Multi-User Spreading Codes Retaining Orthogonality through Unknown Time- and Frequency-Selective Fading

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***Abstract*— Suppression of Multi-User Interference (MUI) and mitigation of time- and frequency-selective effects constitute major challenges in the design of third-generation wireless mobile systems. Relying on block spreading and judiciously chosen time-frequency guard intervals, we propose a multi-user transceiver that eliminates MUI deterministically and guarantees symbol detectability in the presence of unknown time- and frequency-selective fading. Blind channel estimation is also investigated. Simulation results demonstrate the validity of the theoretical results and show improved performance of the proposed transceiver over a multi-user time-frequency RAKE receiver.**

I. Introduction

Recently, various multi-user transceivers have been proposed that eliminate MUI deterministically and guarantee symbol detectability in the presence of unknown frequency-selective fading [2], [8], [3], [9]. They serve as attractive alternatives to multi-user RAKE receivers [10], which are very complex, do not guarantee symbol detectability, and require the knowledge of the spreading codes of all active users. However, all these transceivers are based on the assumption that the channels are time-invariant over the transmitted frame. In a practical scenario, this assumption may not hold true due to high-mobility and carrier frequency and phase drifts. Relying on block spreading and judiciously chosen time-frequency guard intervals, we here propose a multi-user transceiver that eliminates MUI deterministically and guarantees symbol detectability in the presence of unknown time- and frequency-selective fading. It serves as an attractive alternative to multi-user time-frequency RAKE receivers [5], which, like the multi-user RAKE receivers, are very complex, do not guarantee symbol detectability, and require the knowledge of the spreading codes of all active users.

Through judicious block spreading code design at the transmitter, the proposed orthogonal multi-user transceiver transforms a multi-user communication problem into a set of parallel single user communication problems with matched filtering, regardless of the underlying time- and frequency- selective channels. Compared with multi-user time-frequency RAKE receivers, the proposed transceiver has the following advantages:
i) It is in general less complex since it does not operate on the received blocks, but on the output of a multi-user separator, which has a shorter block length.
ii) It guarantees symbol detectability through controlled redundancy.
iii) It does not require any knowledge of the other users. Thus the detection is uncoordinated among users, unlike multi-user time-frequency RAKE receivers. Allowing for minimal coordination among users agrees with CDMA philosophy.

The rest of this paper is organized as follows. In Section II, we introduce the transceiver model that we will use. Section III then describes the proposed transceiver. In Section IV, we investigate blind channel estimation. Finally, simulations are presented in Section V.

Notations: We use upper (lower) bold face letters to denote matrices (column vectors). Superscripts *, ^{T}, and ^{H} represent conjugate, transpose, and Hermitian, respectively. We reserve $\mathrm{E}\{\cdot\}$ for expectation, and $\lfloor\cdot\rfloor$ for integer flooring. We denote the discrete impulse function as $\delta[n]$. The $M\times N$ all-zero matrix is denoted as $\mathbf{0}_{M\times N}$, the $N\times N$ identity matrix as $\mathbf{I}_N$, and the $N\times N$ unitary FFT matrix as $\mathbf{F}_N$. We define $\mathbf{i}_N[n]$ as the $(n+1)$th column of $\mathbf{I}_N$ and $\mathbf{f}_N[n]$ as the $(n+1)$th column of $\mathbf{F}_N$. Furthermore, we define $\mathbf{\Lambda}_N(\nu)$ as the $N\times N$ diagonal matrix with main diagonal $[1, e^{-j2\pi\nu},\ldots,e^{-j2\pi(N-1)\nu}]$. Finally, we define $[\mathbf{A}]_{m,n}$ as the $(m+1,n+1)$th entry of matrix $\mathbf{A}$.

II. Transceiver Model

The block diagram in Fig. 1 describes the CDMA system with block spreading studied in this paper. Before transmission, the uth user's ($u\in\{0,\ldots,U-1\}$) symbol stream $s_u[m]$ is first serial to parallel converted into a stream of $M\times 1$ symbol blocks $\mathbf{s}_u[i] := [s_u[iM],\ldots,s_u[(i+1)M-1]]$, and then spread by an $N\times M$ matrix $\mathbf{C}_u$ to obtain a stream of $N\times 1$ chip blocks $\mathbf{x}_u[i] := \mathbf{C}_u\mathbf{s}_u[i]$. The chip blocks $\mathbf{x}_u[i]$ are finally parallel to serial converted into a chip stream

$$[x_u[iN],\ldots,x_u[(i+1)N-1]] := \mathbf{x}_u[i].$$

After chip rate sampling (the chip rate is denoted as $1/T_c$), the received sample stream can be written as

$$y[n] = \sum_{u=0}^{U-1}\sum_{\nu=-\infty}^{\infty} h_u[n;\nu]x_u[n-\nu] + \eta[n],$$

where $\eta[n]$ is the additive noise and $h_u[n;\nu]$ is the time- and frequency-selective channel for the uth user, including transmit and receive filters. We assume that the channel $h_u[n,\nu]$ can be written as

$$h_u[n,\nu] = \sum_{l=0}^{L}\delta[\nu-l]\sum_{q=-Q}^{Q} e^{-j2\pi qn/N}h_{u,q}[\lfloor n/N\rfloor;l], \quad (1)$$

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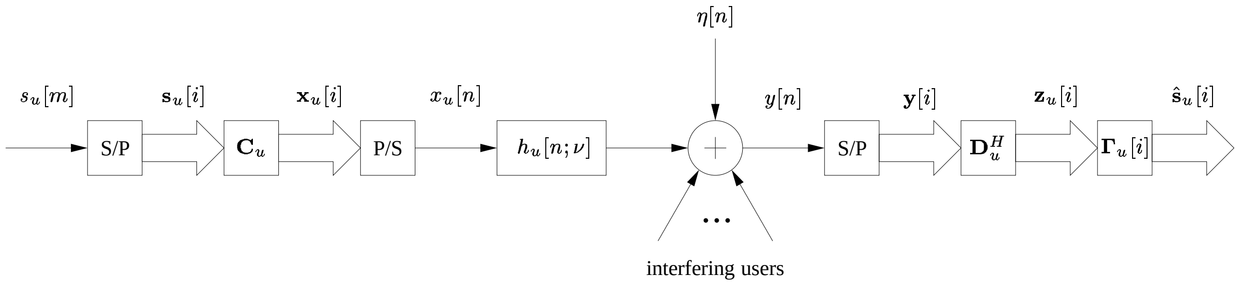

Fig. 1. Discrete-time equivalent baseband system model (only uth user shown).

with $L \ll N$ and $Q \ll N$. This model, which was also used in [1], [6], [5], [7], is valid if N is sufficiently large, the users are quasi-synchronous in time and frequency, LT_c is larger than or equal to the maximal delay spread plus time-offset of all users, and $Q/(NT_c)$ is larger than or equal to the maximal Doppler spread plus frequency-offset of all users. Analytical results revealing the fitting accuracy of this basis expansion model can be found in [4].

At the receiver, we collect received samples into vectors. Defining $\mathbf{y}[i] := [y[iN], \ldots, y[(i+1)N-1]]^T$, and $\boldsymbol{\eta}[i] := [\eta[iN], \ldots, \eta[(i+1)N-1]]^T$, the block channel input/output relationship can be described by (see [8] for time-invariant channels):

$$\mathbf{y}[i] = \sum_{u=0}^{U-1} \sum_{q=-Q}^{Q} \mathbf{\Lambda}_N\left(\tfrac{q}{N}\right)(\mathbf{H}^{(0)}_{N,u,q}[i]\mathbf{C}_u\mathbf{s}_u[i] + \mathbf{H}^{(1)}_{N,u,q}[i]\mathbf{C}_u\mathbf{s}_u[i-1]) + \boldsymbol{\eta}[i],$$

where $\mathbf{H}^{(a)}_{N,u,q}[i]$ represents the $N \times N$ Toeplitz matrix with $[\mathbf{H}^{(a)}_{N,u,q}[i]]_{n,n'} = h_{u,q}[i; n-n'+aN]$.

To eliminate the MUI deterministically and guarantee symbol detectability for the uth user, regardless of the delay and Doppler spread profiles, we despread $\mathbf{y}[i]$ by a matrix $\mathbf{D}_u$ to obtain

$$\mathbf{z}_u[i] := \mathbf{D}_u^H \mathbf{y}[i]. \tag{2}$$

On the MUI-free output $\mathbf{z}_u[i]$, we can then apply any single user equalizer to mitigate the Inter Symbol Interference (ISI). We can, e.g., adopt a linear single-user equalizer $\mathbf{\Gamma}_u[i]$ to obtain $\hat{\mathbf{s}}_u[i] := \mathbf{\Gamma}_u[i]\mathbf{z}_u[i]$.

III. Proposed Transceiver

For the proposed transceiver, we set the symbol block length to $M = PK$ and set the transmitted chip block length to $N = U(P+2Q)(K+L)$. Let us define the $(P+2Q) \times P$ zero-inserting matrix as $\mathbf{T}_1 = [\mathbf{0}_{P\times Q}, \mathbf{I}_P, \mathbf{0}_{P\times Q}]^T$ and the $(K+L) \times K$ zero-inserting matrix as $\mathbf{T}_2 = [\mathbf{I}_K, \mathbf{0}_{K\times L}]^T$. Denoting $\otimes$ as the Kronecker product, we then design $\mathbf{C}_u$ as the $N \times M$ matrix given by

$$\mathbf{C}_u = (\mathbf{G}_u\mathbf{T}_1) \otimes \mathbf{T}_2,$$

and $\mathbf{D}_u$ as the $N \times (P+2Q)(K+L)$ matrix given by

$$\mathbf{D}_u = \mathbf{G}_u \otimes \mathbf{I}_{K+L},$$

where $\mathbf{G}_u$ is the $U(P+2Q) \times (P+2Q)$ matrix defined as

$$\mathbf{G}_u := [\mathbf{f}_{U(P+2Q)}[u(P+2Q)], \ldots, \mathbf{f}_{U(P+2Q)}[(u+1)(P+2Q)-1]].$$

Note that a more general transceiver design for time- and frequency-selective channels is described in [4].

For time-invariant channels, we can set $Q = 0$ and $P = 1$, such that $\mathbf{C}_u = \mathbf{f}_U[u] \otimes \mathbf{T}_2$, which reduces to the Chip Interleaved Block Spread (CIBS) CDMA proposed in [9] with the user signature codes being taken from an FFT matrix. Unlike [9], which focuses on time-invariant channels and only introduces redundancy (by $\mathbf{T}_2$) in the time domain, we here deal with time-varying channels and introduce redundancy both in the time domain (by $\mathbf{T}_2$) and in the frequency domain (by $\mathbf{T}_1$). This is illustrated in Fig. 2, where we show a time-frequency plot of $\{\mathbf{x}_u[i] = \mathbf{C}_u\mathbf{s}_u[i]\}_{u=0}^{U-1}$.

One immediate consequence of the design of $\mathbf{C}_u$ is that Inter Block Interference (IBI) is removed. Because the last L rows of $\mathbf{C}_u$ are zero, $\mathbf{H}^{(1)}_{N,u,q}[i]\mathbf{C}_u = \mathbf{0}_{N\times M}$. We then only need to perform block by block processing based on the following IBI-free blocks:

$$\mathbf{y}[i] = \sum_{u=0}^{U-1} \sum_{q=-Q}^{Q} \mathbf{\Lambda}_N\left(\tfrac{q}{N}\right)\mathbf{H}^{(0)}_{N,u,q}[i]\mathbf{C}_u\mathbf{s}_u[i] + \boldsymbol{\eta}[i]. \tag{3}$$

We next show that the designed transceiver pairs $\{\mathbf{C}_u, \mathbf{D}_u\}_{u=0}^{U-1}$ can indeed achieve deterministic multi-user separation and guaranteed symbol detectability, without knowing the underlying time- and frequency-selective channels.

In the following derivations, we will continuously make use of the formula for the product of Kronecker products of matrices with matching dimensions, which is given by

$$(\mathbf{A}_1 \otimes \mathbf{A}_2)(\mathbf{A}_3 \otimes \mathbf{A}_4) = (\mathbf{A}_1\mathbf{A}_3) \otimes (\mathbf{A}_2\mathbf{A}_4). \tag{4}$$

Let us also introduce the notation $\mathbf{J}_{N,q}$, which represents the $N \times N$ Toeplitz matrix with $[\mathbf{J}_{N,q}]_{n,n'} = \delta[n-n'-q]$. First,

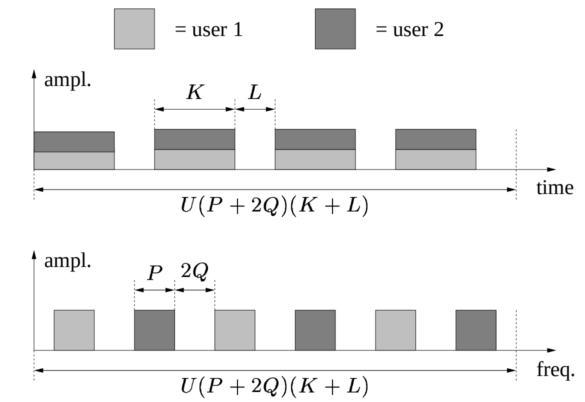

Fig. 2. Time-frequency representation of $\{\mathbf{x}_u[i] = \mathbf{C}_u\mathbf{s}_u[i]\}_{u=0}^{U-1}$.

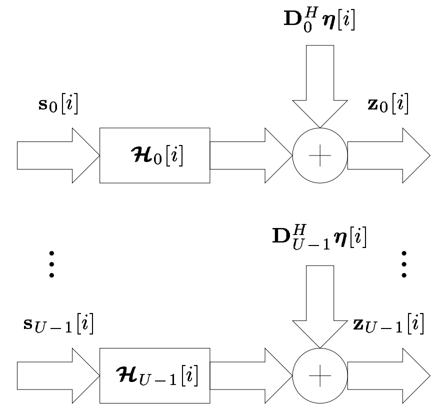

Fig. 3. Resulting set of parallel single user systems.

we note that $\mathbf{H}_{N,u,q}^{(0)}$ can be expressed as (see [9] for details):

$$\mathbf{H}_{N,u,q}^{(0)}[i] = (\mathbf{I}_{U(P+2Q)} \otimes \mathbf{H}_{K+L,u,q}^{(0)}[i] + \mathbf{J}_{U(P+2Q),1} \otimes \mathbf{H}_{K+L,u,q}^{(1)}[i]) \quad (5)$$

Using (5) and the fact that $\mathbf{H}_{K+L,u,q}^{(1)}[i]\mathbf{T}_2 = \mathbf{0}_{(K+L)\times K}$, we obtain:

$$\begin{aligned}
&\mathbf{H}_{N,u,q}^{(0)}[i][(\mathbf{G}_u\mathbf{T}_1) \otimes \mathbf{T}_2] \\
&\quad = (\mathbf{I}_{U(P+2Q)} \otimes \mathbf{H}_{K+L,u,q}^{(0)}[i])[(\mathbf{G}_u\mathbf{T}_1) \otimes \mathbf{T}_2] \\
&\qquad + (\mathbf{J}_{U(P+2Q),1} \otimes \mathbf{H}_{K+L,u,q}^{(1)}[i])[(\mathbf{G}_u\mathbf{T}_1) \otimes \mathbf{T}_2] \\
&\quad = (\mathbf{G}_u\mathbf{T}_1) \otimes (\mathbf{H}_{K+L,u,q}^{(0)}[i]\mathbf{T}_2) + \mathbf{0}_{N\times M} \\
&\quad = [(\mathbf{G}_u\mathbf{T}_1) \otimes \mathbf{I}_{K+L}][\mathbf{I}_P \otimes \underbrace{(\mathbf{H}_{K+L,u,q}^{(0)}[i]\mathbf{T}_2)}_{:= \mathbf{H}_{u,q}[i]}].
\end{aligned} \quad (6)$$

Next, it can be shown that

$$\begin{aligned}
&\mathbf{\Lambda}_N\left(\tfrac{q}{N}\right)[(\mathbf{G}_u\mathbf{T}_1) \otimes \mathbf{I}_{K+L}] \\
&= \left(\mathbf{\Lambda}_{U(P+2Q)}\left(\tfrac{q}{U(P+2Q)}\right) \otimes \mathbf{\Lambda}_{K+L}\left(\tfrac{q}{N}\right)\right) \\
&\qquad \cdot [(\mathbf{G}_u\mathbf{T}_1) \otimes \mathbf{I}_{K+L}] \\
&= \left(\mathbf{\Lambda}_{U(P+2Q)}\left(\tfrac{q}{U(P+2Q)}\right)\mathbf{G}_u\mathbf{T}_1\right) \otimes \underbrace{\mathbf{\Lambda}_{K+L}\left(\tfrac{q}{N}\right)}_{:= \mathbf{\Delta}_q}.
\end{aligned} \quad (7)$$

In the Appendix, we also prove that

$$\mathbf{\Lambda}_{U(P+2Q)}\left(\tfrac{q}{U(P+2Q)}\right)\mathbf{G}_u\mathbf{T}_1 = \mathbf{G}_u\mathbf{J}_{P+2Q,q}\mathbf{T}_1. \quad (8)$$

Defining $\mathbf{J}_q := \mathbf{J}_{P+2Q,q}\mathbf{T}_1$ and using (8), we can further simplify (7) as:

$$\mathbf{\Lambda}_N\left(\tfrac{q}{N}\right)[(\mathbf{G}_u\mathbf{T}_1) \otimes \mathbf{I}_{K+L}] = (\mathbf{G}_u \otimes \mathbf{I}_{K+L})(\mathbf{J}_q \otimes \mathbf{\Delta}_q). \quad (9)$$

From (6), (7), (9), and $\mathbf{G}_u^H\mathbf{G}_{u'} = \delta[u'-u]\,\mathbf{I}_{P+2Q}$, it is finally clear that

$$\begin{aligned}
&\mathbf{D}_u^H \sum_{q=-Q}^{Q} \mathbf{\Lambda}_N\left(\tfrac{q}{N}\right)\mathbf{H}_{N,u',q}^{(0)}[i]\mathbf{C}_{u'} \\
&\qquad = \delta[u'-u] \sum_{q=-Q}^{Q} (\mathbf{J}_q \otimes \mathbf{\Delta}_q)(\mathbf{I}_P \otimes \mathbf{H}_{u,q}[i]).
\end{aligned} \quad (10)$$

Hence, the designed transceiver pairs $\{\mathbf{C}_u, \mathbf{D}_u\}_{u=0}^{U-1}$ eliminate the MUI deterministically, regardless of the delay and Doppler spread profiles. Using (10) to (2) and (3), we arrive at the following single user output:

$$\mathbf{z}_u[i] = \mathbf{D}_u^H\mathbf{y}[i] = \boldsymbol{\mathcal{H}}_u[i]\mathbf{s}_u[i] + \mathbf{D}_u^H\boldsymbol{\eta}[i], \quad (11)$$

where $\boldsymbol{\mathcal{H}}_u[i]$ is the $(P+2Q)(K+L) \times M$ matrix that is given by $\boldsymbol{\mathcal{H}}_u[i] := \sum_{q=-Q}^{Q}(\mathbf{J}_q \otimes \mathbf{\Delta}_q)(\mathbf{I}_P \otimes \mathbf{H}_{u,q}[i]) = \sum_{q=-Q}^{Q}\mathbf{J}_q \otimes (\mathbf{\Delta}_q\mathbf{H}_{u,q}[i])$, which has the following form:

$$\boldsymbol{\mathcal{H}}_u[i] = \begin{bmatrix} \mathbf{\Delta}_{-Q}\mathbf{H}_{u,-Q}[i] & & \\ \vdots & \ddots & \\ \mathbf{\Delta}_Q\mathbf{H}_{u,Q}[i] & & \mathbf{\Delta}_{-Q}\mathbf{H}_{u,-Q}[i] \\ & \ddots & \vdots \\ & & \mathbf{\Delta}_Q\mathbf{H}_{u,Q}[i] \end{bmatrix}.$$

Therefore, a multi-user communication problem has been converted into a set of parallel single user communication problems, as depicted in Fig. 3. More important, since the matrix $\mathbf{D} := [\mathbf{D}_0, \ldots, \mathbf{D}_{U-1}]$ is square unitary, *Maximum Likelihood (ML) optimality is preserved in this multi-user separation step*, as in the case of [9] for time-invariant channels. On the other hand, since the nonzero matrices from the set $\{\mathbf{\Delta}_q\mathbf{H}_{u,q}[i]\}_{q=-Q}^{Q}$ always have full column rank, $\boldsymbol{\mathcal{H}}_u[i]$ always has full column rank. Hence, the proposed choice for $\{\mathbf{C}_u, \mathbf{D}_u\}_{u=0}^{U-1}$ guarantees symbol detectability, regardless of the delay and Doppler spread profiles.

On the MUI-free output $\mathbf{z}_u[i]$, we can then apply any single-user equalizer to mitigate the ISI. Here for brevity, we assume that the additive noise $\boldsymbol{\eta}[i]$ is white with covariance matrix $\boldsymbol{R}_\eta = \sigma_\eta^2 \boldsymbol{I}_N$. Because the matrix $\mathbf{D}_u$ is unitary, the resulting noise $\mathbf{D}_u^H \boldsymbol{\eta}[i]$ is still white. With white noise, we can, e.g., adopt the linear ZF equalizer given by

$$\mathbf{\Gamma}_u^{\mathrm{zf}}[i] = (\boldsymbol{\mathcal{H}}_u^H[i]\boldsymbol{\mathcal{H}}_u[i])^{-1}\boldsymbol{\mathcal{H}}_u^H[i],$$

or the linear MMSE equalizer given by

$$\mathbf{\Gamma}_u^{\mathrm{mmse}}[i] = \left(\sigma_s^2 \boldsymbol{\mathcal{H}}_u^H[i]\boldsymbol{\mathcal{H}}_u[i] + \sigma_\eta^2 \mathbf{I}\right)^{-1} \sigma_s^2 \boldsymbol{\mathcal{H}}_u^H[i],$$

where $\sigma_s^2 = \mathrm{E}\{s[n]s^*[n]\}$. The extension to colored noise is straightforward.

A. Spectral Efficiency

Within the $N \times 1$ received block $\mathbf{y}[i]$, M symbols are transmitted per user. Hence, the spectral efficiency is

$$\mathcal{E} = \frac{UM}{N} = \frac{PK}{(P+2Q)(K+L)}.$$

By tuning the parameters, we can change the spectral efficiency. However, keep in mind that (1) is only valid if LT_c is larger than or equal to the maximal delay spread plus time-offset of all users and $Q/(NT_c)$ is larger than or equal to the maximal Doppler spread plus frequency-offset of all users. Hence, the minimal values for L and Q/N are determined by the chip rate, the propagation environment and the time- and frequency-offsets. Maximum spectral efficiency designs are discussed in [4].

B. Complexity

The spreading operation requires $U(P+2Q)PK$ multiply-add operations per user and block. Hence, the spreading complexity is $\mathcal{O}\{U(P+2Q)PK\}$ per user and block.

The despreading operation requires $U(P+2Q)^2(K+L)$ multiply-add operations per user and block. Hence, the despreading complexity is $\mathcal{O}\{U(P+2Q)^2(K+L)\}$ per user and block. Let us now consider the despreading complexity at the base station (BS), where we need to extract all users' information. At first sight, it seems that the BS's despreading complexity is $\mathcal{O}\{U^2(P+2Q)^2(K+L)\}$ per block. However, by making use of a $U(P+2Q)$-point FFT, the BS's despreading complexity reduces to $\mathcal{O}\{U(P+2Q)\log_2(U(P+2Q))(K+L)\}$ per block.

Finally, the complexity of the single-user equalizer depends on the type of equalizer that is used. For the linear ZF or MMSE equalizer described earlier, we obtain a complexity of $\mathcal{O}\{(P+2Q)(K+L)P^2K^2\}$ per block; or $\mathcal{O}\{(P+2Q)(K+L)PK\}$ per symbol.

IV. Blind Channel Estimation

To perform channel equalization, channel knowledge is required at the receiver. Since the multi-user system is converted into a set of parallel single user systems, single user channel estimation methods can be applied. Here, we will focus on blind channel estimation. However, it is also possible to insert training symbols and use training-based or semi-blind channel estimation at the expense of a decreased spectral efficiency [7].

Suppose that U is a multiple of G and that the users $\{u_{\mathrm{sup}}G + g\}_{g=0}^{G-1}$, with $u_{\mathrm{sup}} \in \{0,\ldots,U/G-1\}$, are actually one and the same user, referred to as the u_{sup}th super user. For this u_{sup}th super user, we can write

$$\underbrace{[\mathbf{z}_{u_{\mathrm{sup}}G}[i],\ldots,\mathbf{z}_{u_{\mathrm{sup}}G+G-1}[i]]}_{\mathbf{Z}_{u_{\mathrm{sup}}}[i]} = \boldsymbol{\mathcal{H}}_{u_{\mathrm{sup}}G}[i]\overbrace{[\mathbf{s}_{u_{\mathrm{sup}}G}[i],\ldots,\mathbf{s}_{u_{\mathrm{sup}}G+G-1}[i]]}^{\mathbf{S}_{u_{\mathrm{sup}}}[i]}.$$

On this data model we can then apply the second deterministic blind single user channel estimation method presented in [6] (method II). Using this method, $\boldsymbol{\mathcal{H}}_{u_{\mathrm{sup}}G}[i]$ is identifiable from $\mathbf{Z}_{u_{\mathrm{sup}}}[i]$ if and only if $\mathbf{S}_{u_{\mathrm{sup}}}[i]$ has full row rank. Hence, identifiability is independent of $\boldsymbol{\mathcal{H}}_{u_{\mathrm{sup}}G}[i]$. Note that for $\mathbf{S}_{u_{\mathrm{sup}}}[i]$ to have full row rank, it is necessary that $M \leq G$.

V. Simulation Results

In this section, we illustrate the ideas presented in this paper with computer simulations. We consider BPSK modulation and additive white Gaussian noise.

Test Case 1: Let's first consider the same fading scenario as in [5] and compare the proposed transceiver with the multi-user time-frequency RAKE receiver [5]. As in [5], we take $N = 63$, $L = 1$, $Q = 1$, and generate a set of independent channels $\{h_u[n;\nu]\}_{u=0}^{U-1}$ (see (1)) with $h_{u,q}[i;l]$ complex Gaussian distributed with variance 0.9 if $q = 0$ and 0.05 if $q = \pm 1$. We assume that the receiver knows the channels $\{h_u[n;\nu]\}_{u=0}^{U-1}$. For the proposed transceiver, N corresponds to the length of the transmitted chip block, while for the multi-user time-frequency RAKE receiver, N corresponds to the CDMA spreading factor. For the proposed transceiver, we consider $U = 7$, $P = 1$, and $K = 2$ (hence, $N = U(P+2Q)(K+L) = 63$). Note that in $N = 63$ chip periods, the proposed transceiver can handle 2 bits per user (the symbol block length is $M = PK = 2$), while the multi-user time-frequency RAKE receiver can only handle 1 bit per user. Keeping this important rate difference in mind, Fig. 4 depicts a comparison between the performance of the proposed transceiver with linear ZF equalization and the performance of the linear ZF multi-user time-frequency RAKE receiver (the latter performance corresponds to the one shown in [5, Fig. 3]). From Fig. 4, we observe that the performance of the proposed transceiver accommodating 7 users (that transmit 2 bits/63 chips) is comparable to the performance of the multi-user time-frequency RAKE receiver accommodating only 2 users (that transmit 1 bit/63 chips), and much better than the performance of the multi-user time-frequency RAKE receiver accommodating 7 users (that transmit 1 bit/63 chips). More specifically, we gain about 4 dB for a BER of 10^{-3}.

Test Case 2: Since the previous fading scenario is rather artificial, let's now consider a more realistic fading scenario. We generate a set of independent channels $\{\tilde{h}_u[n;\nu]\}_{u=0}^{U-1}$, with

$$\tilde{h}_u[n;\nu] = \sum_{l=0}^{L_u-1} \delta[\nu - l] \sum_{q=0}^{Q_{u,l}-1} A_{u,l,q} e^{j\phi_{u,l,q}} e^{-j2\pi f_{u,l,q} n T_c},$$

where L_u is the number of delays for the uth user, $Q_{u,l}$ is the number of Doppler shifts for the lth delay of the uth user, and $A_{u,l,q}$, $\phi_{u,l,q}$, and $f_{u,l,q}$ are the amplitude, phase and Doppler frequency for the qth Doppler shift of the lth delay of the uth user, respectively. We adopt Jakes' model and take $L_u = 3$, $Q_{u,l} = 100$, and $f_{u,l,q} = 80 \cos(2\pi q/Q_{u,l})$ (this corresponds to a relative mobile speed of 96 km/h for a carrier frequency of 900 MHz). Further, we consider $A_{u,l,q} = 1/\sqrt{Q_{u,l}}$ and $\phi_{u,l,q}$ a uniformly distributed random variable in $[0, 2\pi)$. Applying this fading scenario, the channel $\tilde{h}_u[n;\nu]$, defined above, can not exactly be modeled by a channel $h_u[n;\nu]$, defined in (1), but can be approximated (in LS sense) by a channel $h_u[n;\nu]$, when $L \geq 2$ and $Q \geq 80NT_c$ (because $Q/(NT_c)$ should be larger than or equal to the maximal Doppler spread of all users, which is chosen to be 80 Hz). For a transmitted block size of $N = 1000$ and a chip rate of $1/T_c = 20$ kHz, we need $Q \geq 4$. We choose the following transceiver parameters: $P = 17$, $Q = 4$, $K = 8$, $L = 2$ and $U = 4$ (hence, $N = U(P+2Q)(K+L) = 1000$). Again assuming that the receiver knows the channels $\{h_u[n;\nu]\}_{u=0}^{U-1}$, Fig. 5 shows the performance of the proposed transceiver using linear ZF equalization for two scenarios: 1) we use the true channels $\{\tilde{h}_u[n;\nu]\}_{u=0}^{U-1}$ for propagation; 2) we use the approximate channels $\{h_u[n;\nu]\}_{u=0}^{U-1}$ for propagation. The difference in performance between these two scenarios is a measure for the validity of (1). We see that the performance of scenario 1 is comparable to the performance of scenario 2 up to a certain SNR per user. At higher SNR's per user, modeling errors cause the performance of scenario 1 to saturate.

Appendix: Proof of (8)

The matrices in both sides of (8) are $U(P+2Q) \times PK$. We will prove (8) column by column. The ith column of $\mathbf{\Lambda}_{U(P+2Q)}\left(\frac{q}{U(P+2Q)}\right)\mathbf{G}_u\mathbf{T}_1$ is the $(Q+i)$th column of $\mathbf{\Lambda}_{U(P+2Q)}\left(\frac{q}{U(P+2Q)}\right)\mathbf{G}_u$ due to the specific structure of $\mathbf{T}_1$, and thus can be written as:

$$\mathbf{\Lambda}_{U(P+2Q)}\left(\frac{q}{U(P+2Q)}\right)\mathbf{f}_{U(P+2Q)}[u(P+2Q)+Q+i]$$
$$= \mathbf{f}_{U(P+2Q)}[u(P+2Q)+Q+i+q] \quad (12)$$

which can easily be verified.

Similarly, the ith column of $(\mathbf{G}_u\mathbf{J}_{P+2Q,q})\mathbf{T}_1$ is the $(Q+i)$th column of $\mathbf{G}_u\mathbf{J}_{P+2Q,q}$. Because $\mathbf{J}_{P+2Q,q}$ is a column shifting matrix, the $(Q+i)$th column of $\mathbf{G}_u\mathbf{J}_{P+2Q,q}$ will be the $(Q+q+i)$th column of $\mathbf{G}_u$, which is $\mathbf{f}_{U(P+2Q)}[u(P+2Q)+Q+i+q]$. Comparing the latter with (12), we have verified (8).

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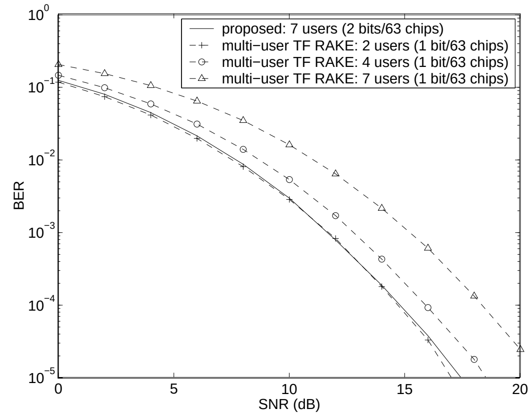

Fig. 4. Average BER as a function of the SNR per user.

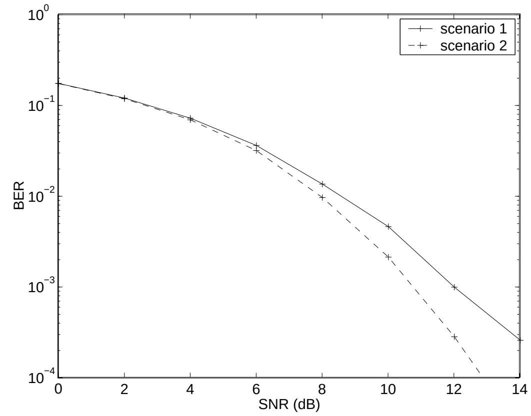

Fig. 5. Comparison between true channels and approximate channels.

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