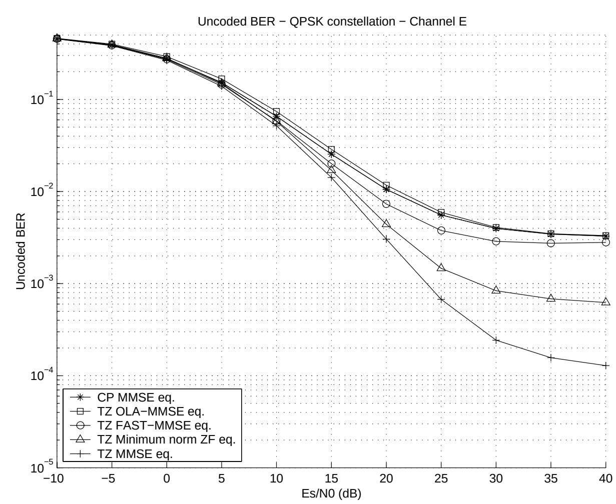

Figure 5: BER for the HiperLAN/2 channel model E