FILTERBANK TRANSCEIVERS OPTIMIZING INFORMATION RATE IN BLOCK TRANSMISSIONS OVER DISPERSIVE CHANNELS

Anna Scaglione[1], *Sergio Barbarossa*[1], *Georgios B. Giannakis*[2]

[1]Infocom Dept., Univ. of Rome "La Sapienza", Roma, ITALY {annas, sergio}@infocom.ing.uniroma1.it

[2] Dept. of Electrical Engr., Univ. of Virginia, Charlottesville, VA 22903-2442, USA e-mail: georgios@virginia.edu.

Abstract: Optimal FIR transmit and receive filterbanks are derived for block based data transmissions over frequency-selective AGN channels by maximizing mutual information subject to a fixed transmit-power constraint. The inherent flexibility of the proposed transceivers is exploited to derive as special cases, zero-forcing and minimum mean-square error solutions. The potential of the proposed scheme is illustrated and compared to Discrete Multi-Tone (DMT).

1. Introduction and Problem Statement

Block transmission is commonly used for communicating over dispersive channels affected by intersymbol interference (ISI). The transmitted data stream is parsed into consecutive equal-size blocks and redundancy is added to each block in order to remove inter-block interference and devise simple and effective schemes for canceling the ISI. Examples of block transmissions include orthogonal frequency division multiplexing (OFDM) or coded-OFDM (COFDM) [16], used for audio and video digital terrestrial broadcasting in Europe, and discrete multitone (DMT) transceivers [2], adopted for high bit rate digital subscriber loop (HDSL) and asymmetric digital subscriber loop (ADSL). When the channel is known at the transmitter side via feedback channels, as in HDSL/ADSL applications, the input data stream can be precoded in order to optimize the system performance [3]. Multirate precoding for ISI cancellation using filterbanks was proposed in [15]. Indeed, precoding is an old idea dating back to the early works of Tomlinson and Harashima [14], [6]. However, the main objective in high bit rate transmissions is maximization of the mutual information between transmitter and receiver, for given performance specifications and limited resources – tasks entailing more than simple mitigation of ISI. Optimality in the sense of maximizing mutual information was proved theoretically for ideal decision feedback equalizers (DFE) in [10], assuming PAM signaling, error-free decisions, and infinite-length feedforward equalizers. An alternative approach is the so called vector coding (VC), that utilizes a bank of filters whose impulse responses are the eigenvectors of an appropriately defined channel matrix [8]. Optimality can be reached with these systems asymptotically, as block sizes tend to infinity, by proper allocation of power and number of bits along the subchannels [7]. In spite of the undoubtful interest of asymptotics, it is practically important, to derive systems leading to the maximum mutual information, for finite size block transmissions. Relatively small size blocks are highly desirable to avoid excessive decoding delays, storage requirements, and computational load. The optimal scheme for finite block-lengths may significantly differ from the asymptotic solution, especially for small size blocks. Indeed, it was proved in [1] that maximum mutual information with finite size blocks can be achieved by shaping appropriately the correlation matrix of the transmitted block. However, such an optimum correlation matrix turns out to be non-Toeplitz, and thus spectral pulse shaping using a linear time-invariant (LTI) filter proposed in [1] can only be approximate, as recognized in [1].

In this work, we prove that the optimal correlation matrix can be induced *exactly*, irrespective of the non-Toeplitz structure of the optimal spectral shaping matrix, using an FIR multirate filterbank that introduces minimal redundancy on the input bit stream. We adopt the precoding (decoding) structure based on multirate filterbanks proposed in [11], [12], and derive the optimal transmit-receive filterbank pair which maximizes mutual information between transmitter and receiver, subject to a limited transmit-power, for any finite blocklength. The optimization herein is performed for FIR channels and the extension to ARMA channels, adopted for parsimonious modeling of copper twisted pair channels encountered in ADSL/HDSL applications [2], can be found in [13]. The proposed transceivers convert the frequency selective channel into M independent parallel flat fading subchannels - a decomposition reached also in [8]. However, our solution stems from maximizing a mutual information criterion and possesses inherent flexibility that yields as special cases zero-forcing (ZF) and minimum mean-square error (MMSE) receivers, *within the class of filterbanks maximizing the information rate.*

2. Filterbank transceiver model

Fig. 1 shows the discrete-time multirate equivalent model of our baseband communication system using filterbank precoders. Successive advancing and down-sampling by M per branch creates blocking, or, conversion of the serial data stream $s(n)$ to M parallel

This work was supported by NSF-NCR grant no. 980350.

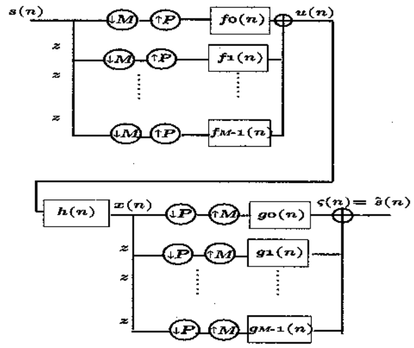

Figure 1: - Multirate discrete-time equivalent transmitter/channel/receiver model

substreams $s_m(n) := s(nM + m)$, where $s_m(n)$ denotes the mth symbol in the nth block of M symbols. Upsamplers by P insert $P - 1$ zeros, and the mth upsampler's output is $\sum_{i=-\infty}^{\infty} s(iM + m)\delta(n - iP)$, where $\delta(n)$ denotes Kronecker's delta. With $P > M$, the ratio $(P - M)/P$ represents the amount of redundancy introduced per transmitted block. At the receiver, the rate is reduced by the same amount such that the overall rate remains unchanged. Indicating by $\{f_m(n)\}_{m=0}^{M-1}$ the impulse responses of filters at each branch of the transmit filterbank, our precoder's output is:

$$u(n) = \sum_{m=0}^{M-1} \sum_{i=-\infty}^{\infty} s(iM + m) f_m(n - iP) \ . \quad (1)$$

From an input-output (I/O) point of view, our transmit-filterbank precoder takes size-M blocks of $s(n)$, vector filters them, and maps them to size-P blocks of $u(n)$. After passing through the linear time-invariant (LTI) channel $h(l)$, the data received in additive Gaussian noise (AGN), $v(n)$, are:

$$y(n) = x(n) \ + \ v(n) = \sum_{l=-\infty}^{\infty} h(l) u(n - l) + v(n) \quad (2)$$

$$= \sum_{m=0}^{M-1} \sum_{i=-\infty}^{\infty} s(iM+m) \sum_{l=-\infty}^{\infty} h(l) f_m(n-l-iP) + v(n).$$

A mapping mirror to (1) takes place at the receiver where size-P blocks of $y(n)$ are mapped to size-M blocks of $\hat{s}(n)$ after being filtered through the receive-filterbank:

$$\hat{s}(n) = \sum_{p=0}^{P-1} \sum_{j=-\infty}^{\infty} y(jP + p) g_p(n - jM) \ . \quad (3)$$

Substituting (2) into (3) leads to a rather cumbersome I/O relationship. However, (1)-(3) can be expressed in compact form using a matrix representation. Denote by $\boldsymbol{s}(n)$ and $\hat{\boldsymbol{s}}(n)$ the $M \times 1$ vectors $\boldsymbol{s}(n) := (s(nM), s(nM + 1), \cdots, s(nM + M - 1))^T$ and $\hat{\boldsymbol{s}}(n) := (\hat{s}(nM), \hat{s}(nM+1), \cdots, \hat{s}(nM+M-1))^T$ respectively, and by $\boldsymbol{u}(n)$, $\boldsymbol{y}(n)$ the $P \times 1$ vectors $\boldsymbol{u}(n) := (u(nP), u(nP + 1), \cdots, u(nP + P - 1))^T$ and $\boldsymbol{y}(n) := (y(nP), y(nP + 1), \cdots, y(nP + P - 1))^T$, respectively. It can be readily verified that (1) and (3) can be cast into the two following equivalent block-relationships:

$$\boldsymbol{u}(n) = \sum_{i=-\infty}^{\infty} \boldsymbol{F}_i \boldsymbol{s}(n-i) \ , \ \hat{\boldsymbol{s}}(n) = \sum_{j=-\infty}^{\infty} \boldsymbol{G}_j \boldsymbol{y}(n-j), \quad (4)$$

where $P \times M$ and $M \times P$ matrices $\boldsymbol{F}_i$ and $\boldsymbol{G}_j$ have entries:

$$\{\boldsymbol{F}_i\}_{k,m} = f_m(iP+k) \quad , \quad \{\boldsymbol{G}_j\}_{k,p} = g_p(jM+k). \quad (5)$$

An FIR filterbank has filters $\{f_m(n)\}_{m=0}^{M-1}$ ($\{g_p(n)\}_{p=0}^{P-1}$) that are FIR, which renders the infinite sums in (1) and (3) finite. In order to generalize our matrix formulation to the LTI-channel I/O relationship, let the $P \times 1$ vector $\boldsymbol{x}(n) := (x(nP), x(nP + 1), \cdots, x(nP + P - 1))^T$, denote the noise-free block of the channel output and $\boldsymbol{v}(n)$ the corresponding AGN vector with zero-mean and covariance matrix $\boldsymbol{R}_{vv}$, assumed to be full rank. The received data block is then given by:

$$\boldsymbol{y}(n) \ = \ \boldsymbol{x}(n) + \boldsymbol{v}(n) \ = \ \sum_{l=-\infty}^{\infty} \boldsymbol{H}_l \boldsymbol{u}(n-l) + \boldsymbol{v}(n) \ , \quad (6)$$

where the $P \times P$ matrices $\boldsymbol{H}_l$ are defined as $\{\boldsymbol{H}_l\}_{n,k} = h(lP + n - k)$. The input/output relationship (4) can thus be cast in matrix notation as:

$$\hat{\boldsymbol{s}}(n) = \sum_{j,l,i=-\infty}^{\infty} \boldsymbol{G}_j \boldsymbol{H}_l \boldsymbol{F}_i \boldsymbol{s}(n-l-i-j) + \sum_{j=-\infty}^{\infty} \boldsymbol{G}_j \boldsymbol{v}(n-j). \quad (7)$$

The transmission scheme in Fig. 1 offers degrees of freedom that can be used effectively to improve system performance. In particular, it is shown in [11] that an FIR filterbank at the receiver can equalize exactly an FIR channel (irrespective of its zero locations) provided that $P > M$. In the following we will assume:

(a0) Channel $h(l)$ is Lth order FIR with $h(0), h(L) \neq 0$.

(a1) (P, M, L) are chosen such that the triplet (P, M, L) satisfies: $P = M + L$, and $M > L$.

(a2) Transmit filters $\{f_m(n)\}_{m=0}^{M-1}$ are of order $< P$, and receive filters $\{g_p(n)\}_{p=0}^{P-1}$ are of order $< M$.

Under (a2), we have that $\boldsymbol{F}_i = \boldsymbol{F}_0 \delta(i)$ and $\boldsymbol{G}_j = \boldsymbol{G}_0 \delta(j)$. Because $P > L$, we infer from (a0) that $\boldsymbol{H}_l = \boldsymbol{H}_0 \delta(l) + \boldsymbol{H}_1 \delta(l-1)$, which together with (a1) implies that interblock interference due to the channel entails no more than two successive blocks, namely $\boldsymbol{s}(n)$ and $\boldsymbol{s}(n-1)$; thus, (7) becomes:

$$\hat{\boldsymbol{s}}(n) = \boldsymbol{G}_0 \boldsymbol{H}_0 \boldsymbol{F}_0 \boldsymbol{s}(n) + \boldsymbol{G}_0 \boldsymbol{H}_1 \boldsymbol{F}_0 \boldsymbol{s}(n-1) + \boldsymbol{G}_0 \boldsymbol{v}(n) \ , \quad (8)$$

where matrix $\boldsymbol{F}_0$ is $P \times M$, $\boldsymbol{G}_0$ is $M \times P$, and $\boldsymbol{H}_0, \boldsymbol{H}_1$ are square $P \times P$ matrices.

For perfect, or, zero-forcing (ZF) reconstruction of $\boldsymbol{s}(n)$ from $\hat{\boldsymbol{s}}(n)$, two options can be pursued [11]: i) force the last L samples of the transmit filters to be zero, so that $\boldsymbol{F}_0 = (\boldsymbol{F}^T\ \boldsymbol{0})^T$ with $\boldsymbol{F}$ an $M \times M$ matrix and $\boldsymbol{0}$ an $L \times M$ block of zeros, and let $\boldsymbol{G}_0 = \boldsymbol{G}$, where $\boldsymbol{G}$ is a $M \times P$ matrix (we term this option the trailing transmitter zeros approach, or, TZ for short); ii) force the first L filters of the receive filterbank to be zero, so that $\boldsymbol{G}_0 = (\boldsymbol{0}\ \boldsymbol{G})$, where now $\boldsymbol{G}$ is an $M \times M$ matrix, whereas $\boldsymbol{F}_0 = \boldsymbol{F}$ and $\boldsymbol{F}$ has now dimensionality $P \times M$ (correspondingly, we call this option the leading receiver zeros approach, or, LZ for short). From an implementation point of view, ii) may be preferred over i) because transmission need not be paused after each block. Although the dimensionalities of $\boldsymbol{F}$ and $\boldsymbol{G}$ will vary for options i) and ii), for brevity we will maintain similar notation for both cases, because if $\boldsymbol{H}$ is defined appropriately, one can adopt a common form for (8):

$$\hat{\boldsymbol{s}}(n) = \boldsymbol{G}\boldsymbol{H}\boldsymbol{F}\boldsymbol{s}(n) + \boldsymbol{G}\boldsymbol{v}(n)\ . \quad (9)$$

Specifically, for case i) of trailing transmitter zeros, $P \times M$ matrix $\boldsymbol{H}$ will be defined as the Toeplitz matrix whose first column is $(h(0), \cdots, h(L), 0, \cdots, 0)$ and the first row is $(h(0), 0, \cdots, 0)$, while for case ii) of leading receiver zeros (LZ), $\boldsymbol{H}$ will denote the $M \times P$ Toeplitz matrix whose first row is $(h(L), \cdots, h(0), 0, \cdots, 0)$ and the first column is $(h(L), 0, \cdots, 0)$. In [11], ZF equalizing filterbanks are developed even when $P = M + 1$ (and $L > 1$), but finite memory is then required in (7) which increases complexity at the receiver. Interestingly, ZF is established in [11] with no statistical assumptions on $s(n)$ and $v(n)$. In this paper however, we will assume that:

(a3) Input $\boldsymbol{s}(n)$ and AGN $\boldsymbol{v}(n)$ are generally complex, mutually uncorrelated, stationary with full rank covariance matrices $\boldsymbol{R}_{ss}$ and $\boldsymbol{R}_{vv}$, respectively ($\sigma_{ss}^2 \boldsymbol{I}$ and $\sigma_{vv}^2 \boldsymbol{I}$ when white).

3. Filterbanks maximizing information rate

Starting with (9), we seek the filterbank pair $(\boldsymbol{F}, \boldsymbol{G})$ that for given $\boldsymbol{H}$, $\boldsymbol{R}_{ss}$, and $\boldsymbol{R}_{vv}$, maximizes the information rate, subject to a limited average transmitted power. We can write (9) equivalently as $\hat{\boldsymbol{s}}(n) = \boldsymbol{T}\boldsymbol{u}(n) + \boldsymbol{w}(n)$, where $\boldsymbol{T} := \boldsymbol{G}\boldsymbol{H}$ and $\boldsymbol{w}(n) := \boldsymbol{G}\boldsymbol{v}(n)$. Vector $\boldsymbol{u}(n)$ is our channel's block input, and $\hat{\boldsymbol{s}}(n) = \boldsymbol{T}\boldsymbol{u}(n) + \boldsymbol{w}(n)$ denotes the received block. The starting point in maximizing the information rate is to express the mutual information between channel input $\boldsymbol{u}(n)$ and receive-filterbank output $\hat{\boldsymbol{s}}(n)$ as a function of matrices $\boldsymbol{F}$ and $\boldsymbol{G}$. We will borrow a result derived in [1, Thm. 1], and state it without proof in a slightly more general form that allows for colored input and noise vectors.

Lemma 1: *Consider the finite-dimensional vector model $\hat{\boldsymbol{s}} = \boldsymbol{T}\boldsymbol{u} + \boldsymbol{w}$, where $\boldsymbol{u}$ and $\boldsymbol{w}$ are zero-mean independent vectors with covariance matrices $\boldsymbol{R}_{uu}$ and $\boldsymbol{R}_{ww}$, and $\boldsymbol{w}$ is (in general complex and circularly) Gaussian. The normalized (per input symbol) mutual information, $I(\boldsymbol{u}; \hat{\boldsymbol{s}})$, between any block $\boldsymbol{u}$ of P channel input symbols and the corresponding block $\hat{\boldsymbol{s}}$ of M receiver output symbols is maximized when $\boldsymbol{u}$ is Gaussian, and is given by* [1]:

$$I(\boldsymbol{u}; \hat{\boldsymbol{s}}) = \frac{1}{P} \log_2 |(\boldsymbol{R}_{uu}^{\dagger} + \boldsymbol{T}^H \boldsymbol{R}_{ww}^{-1} \boldsymbol{T}) \boldsymbol{R}_{uu}|\ . \quad (10)$$

The matrix $(\boldsymbol{R}_{uu}^{\dagger} + \boldsymbol{T}^H \boldsymbol{R}_{ww}^{-1} \boldsymbol{T}) \boldsymbol{R}_{uu}$ may be rank deficient and, in such a case, as in the evaluation of the entropy of Gaussian random vectors having a singular covariance matrix, the determinant has to be substituted by the product of the nonzero singular values of $(\boldsymbol{R}_{uu}^{\dagger} + \boldsymbol{T}^H \boldsymbol{R}_{ww}^{-1} \boldsymbol{T}) \boldsymbol{R}_{uu}$ [1, Appendix II]. As expected intuitively, spectral shaping of the transmitted blocks (described by $\boldsymbol{R}_{uu}$ in (10)) affects mutual information and thus capacity and information rate of our block transmission through the channel. Without specifying the receiver structure and assuming AWGN, the mutual information $I(\boldsymbol{u}; \boldsymbol{y})$ was maximized in [1] with respect to $\boldsymbol{R}_{uu}$. The optimum $\boldsymbol{R}_{uu}$ was found to be non-Toeplitz, which corresponds to a nonstationary $u(n)$ and indicates that the desired spectral shaper must be time-varying since $s(n)$ is stationary. LTI lattice structures were proposed in [1] to *approximate* the desired time-varying transmitter.

The spectral shaper in our setup is the transmit filterbank $\boldsymbol{F}$ which induces the linear periodically varying I/O relationship (1). Interestingly, $\boldsymbol{F}$ will turn out to offer the *exact* spectral shaper leading to the optimum $\boldsymbol{R}_{uu}$ sought by [1]. Along with the optimum $\boldsymbol{G}$, the optimum $\boldsymbol{F}$ will be derived in closed form as a result of maximizing (10) and will thus achieve the maximum information rate for block transmissions. Our optimization result is summarized in the following (see [13] for the proof).

Theorem 1: *Suppose* **(a0)-(a2)**, **(a3)** *hold true, and let the transmit power $\mathcal{P}_0 := tr(\boldsymbol{F}\boldsymbol{R}_{ss}\boldsymbol{F}^H)$, the channel matrix $\boldsymbol{H}$, the input symbol covariance matrix $\boldsymbol{R}_{ss}$ and the noise covariance matrix $\boldsymbol{R}_{vv}$, be given. Denoting by $\boldsymbol{U}$, $\boldsymbol{V}$ the unitary matrices, and by $\boldsymbol{\Delta}$, $\boldsymbol{\Lambda}$ the diagonal matrices resulting from the eigen- decompositions*[2]:

$$\boldsymbol{R}_{ss} = \boldsymbol{U}\boldsymbol{\Delta}\boldsymbol{U}^H, \qquad \boldsymbol{H}^H \boldsymbol{R}_{vv}^{-1} \boldsymbol{H} = \boldsymbol{V}\boldsymbol{\Lambda}\boldsymbol{V}^H\ , \quad (11)$$

the optimum $(\boldsymbol{F}, \boldsymbol{G})$ filterbank pair maximizing (10) *is:*

$$\boldsymbol{F}_{opt} = \boldsymbol{V}\boldsymbol{\Phi}\boldsymbol{U}^H, \qquad \boldsymbol{G}_{opt} = \boldsymbol{U}\boldsymbol{\Gamma}\boldsymbol{\Lambda}^{-1}\boldsymbol{V}^H\boldsymbol{H}^H\boldsymbol{R}_{vv}^{-1}\ , \quad (12)$$

where $\boldsymbol{\Gamma}$ denotes an arbitrary invertible matrix and $\boldsymbol{\Phi}$ is a diagonal matrix with entries:

$$\phi_{ii} = \sqrt{\max\left(\frac{\mathcal{P}_0 + tr(\boldsymbol{\Lambda}^{-1})}{M\delta_{ii}} - \frac{1}{\lambda_{ii}\delta_{ii}}\ , 0\right)}\ , \quad (13)$$

[1] We adopt hereafter the following notation: $|\boldsymbol{A}|$ denotes the determinant of $\boldsymbol{A}$, the superscript ^{H} denotes transposition and conjugation, and $\dagger$ denotes pseudoinverse.

[2] In the LZ case, matrix $\boldsymbol{H}^H \boldsymbol{R}_{vv}^{-1} \boldsymbol{H}$ in (11) is rank deficient, and $\boldsymbol{\Lambda}$ has to be slightly modified (see [13]).

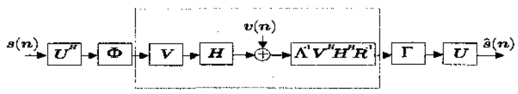

Figure 2: Optimal transceiver: matrix model.

and $\lambda_{ii}(\delta_{ii})$ is the ith diagonal entry of $\mathbf{\Lambda}(\mathbf{\Delta})$. □

First, let us interpret Theorem 1 with special cases. If $s(n)$ is white with unit variance, then $\boldsymbol{R}_{ss} = \boldsymbol{I}$ and $\boldsymbol{F}_{opt} = \boldsymbol{V}\mathbf{\Phi}$. In such a case, the impulse response of the ith transmit filter is $\mathbf{f}_{i,opt} = \phi_{ii}\boldsymbol{v}_i$, where $\boldsymbol{v}_i$ is the ith column of $\boldsymbol{V}$, and thus $|\phi_{ii}|^2$ represents the power assigned to the ith filter (recall that matrix $\mathbf{\Phi}$ is diagonal). If $\boldsymbol{v}(n)$ is white, $\boldsymbol{R}_{vv} = \sigma_{vv}^2\boldsymbol{I}$ and $\boldsymbol{v}_i$ in (11) corresponds to the ith eigenvector of the channel matrix $\boldsymbol{H}^H\boldsymbol{H}$. Note that the optimum pair $(\boldsymbol{F}_{opt}, \boldsymbol{G}_{opt})$ is non-unique, and matrix $\mathbf{\Gamma}$ in (12) offers degrees of freedom which can be exploited to satisfy added requirements. For example, judicious selections of $\mathbf{\Gamma}$, yield the zero-forcing (ZF), or, the Minimum Mean-Square Error (MMSE) receive-filterbank, as testified by the following corollaries.

Corollary 1: *Under (a0)-(a3), the ZF receiver filterbank that maximizes mutual information in (10) under fixed transmitted power $\mathcal{P}_0$, is obtained by setting $\mathbf{\Gamma} = \mathbf{\Phi}^\dagger$. The corresponding receive-filterbank matrix is:*

$$\boldsymbol{G}_{opt}^{zf} = (\boldsymbol{R}_{vv}^{-1/2}\boldsymbol{H}\boldsymbol{F}_{opt})^\dagger \boldsymbol{R}_{vv}^{-1/2}. \qquad (14)$$

Proof: Recalling (9), the ZF condition requires that $\boldsymbol{GHF} = \boldsymbol{I}$. From (11) and (12), we obtain

$$\boldsymbol{GHF} = \boldsymbol{U}\mathbf{\Gamma}\mathbf{\Lambda}^{-1}\boldsymbol{V}^H\boldsymbol{H}^H\boldsymbol{R}_{vv}^{-1}\boldsymbol{H}\boldsymbol{V}\mathbf{\Phi}\boldsymbol{U}^H = \boldsymbol{U}\mathbf{\Gamma}\mathbf{\Phi}\boldsymbol{U}^H. \qquad (15)$$

Therefore, the choice $\mathbf{\Gamma}_{zf} = \mathbf{\Phi}^\dagger$ leads to the ZF solution. Substituting $\mathbf{\Gamma}_{zf}$ into (12), yields (14).□

Corollary 2: *Under (a0)-(a3), the MMSE receiver filterbank that maximizes mutual information in (10) under fixed transmitted power $\mathcal{P}_0$, is obtained by setting $\mathbf{\Gamma}_{mmse} = \mathbf{\Delta}\mathbf{\Phi}^H(\mathbf{\Lambda}^{-1} + \mathbf{\Phi}^H\mathbf{\Delta}\mathbf{\Phi})^{-1}$. The corresponding receive filterbank matrix is:*

$$\boldsymbol{G}_{opt}^{mmse} = \boldsymbol{R}_{ss}\boldsymbol{F}^H\boldsymbol{H}^H(\boldsymbol{R}_{vv} + \boldsymbol{H}\boldsymbol{F}_{opt}\boldsymbol{R}_{ss}\boldsymbol{F}_{opt}^H\boldsymbol{H}^H)^{-1}. \qquad (16)$$

Proof: Using (9), the MSE $\mathcal{E} := tr\{E([\hat{\boldsymbol{s}}(n) - \boldsymbol{s}(n)][\hat{\boldsymbol{s}}(n) - \boldsymbol{s}(n)]^H)\}$, can be decomposed into a residual ISI term plus an output noise power term as follows:

$$\mathcal{E} = tr((\boldsymbol{GHF}-\boldsymbol{I})\boldsymbol{R}_{ss}(\boldsymbol{GHF}-\boldsymbol{I})^H) + tr(\boldsymbol{G}\boldsymbol{R}_{vv}\boldsymbol{G}^H), \qquad (17)$$

where $tr(\boldsymbol{T})$ indicates the trace of matrix $\boldsymbol{T}$. Substituting (12) into (17), we find

$$\mathcal{E} = tr((\mathbf{\Gamma}\mathbf{\Phi} - \boldsymbol{I})\mathbf{\Delta}(\mathbf{\Gamma}\mathbf{\Phi} - \boldsymbol{I})^H) + tr(\mathbf{\Gamma}\mathbf{\Lambda}^{-1}\mathbf{\Gamma}^H)\ . \qquad (18)$$

The matrix $\mathbf{\Gamma}_{mmse}$ minimizing (18) is found by equating to zero the gradient of (18) with respect to $\mathbf{\Gamma}$:

$$\mathbf{\Gamma}_{mmse} = \mathbf{\Delta}\mathbf{\Phi}^H(\mathbf{\Lambda}^{-1} + \mathbf{\Phi}^H\mathbf{\Delta}\mathbf{\Phi})^{-1}. \qquad (19)$$

The corresponding receive filterbank matrix is given by (16).□ Further insight is gained on our optimal transceivers from Fig. 3, where cascaded matrices implement the $\boldsymbol{F}_{opt}$ and $\boldsymbol{G}_{opt}$ filterbanks of (12). If $\bar{\boldsymbol{s}}(n) := \boldsymbol{U}^H\boldsymbol{s}(n)$ denotes the $\boldsymbol{U}^H$ block output, it follows easily from (11) that $\boldsymbol{R}_{\bar{s}\bar{s}} = \mathbf{\Delta}$, i.e., components $\bar{s}_i(n)$ are mutually uncorrelated and each with variance $\sigma_{\bar{s}_i\bar{s}_i}^2 = \delta_{ii}$. Hence, the block $\boldsymbol{U}^H$ decorrelates the entries of our (possibly colored) input vector $\boldsymbol{s}(n)$. The next diagonal block $\mathbf{\Phi}$ can be decomposed as [c.f. (13)]: $\mathbf{\Phi} = \mathbf{D}_\phi\mathbf{\Delta}^{-1/2}$, where $\mathbf{D}_\phi$ depends only on $\mathcal{P}_0$ and $\mathbf{\Lambda}$. With the decomposition $\mathbf{\Phi}\boldsymbol{U}^H = \mathbf{D}_\phi(\mathbf{\Delta}^{-1/2}\boldsymbol{U}^H)$, we identify that the first part of our transmit filterbank $\boldsymbol{F}_{opt}$, performs prewhitening, while the second part, namely $\boldsymbol{V}\mathbf{D}_\phi$, tunes the transmit filters according to the eigenstructure of the propagation channel which depends on the ISI matrix and the AGN covariance $\boldsymbol{R}_{vv}$ (c.f. (11)). When the AGN is white, $\boldsymbol{V}$ in (11) is formed by the left singular vectors of the channel matrix $\boldsymbol{H}$ (or the eigenvectors of $\boldsymbol{H}^H\boldsymbol{H}$) and the corresponding part of the precoder filterbank, $\boldsymbol{V}\mathbf{D}_\phi$ is composed of nothing but transmit filters each with impulse response $\boldsymbol{v}_i$ (the ith column of $\boldsymbol{V}$) and gain $d_{\phi_{ii}} := \phi_{ii}\sqrt{\delta_{ii}}$ as in (13).

Consider now the multichannel equivalent of the cascaded matrix systems inside the box of Fig. 3. Using (11), we find that its vector transfer function is

$$\mathbf{\Lambda}^{-1}\boldsymbol{V}^H\boldsymbol{H}^H\boldsymbol{R}_{vv}^{-1}\boldsymbol{H}\boldsymbol{V} = \mathbf{\Lambda}^{-1}\boldsymbol{V}^H\boldsymbol{V}\mathbf{\Lambda}\boldsymbol{V}^H\boldsymbol{V} = \boldsymbol{I}\ , \qquad (20)$$

which implies that if we select a diagonal $\mathbf{\Gamma}$, the matrix (or block) channel between the outer blocks $\boldsymbol{U}^H$ and $\boldsymbol{U}$ is described by the diagonal matrix $\mathbf{\Gamma}\mathbf{\Phi}$. Furthemore, the covariance matrix of the transformed noise $\beta(n) := \mathbf{\Lambda}^{-1}\boldsymbol{V}^H\boldsymbol{H}^H\boldsymbol{R}_{vv}^{-1}\boldsymbol{v}(n)$ at the output of the block in Fig. 3 is

$$\boldsymbol{R}_{\beta\beta} = \mathbf{\Lambda}^{-1}\boldsymbol{V}^H\boldsymbol{H}^H\boldsymbol{R}_{vv}^{-1}\boldsymbol{R}_{vv}\boldsymbol{R}_{vv}^{-1}\boldsymbol{H}\boldsymbol{V}\mathbf{\Lambda}^{-1} = \mathbf{\Lambda}^{-1}\ , \qquad (21)$$

justifying the de-correlation and thus independence (since $\beta(n)$ is AGN) among the subchannels. We have thus established that:

Corollary 3: *With $\mathbf{\Gamma}$ diagonal, the mutual information maximizing filterbank transceivers of Theorem 1 render the block transmission ISI channel model equivalent to M independent parallel ISI-free subchannels each with flat fading gains $\phi_{ii}\gamma_{ii}$ and uncorrelated AGN samples $\beta_i(n)$ with variance $1/\lambda_{ii}$; i.e.,*

$$\hat{\bar{s}}_i(n) = \phi_{ii}\,\gamma_{ii}\,\bar{s}_i(n) + \gamma_{ii}\,\beta_i(n)\ . \qquad (22)$$

Because $\bar{s}_i(n)$ has variance δ_{ii}, the SNR_i at the output of the ith subchannel is:

$$SNR_i = \frac{\delta_{ii}|\phi_{ii}|^2|\gamma_{ii}|^2}{\lambda_{ii}^{-1}|\gamma_{ii}|^2} = \delta_{ii}|\phi_{ii}|^2\lambda_{ii}\ . \qquad (23)$$

The independence of the parallel subchannels in Corollary 3, implies a corresponding decomposition of the maximum mutual information in (10) as (see [13]):

$$I(\boldsymbol{u};\hat{\boldsymbol{s}}) = \frac{1}{P}\sum_{i=1}^{M}\log_2(1 + SNR_i)\ , \qquad (24)$$

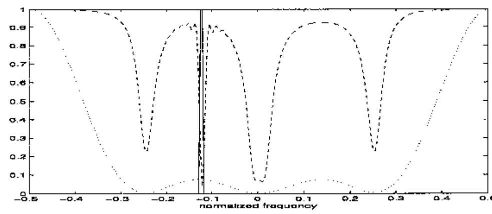

Figure 3: - Average of the optimal transmit filters transfer functions (dashed line), channel tranfer function (dotted line), and interference (solid line).

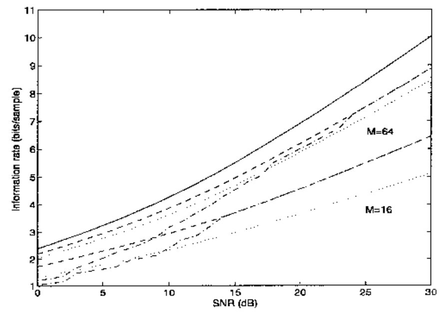

Figure 4: Channel capacity (solid line) and information rates (bits/sample) with optimal filterbank, loading algorithm #1 (dashed-dotted) and #2 (dashed), and with DMT (dotted).

where SNR_i and ϕ_{ii} are given by (23) and (13), respectively. Interestingly, the SNR_i and $I(\boldsymbol{u};\hat{\boldsymbol{s}})$ do not depend on the matrix $\boldsymbol{\Gamma}$, which can thus be selected according to Corollaries 1 or 2, without affecting the maximum information rate.

As the block length $M \to \infty$, the energy per subchannel suggested by (13) corresponds to the well known "water-pouring" (or "water-filling") principle (see e.g., [4, Ch. 8]), which under *ideal* DFE *conditions* (correct decisions entering the feedback loop) was reached also by [9] in the context of finite length block codes. Note though that in contrast to our FIR filterbank transceivers, the ideal ZF-DFE entails IIR feedforward filtering. Trailing transmitter zeros and transmit filters corresponding to eigenvectors of the channel matrix were also derived in [8] relying on a ZF/constrained power criterion for white input and noise processes. In this work, in addition to treating more general TZ/LZ cases, we derived the optimal filterbanks by directly maximizing the information rate and obtained the ZF and MMSE receivers, within the class of filterbanks maximizing the information rate.

Example 1: An example of application is reported in Fig. 3, dealing with block ($P = 64$) transmission over a frequency selective channel characterized by an FIR filter with zeros at $0.9, 0.9j, -0.9j$, in the presence of additive white Gaussian noise $v_{AGN}(n)$ at SNR=10 dB, plus stationary Gaussian narrowband interference $v_{NBI}(n)$, generated to be independent of $v_{AGN}(n)$. With $v(n) := v_{AGN}(n) + v_{NBI}(n)$, we applied Theorem 1 to obtain our transmit filters $\{f_m(n)\}_{m=0}^{M-1}$ from the columns of $\boldsymbol{F}_{opt}$ in (12). Fig. 3 shows the normalized channel transfer function $|H(f)|/\ |H(0)|$ (dotted line), the interference spectral density (solid line), and the average of the optimal filter magnitudes (dashed line). As expected, the optimal filters do not allocate any power at the frequency bin containing the interference and allocate most of the power in the frequencies where the channel gain (SNR) is higher, as predicted by the water-filling principle [4].

Example 2: As with DMT, proper allocation of power and number of bits per subchannel is necessary to maximize the information rate, subject to a fixed average transmit power and an upper bounded bit error rate. We compare now the information rates achievable with two loading algorithms detailed in [13], based on the optimal filterbanks described before, with DMT, when both $s(n)$ and $v(n)$ are white. We compute the information rate pertaining to DMT using (10), with $\boldsymbol{R}_{uu} = \boldsymbol{F}\boldsymbol{F}^H$, $\boldsymbol{T} = \boldsymbol{G}\boldsymbol{H}$, $\boldsymbol{R}_{ww} = \boldsymbol{G}\boldsymbol{R}_{vv}\boldsymbol{G}^H$, and using a $P \times M$ IDFT precoding matrix $\boldsymbol{F}$, with cyclic prefix $L = P - M$ [2]. Also in this case, as with (10), the determinant is computed as the product of the nonzero singular values of the matrix $(\boldsymbol{R}_{uu}^{\dagger} + \boldsymbol{T}^H\boldsymbol{R}_{ww}^{-1}\boldsymbol{T})\boldsymbol{R}_{uu}$ and an iterative loading algorithm is used to redistribute the number of bits per channel when some singular values are less than one. [2]. The comparison reported in Fig. 4 refers to on an FIR channel of order $L = 5$, with zeros at $0.9, \pm 0.9j$, $\exp(\pm j\pi/4)$. Two block sizes $M = 16$ and $M = 64$ were used to compare their impact on the performance. Fig. 4 shows the channel capacity (solid line) computed as in [4, p. 388], and the information rates (in bits per sample) achievable with DMT (dotted line) and with the loading algorithms #1 (dashed-dotted line) and #2 (dashed line). From Fig. 4, we observe that the information rates achievable with DMT and with the optimal filters tend to coincide and approach closely the channel capacity as the block size increases, but the optimal filterbank outperforms DMT. The asymptotic (as $M \to \infty$) convergence of the two algorithms is justified because the complex exponentials used for DMT filters coincide with the channel eigenvectors, arising from the filterbank optimization of Theorem 1. For relatively large blocks, the sub-optimality of DMT is compensated by the computational advantage it has over the optimal filterbank because the former is FFT-based while the latter requires eigendecompositions of large matrices. Conversely, when small size blocks are used, the added flexibility gained with the optimal filters relative to complex exponentials is helpful to approach the theoretical information rate more closely. The relative improvement achieved

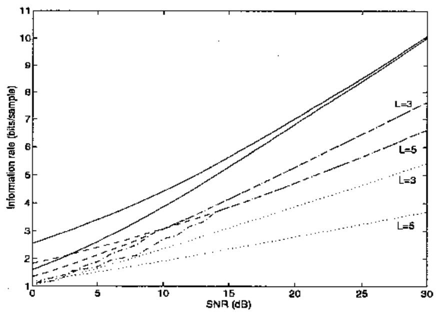

Figure 5: Channel capacity (solid line) and information rates (bits/sample), with optimal filterbank, loading algorithm #1 (dashed-dotted) and #2 (dashed), and with DMT (dotted).

with the optimal filters over the DMT increases also as the channel's frequency selectivity increases. As an example, Fig. 5 shows the information rates obtained with the optimal filterbank and with DMT, using blocks of size $M = 32$, considering two kinds of channels: one with $L = 3$ zeros at $1, \pm j$ and the other with $L = 5$ zeros at $1, \pm j, \pm \exp(j\pi/4)$. From Fig. 5, we observe that the advantage of using the optimal filterbank is more pronounced as the number of zeros on the unit circle increases. In fact, when channel zeros are uniformly spaced on the unit circle, DMT avoids transmission over the corresponding subchannels and distributes the available power on the remaining channels. In contrast, depending on the channel's eigen-characteristics, the optimal approach reshapes all the transmit-filters and this extra flexibility offers the aforementioned improvement over the DMT. In summary, we deduce that both optimal and DMT schemes allow reliable transmission at information rates approaching channel capacity, as the block size increases, but the optimal filterbank offers advantages with respect to DMT, which are more evident for small block sizes and highly selective channels.

4. Conclusions

In this paper we proposed multirate filterbank transceivers guaranteeing block data transmission at the maximum information rate, subject to a fixed average transmit power. Within the class of filters maximizing the information rate, the inherent flexibility of the proposed structure was exploited to derive zero-forcing and minimum mean-square error receive filterbanks. The proposed transceivers outperform DMT for small size blocks transmitted through highly frequency selective channels, at the expense of added complexity required to perform (non-FFT based) eigen- decomposition. Works are in progress to develop decision-feedback filterbank transceivers and extend the proposed approach to time-varying transmission channels. In fact, the multirate filterbank approach may be particularly useful for imposing transmit-correlation matrices with non-Toeplitz structures.

1. REFERENCES

[1] N. Al-Dhahir, J.M. Cioffi, "Block transmission over dispersive channels: Transmit filter optimization and realization, and MMSE-DFE receiver performance," *IEEE Trans. Inform. Theory*, pp. 137-160, Jan. 1996.

[2] J. S. Chow, J. C. Tu, J. M. Cioffi, "A discrete multitone transceiver system for HDSL applications," *IEEE Journal on Selected Areas in Commun.*, pp. 895-908, Aug. 1991.

[3] G. D. Forney, Jr., M. V. Eyuboğlu "Combined equalization and coding using precoding", *IEEE Communications Magazine*, pp. 25-34, Dec. 1991.

[4] R. Gallager, *Elements of Information Theory*, New York: Wiley, 1968.

[5] G. B. Giannakis, "Filterbanks for blind channel identification and equalization," *IEEE Signal Processing Letters*, pp. 184-187, vol. 4, no. 6, June 1997.

[6] H. Harashima, H. Miyakawa, "Matched-transmission technique for channels with intersymbol interference", *IEEE Trans. on Comm.*, pp.774-780, Aug. 1972.

[7] I. Kalet, "The multitone channel", *IEEE Trans. on Comm.*, vol. 37, pp.119-124, Febr. 1989.

[8] S. Kasturia, J. T. Aslanis, and J. M. Cioffi, "Vector Coding for Partial Response Channels," *IEEE Trans. on Info. Theory*, vol. 36, pp. 741-761, July 1990.

[9] J. W. Lechleider, "The optimum combination of block codes and receivers for arbitrary channels," *IEEE Trans. on Comm.*, pp. 615-621, May 1990.

[10] R. Price, "Nonlinear feedback equalized PAM versus capacity for noisy filter channels", *Proc. of Intl. Conf. on Communications*, pp. 22.12-22.17, 1972.

[11] A. Scaglione, G. B. Giannakis, and S. Barbarossa, "Redundant Filterbank Precoders and Equalizers, Part I: Unification and Optimal Designs," *IEEE Trans. on Signal Proc.*, 1997 (submitted); see also *Proc. of ICC'98*, pp. 21-25, Atlanta (GA), June 1998.

[12] A. Scaglione, G. B. Giannakis, and S. Barbarossa, "Redundant Filterbank Precoders and Equalizers, Part II: Blind Channel Estimation, Synchronization and Direct Equalization," *IEEE Trans. on Signal Proc.*, 1997 (submitted); see also *Proc. of Intl. Conf. on ASSP*, vol. VI, pp. 3501-3504, Seattle, WA, May 12-15, 1998.

[13] A. Scaglione, S. Barbarossa, and G. B. Giannakis, "Filterbank Transceivers Optimizing Information Rate in Block Transmissions over Dispersive Channels," *IEEE Trans. on Info. Theory*, 1999 (to appear).

[14] M. Tomlinson, "New automatic equaliser employing modulo arithmetic", *Electron. Lett.*, pp.138-139, Mar. 1971.

[15] X. Xia, "New Precoding for Intersymbol Interference Cancellation Using Nonmaximally Decimated Multirate Filterbanks with Ideal FIR Equalizers", *IEEE Trans. on Signal Proc.*, pp.2431-2441, Oct. 1997.

[16] W. Y. Zou, Y. Wu, "COFDM: An Overview", *IEEE Trans. on Broadcasting*, Vol. 41, No. 1, pp.1-8, March 1995.