INFORMATION RATE MAXIMIZING FIR TRANSCEIVERS: FILTERBANK PRECODERS AND DECISION–FEEDBACK EQUALIZERS FOR BLOCK TRANSMISSIONS OVER DISPERSIVE CHANNELS

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Abstract

Optimal FIR transmit-filterbanks and non-linear DFE receivers are derived for block transmission systems. Subject to an average transmitted power constraint, the mutual information rate is maximized by joint optimization of the transmit and receive filters. The FIR transmit filterbank obeys the "water-filling" principle; moreover, transmit-induced redundancy in the form of trailing zeros, and proper design of the receive filters provides degrees of freedom which can be exploited for the equalization of frequency-selective channels. Simulations illustrate the merits of our designs.

Keywords: block transceivers, decision feedback equalization, information rate maximization.

I. Introduction

Block transmission systems have been proposed as an effective means of communicating over frequency selective channels [3, 5]. It has been shown that most of the currently used schemes (e.g., orthogonal frequency division multiplexing, OFDM) can be modeled using the unifying framework of [5], which is based on filterbank transmitters and receivers (transceivers). Transmit induced redundancy, in the form of trailing zeros ("guard interval"), is used to eliminate interblock interference (IBI) as well as intersymbol interference (ISI) within each block. Moreover, the redundancy can be exploited to facilitate blind channel estimation and block synchronization [6]. At the receiver end, non-linear decision-feedback (DFE) equalizers, which capitalize on the finite input alphabet, improve the bit error rate (BER) performance of the system (e.g., [1, 7]).

As the demand for high-data transmission rates is on the rise, the design of precoder-receiver pairs which maximize the mutual information rate has received considerable attention lately [1, 4, 9]. Our focus herein is on deriving the optimal transceiver which maximizes the number of information bits per second under the constraint of fixed power at the transmitter. In [1], the optimal transmitted block correlation matrix (which maximizes the mutual information) was derived; however, the non-Toeplitz structure of the optimal correlation matrix implies that it can only be approximated at the output of a linear time invariant (LTI) transmit filter output. In [4], it was shown that a filterbank transmitter, which functions as a time varying filter, is capable of producing the optimal correlation matrix. However, maximizing the mutual information is a joint transmitter-receiver optimization problem. In [4], linear receivers were assumed, but from a practical point of view, non-linear DFE receivers offer the potential of improving the BER performance. Therefore, the problem of jointly optimizing the filterbank transmitter and the DFE receiver is well-motivated.

In [9], the joint optimization problem was addressed, but the solution was given by an iterative procedure (in the frequency domain), and IIR transmit-receive filters were assumed. In this paper, we derive the optimal FIR transmit and DFE receiver pair which maximizes the mutual information rate under a transmitted power constraint. Though our solution capitalizes on techniques developed in [9], unlike [9] the solution is given in closed form. Moreover, due to the FIR nature of our filters, the optimal transmitter/receiver pair can be realized exactly. We also study the effect of the amount of transmit-induced redundancy on the structure of the DFE receiver. We reach the interesting conclusion that the combination of sufficient redundancy (which obviates IBI) and the optimal precoder renders the feedback part of the DFE receiver unnecessary. In other words, the DFE receiver can be replaced by a linear receiver. On the other hand, when IBI is present, we derive DFE receivers which are shown to exhibit better BER performance than their linear counterparts.

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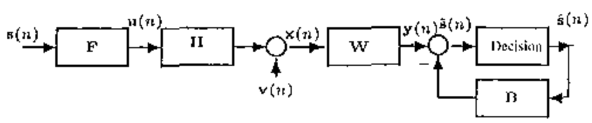
II. Modeling

Fig. 1 depicts the discrete-time model of a baseband block transmission communication system. The transmitted data are parsed into blocks using the advance elements and downsamplers. The transmit filters $\{f_m(n)\}_{m=0}^{M-1}$ are FIR of maximum order $P-1$. The FIR channel $\{h(n)\}$ includes multipath effects and transmit/receive filters. The input to the upsampler of the mth branch is $s_m(n):=s(nM+m)$, which represents the m-th symbol in the n-th block of M symbols. With the insertion of $P-1$ zeros, the corresponding upsampler's output is: $\sum_i s_m(i)\delta(n-iP)$, where $\delta(n)$ denotes Kronecker's delta. The transmitted sequence is: $u(n)=\sum_{m=0}^{M-1} u_m(n)=\sum_i\sum_{m=0}^{M-1} s_m(i)f_m(n-iP)$. The received samples, $x(n)=\sum_l h(l)u(n-l)+v(n)$, are corrupted by additive zero-mean stationary noise with covariance matrix $\mathbf{R}_{vv}$ ($=\sigma_{vv}^2\mathbf{I}_{P\times P}$ when the noise is white). We will assume that:

(a0) Channel $h(l)$ is Lth order FIR with $h(0),h(L)\neq 0$.

(a1) (P,M,L) are chosen such that the triplet (P,M,L) satisfies: $P=M+L$, and $M>L$.

(a2) The last L samples of the filters $\{f_m(n)\}_{m=0}^{M-1}$ are zero, and the $P\times M$ matrix $\mathbf{F}$ (with mth column $\mathbf{f}_m:=(f_m(0)\dots f_m(M-1)0\dots 0)^T$) has full column rank M.



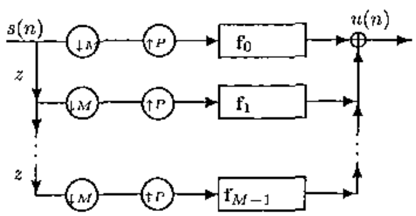


Figure 1: Block Transmission System and Transmitter Structure

Under (a0)-(a2) and thanks to the FIR nature of $h(n)$ and $f_m(n)$, the p-th polyphase component $x_p(n):=x(nP+p)$ (delayed and downsampled version) of $x(n)$ is:

$$x_p(n)=\sum_{m=0}^{M-1} s_m(n)\sum_{l=0}^{L} h(l)f_m(p-l)+v(nP+p), \quad (1)$$

which shows that no inter-block interference arises due to ISI. Let us define the $M\times 1$ vectors $\mathbf{s}(n):=(s_0(n)\quad s_1(n)\quad \dots s_{M-1}(n))^T$, and $\mathbf{u}(n):=(u(nP)u(nP+1)\dots u(nP+M-1))^T$; the $P\times 1$ vectors $\mathbf{x}(n):=(x(nP)x(nP+1)\dots x(nP+P-1))^T$, and $\mathbf{v}(n):=(v(nP)v(nP+1)\dots v(nP+P-1))^T$, and the $P\times M$ precoder matrix $\mathbf{F}:=(\mathbf{f}_0\dots\mathbf{f}_{M-1})$; note that the M last rows of $\mathbf{F}$ are 0 (trailing precoder zeros assumed in (a2)). We introduce the $P\times P$ Toeplitz lower triangular matrix $\mathbf{H}$ with first column $(h(0)\cdots h(L)0\cdots 0)^T$. Based on these definitions, we can cast (1) in matrix form:

$$\mathbf{x}(n)=\mathbf{H}\mathbf{u}(n)+\mathbf{v}(n)=\mathbf{H}\mathbf{F}\mathbf{s}(n)+\mathbf{v}(n). \quad (2)$$

With $\mathbf{R}_{ss}:=E\{\mathbf{s}(n)\mathbf{s}^H(n)\}$ denoting the input covariance matrix, the average transmitted power is $\mathcal{P}_o=\mathrm{tr}E\{\mathbf{F}\mathbf{R}_{ss}\mathbf{F}^H\}$.

The decision feedback equalizer consists of the feedforward filterbank represented by the $M\times P$ matrix $\mathbf{W}$, the decision making device and the feedback filterbank represented by the $M\times M$ matrix $\mathbf{B}$. By defining the $M\times 1$ vectors: $\mathbf{y}(n):=(y(nM)\quad y(nM+1)\quad \dots y(nM+M-1))^T$, $\tilde{\mathbf{s}}(n):=(\tilde{s}(nM)\ \tilde{s}(nM+1)\dots\tilde{s}(nM+M-1))^T$, $\hat{\mathbf{s}}(n):=(\hat{s}(nM)\quad \hat{s}(nM+1)\quad \dots\hat{s}(nM+M-1))^T$, we can write in matrix form (see also Fig. 1):

$$\begin{aligned}\mathbf{y}(n) &= \mathbf{W}\mathbf{x}(n)=\mathbf{W}\mathbf{H}\mathbf{F}\mathbf{s}(n)+\mathbf{W}\mathbf{v}(n)\\ \tilde{\mathbf{s}}(n) &= \mathbf{y}(n)-\mathbf{B}\hat{\mathbf{s}}(n)\\ \hat{\mathbf{s}}(n) &= Q(\tilde{\mathbf{s}}(n)),\end{aligned} \quad (3)$$

where $Q(\cdot)$ is the quantizer used by the decision making device. Note that $\mathbf{B}$ is chosen to be upper triangular which makes successive cancellation possible. By successive cancellation we mean that for every block indexed by n, first the $(M-1)$th symbol is recovered; then the estimate $\hat{s}(nM+M-1)$ is weighted by the last column of $\mathbf{B}$ and is removed from $\mathbf{y}(n)$ so that the remaining symbols can be recovered. When this is done, the $(M-2)$nd symbol is recovered, and the estimate $\hat{s}(nM+M-2)$ is removed from $\mathbf{y}(n)$. This procedure is carried out until all the symbols of the current block n have been recovered.

For the performance evaluation of the transceiver design, we use two figures of merit based on the error $\mathbf{e}(n):=\mathbf{s}(n)-\tilde{\mathbf{s}}(n)$ at the input of the decision device. The first one is the mean square error (MSE), which is given by the trace of the error covariance matrix $\mathrm{tr}E\{\mathbf{e}(n)\mathbf{e}^H(n)\}$. The second one is the geometric mean square error (GMSE), which is given by the determinant of the error covariance matrix $|E\{\mathbf{e}(n)\mathbf{e}^H(n)\}|$. Our problem is to select the optimal filterbank triplet $(\mathbf{F},\mathbf{W},\mathbf{B})$ that minimizes GMSE and MSE for a prescribed transmit-power $\mathcal{P}_o$.

III. Optimal Transceiver Filterbanks

The objective is to design the transmit and receive filters so that the information rate of the overall system is maximized. In [9], it was shown that the mutual information is a monotically decreasing function of the geometric mean square error (GMSE). For a given transmitter there is a family of receivers which minimize the GMSE. A significant observation that was made in [9] is that the minimum mean square error (MMSE) receiver, which attempts to minimize the MSE, belongs to this family. Unlike the minimum GMSE criterion, the MMSE criterion yields a unique receiver for a fixed transmitter [3, 5, 7]. Building upon these results, we next describe how it is possible to derive in our block transmission framework the information rate maximizing FIR transceiver under a fixed power constraint.

First, we show that the FIR MMSE receiver belongs to the class of FIR receivers which minimize the GMSE for a given FIR transmitter. For the MMSE receiver the feedforward filter $\mathbf{W}$ is a function of the transmitter $\mathbf{F}$ and the feedback filter $\mathbf{B}$ [7]. This allows us to express the GMSE as a function of $\mathbf{B}$ and $\mathbf{F}$. By taking advantage of the special structure of $\mathbf{B}$ (which is strictly upper triangular), we express the GMSE as a function of only the precoder filter $\mathbf{F}$. As a result, we are able to derive the optimal precoder filter $\mathbf{F}_{\text{opt}}$ under a fixed power constraint. Having established that the MMSE receiver belongs to the class of receivers which minimize the GMSE, we conclude that the optimal transceiver is composed of the optimal transmitter $\mathbf{F}_{\text{opt}}$, and the MMSE receiver pair $(\mathbf{W},\mathbf{B})$ which corresponds to $\mathbf{F}_{\text{opt}}$. Thus, we have proved the following:

Lemma: *Let $\mathbf{U}$ be the upper triangular matrix and $\mathbf{D}$ the diagonal matrix obtained from the Cholesky factorization of $\mathbf{R}_{ss}+(\mathbf{HF})^{\mathcal{H}}\mathbf{R}_{vv}^{-1}(\mathbf{HF})=\mathbf{U}^{\mathcal{H}}\mathbf{D}\mathbf{U}$. For a given channel matrix $\mathbf{H}$, precoder $\mathbf{F}$, input covariance $\mathbf{R}_{ss}$, and additive Gaussian noise with covariance matrix $\mathbf{R}_{vv}$, the MMSE DFE receiver pair $(\mathbf{W},\mathbf{B})$ is given by:*

$$\begin{aligned}\mathbf{W}&=(\mathbf{B}+\mathbf{I})\mathbf{R}_{ss}(\mathbf{HF})^{\mathcal{H}}(\mathbf{R}_{vv}+(\mathbf{HF})\mathbf{R}_{ss}(\mathbf{HF})^{\mathcal{H}})^{-1}\\ \mathbf{B}&=\mathbf{U}-\mathbf{I}\end{aligned},$$

The resulting GMSE is given by:

$$|\mathbf{R}_{ss}^{-1}+(\mathbf{HF})^{\mathcal{H}}\mathbf{R}_{vv}^{-1}(\mathbf{HF})|^{-1}.$$

Utilizing the results of [8], we have shown:

Theorem: *Under the assumptions (a0)–(a2), and given the channel matrix $\mathbf{H}$, transmit power $\mathcal{P}_o$, input covariance matrix $\mathbf{R}_{ss}$, and noise covariance matrix $\mathbf{R}_{vv}$, the optimal transmitter $\mathbf{F}_{\text{opt}}$ is given by $\mathbf{F}_{\text{opt}}=\mathbf{V}_1\mathbf{\Delta}\mathbf{V}_2^{\mathcal{H}}$, where $\mathbf{V}_1$ diagonalizes the matrix $\mathbf{H}^{\mathcal{H}}\mathbf{R}_{vv}^{-1}\mathbf{H}$, $\mathbf{V}_2$ diagonalizes the matrix $\mathbf{R}_{ss}$,*

$$\begin{aligned}\mathbf{V}_1^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{R}_{vv}^{-1}\mathbf{H}\mathbf{V}_1 &= \mathbf{\Lambda}=\text{diag}(\lambda_1,\lambda_2,\ldots,\lambda_P)\\ \mathbf{V}_2^{\mathcal{H}}\mathbf{R}_{ss}\mathbf{V}_2 &= \mathbf{K}=\text{diag}(k_1,k_2,\ldots,k_M),\end{aligned}$$

and $\mathbf{\Delta}$ is a diagonal matrix with entries given by:

$$|\delta_i|^2=\begin{cases}\frac{1}{rk_i}(\mathcal{P}_o+\sum_{j=1}^{r}\lambda_j^{-1})-(k_i\lambda_i)^{-1} & : i\le r\\ 0 & : i>r\end{cases},\quad (4)$$

where $r\le M$ is the largest integer satisfying
$\lambda_r^{-1}\le r^{-1}(\mathcal{P}_o+\sum_{j=1}^{r}\lambda_j^{-1})$. □

At this point two remarks are due. First, the optimal precoder $\mathbf{F}_{\text{opt}}$ is the same as the precoder of [4], which assumes linear (non-DFE) receivers. Note that the precoder of [4] has been shown to obey the "water-filling" principle. At first sight, it may seem quite strange that the optimal transmitter is the same for the two cases of linear and DFE receivers. An intuitive explanation would be that it is the precoder which basically determines how the information is transmitted over the channel. Formally, using the data processing inequality [2, pp. 32–33], we obtain that $I(\mathbf{u},\mathbf{x})\ge I(\mathbf{u},\hat{\mathbf{s}})$, where $I(\mathbf{u},\mathbf{x})$ ($I(\mathbf{u},\hat{\mathbf{s}})$) is the mutual information between $\mathbf{u}$ and $\mathbf{x}$ ($\mathbf{u}$ and $\hat{\mathbf{s}}$). In the absence of noise and under the assumption of perfect equalization at the receiver, we have that $I(\mathbf{u},\mathbf{x})=I(\mathbf{u},\hat{\mathbf{s}})$, which explains why the optimal transmitter, which maximizes $I(\mathbf{u},\mathbf{x})$, is the same for the case of a linear and the case of a DFE receiver.

The second remark is that the optimal precoder diagonalizes the channel in the sense that the channel is essentially transformed to M parallel subchannels. As a direct result, it can be seen that both $\mathbf{W}$ and $\mathbf{B}+\mathbf{I}$ are diagonal. As $\mathbf{B}+\mathbf{I}$ is diagonal and $\mathbf{B}$ is strictly upper triangular, we obtain that $\mathbf{B}=\mathbf{0}$, which implies that the DFE receiver is identical to the linear receiver. This surprising result holds because the IBI has been canceled by the redundancy of L trailing zeros. Therefore, we reach the interesting conclusion that for block transmission systems there is no need to use the DFE receiver structure when the optimal precoder is used and IBI is absent. However, the situation is different in the presence of IBI.

IV. Presence of IBI

According to assumptions (a0)–(a2), IBI can be eliminated by choosing $P\ge M+L$. Given the fact there are channels where L can be quite high (for example in DSL applications the channel may have 100 taps), having L trailing zeros may lead to a substantial decrease of the information rate, unless high values for M are assumed (which will lead however to longer decoding delays). This imposes an inherent trade-off between longer blocks (i.e., decoding delays) and information rate. One can dispense with this trade-off by using a smaller number of trailing zeros; this will lead to IBI. IBI can be removed at the receiver using a more complex structure. Hence, the trade-off "longer blocks vs. information rate" can be replaced by the trade-off "small number of trailing zeros vs. receiver complexity".

When the block size P is chosen so that $M<P<M+L$, the received data $\mathbf{x}(n)$ are given by:

$$\begin{aligned}\mathbf{x}(n) &= \mathbf{H}_0\mathbf{u}(n)+\mathbf{H}_1\mathbf{u}(n-1)+\mathbf{v}(n)\\ &= \mathbf{H}_0\mathbf{F}\mathbf{s}(n)+\mathbf{H}_1\mathbf{F}\mathbf{s}(n-1)+\mathbf{v}(n),\end{aligned} \tag{5}$$

where $\mathbf{H}_0$ is a $P\times P$ lower triangular Toeplitz with first column $(h(0)\ldots h(L)0\ldots 0)^T$ and $\mathbf{H}_1$ is $P\times P$ upper triangular Toeplitz with first row $(0\ldots 0\,h(L)\ldots h(1))$.

The linear zero-forcing (ZF) receiver in [5] is composed of M FIR filters of length QM, where Q is chosen so that $P\geq M+\lceil L/Q\rceil$. We have proved in [7] that both a ZF and an MMSE DFE receiver also exist. The ZF receiver first removes the IBI from the received data $\mathbf{x}(n)$ to obtain $\mathbf{x}'(n):=\mathbf{x}(n)-\mathbf{H}_1\mathbf{F}\hat{\mathbf{s}}(n-1)$, and the second part of the receiver is identical to the ZF DFE receiver when IBI is absent. On the other hand, for the recovery of the transmitted block $\mathbf{s}(n)$, the MMSE DFE receiver utilizes the information provided by the received blocks $\mathbf{x}(n-1)$, $\mathbf{x}(n)$, $\mathbf{x}(n+1)$, and the decided blocks $\hat{\mathbf{s}}(n-1)$, $\hat{\mathbf{s}}(n)$. In other words, (3) becomes:

$$\begin{aligned}\mathbf{y}(n) &= \mathbf{W}_{-1}\mathbf{x}(n+1)+\mathbf{W}_0\mathbf{x}(n)+\mathbf{W}_1\mathbf{x}(n-1)\\ \tilde{\mathbf{s}}(n) &= \mathbf{y}(n)-\mathbf{B}_0\hat{\mathbf{s}}(n)-\mathbf{B}_1\hat{\mathbf{s}}(n-1)\\ \hat{\mathbf{s}}(n) &= Q(\tilde{\mathbf{s}}(n)).\end{aligned}$$

To derive the settings of the MMSE DF receiver, we introduce the vectors $\bar{\mathbf{y}}(n):=(\mathbf{y}^T(n+1)\,\mathbf{y}^T(n)\,\mathbf{y}^T(n-1))^T$, and $\bar{\mathbf{s}}(n):=(\mathbf{s}^T(n+1)\,\mathbf{s}^T(n)\,\mathbf{s}^T(n-1))^T$. Under the assumption that $E\{\mathbf{s}(n)\mathbf{s}^{\mathcal{H}}(m)\}=\mathbf{R}_{ss}\delta(n-m)$, and $E\{\mathbf{v}(n)\mathbf{v}^{\mathcal{H}}(m)\}=\mathbf{R}_{vv}\delta(n-m)$, it can be verified that for $\mathbf{R}_{\bar{s}\bar{s}}:=E\{\bar{\mathbf{s}}(n)\bar{\mathbf{s}}^{\mathcal{H}}(n)\}$, $\mathbf{R}_{\bar{y}\bar{y}}:=E\{\bar{\mathbf{y}}(n)\bar{\mathbf{y}}^{\mathcal{H}}(n)\}$:

$$\mathbf{R}_{\bar{s}\bar{s}}=\begin{pmatrix}\mathbf{R}_{ss} & \mathbf{0}_{M\times M} & \mathbf{0}_{M\times M}\\ \mathbf{0}_{M\times M} & \mathbf{R}_{ss} & \mathbf{0}_{M\times M}\\ \mathbf{0}_{M\times M} & \mathbf{0}_{M\times M} & \mathbf{R}_{ss}\end{pmatrix},$$

and $\mathbf{R}_{\bar{y}\bar{y}}=\mathbf{H}_F\mathbf{R}_{\bar{s}\bar{s}}\mathbf{H}_F^{\mathcal{H}}+\mathbf{R}_{\bar{v}\bar{v}}$, where:

$$\mathbf{H}_F:=\begin{pmatrix}\mathbf{H}_0\mathbf{F}_0 & \mathbf{H}_1\mathbf{F}_0 & \mathbf{0}_{P\times M}\\ \mathbf{0}_{P\times M} & \mathbf{H}_0\mathbf{F}_0 & \mathbf{H}_1\mathbf{F}_0\\ \mathbf{0}_{P\times M} & \mathbf{0}_{P\times M} & \mathbf{H}_0\mathbf{F}_0\end{pmatrix}, \text{ and}$$

$$\mathbf{R}_{\bar{v}\bar{v}}:=\begin{pmatrix}\mathbf{R}_{vv} & \mathbf{0}_{P\times P} & \mathbf{0}_{P\times P}\\ \mathbf{0}_{P\times P} & \mathbf{R}_{vv} & \mathbf{0}_{P\times P}\\ \mathbf{0}_{P\times P} & \mathbf{0}_{M\times M} & \mathbf{R}_{vv}+\mathbf{H}_1\mathbf{F}_0\mathbf{R}_{ss}(\mathbf{H}_1\mathbf{F}_0)^{\mathcal{H}}\end{pmatrix}.$$

Then, it is proved in [7] that the feedforward filter is given by

$$(\mathbf{W}_{-1}\,\mathbf{W}_0\,\mathbf{W}_1)=(\mathbf{0}_{M\times M}\,\mathbf{U}_{22}\,\mathbf{U}_{23})\mathbf{R}_{\bar{s}\bar{s}}\mathbf{H}_F^{\mathcal{H}}\mathbf{R}_{\bar{y}\bar{y}}^{-1},$$

and the feedback filter is given by

$$\mathbf{B}_0+\mathbf{I}_{M\times M}=\mathbf{U}_{22},\ \mathbf{B}_1=\mathbf{U}_{23},$$

where $\mathbf{U}_{22}$, $\mathbf{U}_{23}$ are $M\times M$ submatrices of the $3M\times 3M$ matrix $\mathbf{U}_{3M\times 3M}$:

$$\mathbf{U}_{3M\times 3M}=\begin{pmatrix}\mathbf{U}_{11} & \mathbf{U}_{12} & \mathbf{U}_{13}\\ \mathbf{0}_{M\times M} & \mathbf{U}_{22} & \mathbf{U}_{23}\\ \mathbf{0}_{M\times M} & \mathbf{0}_{M\times M} & \mathbf{U}_{33}\end{pmatrix},$$

and $\mathbf{U}_{3M\times 3M}$ is given by the Cholesky decomposition $\mathbf{R}_{\bar{s}\bar{s}}+\mathbf{H}_F^{\mathcal{H}}\mathbf{R}_{\bar{v}\bar{v}}^{-1}\mathbf{H}_F=\mathbf{U}_{3M\times 3M}^{\mathcal{H}}\mathbf{D}_{3M\times 3M}\mathbf{U}_{3M\times 3M}$.

V. Simulations

As the optimal precoder has already been shown to obey the "water-filling" principle [4], here we are interested in studying the BER performance of our transceiver system for the two cases where IBI is absent/present. We use Monte Carlo simulations assuming BPSK modulation, and we present two examples: in the first example we assume $P=M+L$ (which results in absence of IBI) and we compare the optimal precoder to the OFDM precoder. Specifically, we used $M=32$, $P=36$ for an FIR channel of order $L=4$ with zeros at 1, $0.9\exp(j9\pi/20)$, $1.1\exp(-j9\pi/20)$, -0.8. Figure 2 depicts the BER performance as a function of E_b/N_o, where $E_b=E_s=\frac{1}{M}\mathrm{tr}\{\mathbf{F}\mathbf{F}^{\mathcal{H}}\}$. The input correlation matrix is taken to be $\mathbf{R}_{ss}=\mathbf{I}_{M\times M}$, and the additive noise is simulated as zero-mean white with autocorrelation matrix $\mathbf{R}_{vv}=\sigma_v^2\mathbf{I}_{P\times P}$, $\sigma_v^2=N_o$. The OFDM precoder matrix is $\mathbf{F}=[\mathbf{F}]_{m,n}$ with trailing zeros; i.e.,

$$\mathbf{F}_{m,n}=\begin{cases} e^{j\frac{2\pi}{M}mn}: 0\leq m\leq M-1, 0\leq n\leq M-1\\ 0 \quad : M\leq m\leq P-1, 0\leq n\leq M-1\end{cases}.$$

It is clear that the optimal precoder outperforms the OFDM precoder. We also note that for the optimal precoder, the performance of the linear and DFE receiver is identical as expected.

In the second example we study the performance of linear and DFE receivers when IBI is present. Using the channel of example 1 and by taking $M=32$, $P=34$, we illustrate in Figure 3 the BER performance of the three receivers when the optimal precoder is used. We deduce from Figure 3 that the DFE receivers outperform the linear receiver. Therefore, when IBI is present a DFE receiver does offer performance improvement. Though the optimal precoder has diagonalized the channel, there is still ISI in each of the independent subchannels. A DFE receiver exploits the finite alphabet of the input symbols to remove the ISI from each of the subchannels, which gives rise to better BER performance.

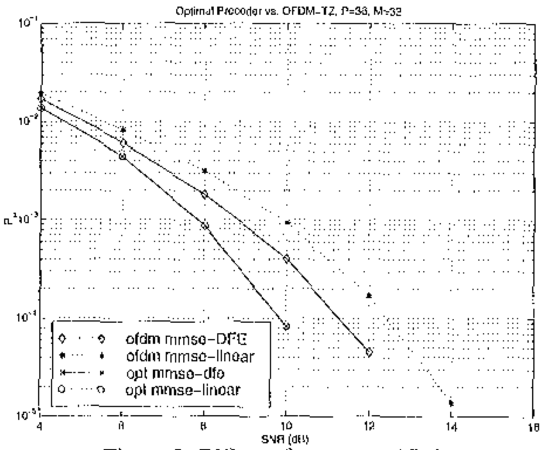


Figure 2: BER performance, IBI absent

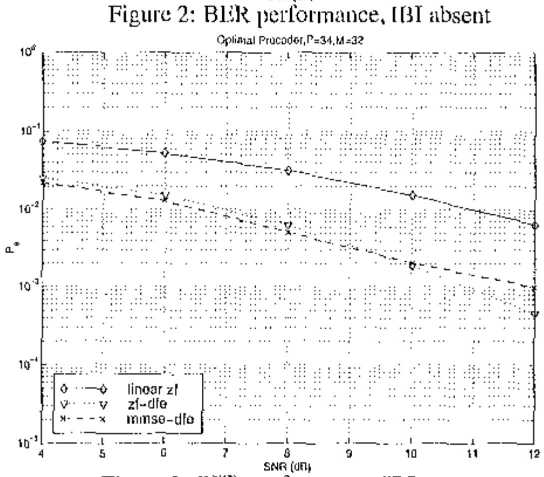


Figure 3: BER performance, IBI present

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