Space-Time-Multipath Coding Using Digital Phase Sweeping*

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Abstract—**We propose novel space-time multipath (STM) coded multi-antenna transmissions over frequency-selective Rayleigh fading channels. We develop STM coded systems that guarantee the maximum possible space-multipath diversity without rate loss for any number of transmit-antennae, and with large coding gains within the class of linearly coded systems. By incorporating subchannel grouping, we also enable desirable tradeoffs between performance and complexity. The merits of our design are confirmed by corroborating simulations, and comparisons with existing approaches.**

I. Introduction

Broadband wireless communications call for high data-rate and high performance. When the symbol duration is smaller than the channel delay spread, frequency-selective propagation effects arise. Therefore, it is important for broadband wireless applications to design single- or multi-antenna systems that account for frequency-selective multipath channels.

Space-time (ST) coded multi-antenna transmissions over flat fading channels have been well documented; see e.g., [10]. ST coding for frequency-selective channels has also been pursued recently using single-carrier [1,13], or, multi-carrier transmissions [2,5,6]. The code designs in [2,6] do not guarantee full space-multipath diversity. Those in [5,13] guarantee full diversity, but as they rely on ST block codes [10], they incur rate loss up to 50%, when the number of transmit antennas is greater than two.

Delay diversity schemes transmit one symbol over two antennas in two different time slots [3,8,9]. Related to delay diversity is a so-termed phase sweeping transmission that creates time-variations to an originally slow-fading channel [4]. Unfortunately, both analog phase-sweeping and delay-diversity approaches [3,4,8,9] consume extra bandwidth, and they do not enjoy joint space-multipath diversity.

In this paper, we design a multi-carrier space-time multipath (STM) coded system, which guarantees full space-multipath diversity and large coding gains with high bandwidth efficiency.

Notation: Upper (lower) bold face letters will be used for matrices (column vectors). Superscript $^{\mathcal{H}}$ will denote Hermitian, * conjugate, ^{T} transpose, and † pseudo-inverse. We will reserve $\otimes$ for the Kronecker product, and $\mathrm{E}[\cdot]$ for expectation. We will use $[\mathbf{A}]_{k,m}$ to denote the $(k+1, m+1)$st entry of a matrix $\mathbf{A}$, $\mathrm{tr}(\mathbf{A})$ for its trace, and $[\mathbf{x}]_m$ to denote the $(m+1)$st entry of the column vector $\mathbf{x}$; $\mathbf{I}_N$ will denote the $N \times N$ identity matrix, and $\mathbf{F}_N$ the $N \times N$ normalized (unitary) FFT matrix; $\mathrm{diag}[\mathbf{x}]$ will stand for a diagonal matrix with $\mathbf{x}$ on its main diagonal.

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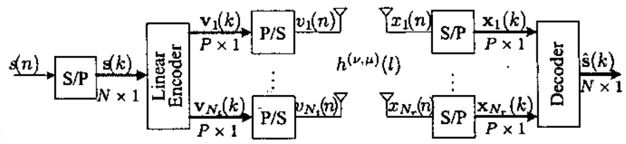

Fig. 1. A discrete-time model of linearly coded systems

II. System Model

Figure 1 depicts a multi-antenna wireless system with N_t transmit- and N_r receive-antennas. The information bearing symbols $\{s(n)\}$ are drawn from a finite alphabet $\mathcal{A}_s$, and parsed into blocks of size $N \times 1$: $\mathbf{s}(k) := [s(kN), \ldots, s((k+1)N-1)]^T$. If the mapping from $\mathbf{s}(k)$ to $\mathbf{v}_\mu(k)$ satisfies

$$\mathbf{v}_\mu(k) = \sum_{n=0}^{N-1} \mathbf{a}_n^{(\mu)}[\mathbf{s}(k)]_n + \mathbf{b}_n^{(\mu)}[\mathbf{s}(k)]_n^*, \;\; \forall \mu \in [1, N_t], \quad (1)$$

where $\mathbf{a}_n^{(\mu)}$ and $\mathbf{b}_n^{(\mu)}$ are $P \times 1$ vectors, then we call this ST transmitter a *linearly coded* one. Notice that not only symbols but also their complex conjugates are linearly combined to form the codeword $\mathbf{v}_\mu(k)$ transmitted from the μth antenna during the kth block interval.

The fading channel between the μth transmit- and the νth receive-antenna is assumed to be frequency-selective but time-flat. It is described by the discrete-time baseband equivalent impulse response vector:

$$\mathbf{h}^{(\nu,\mu)} := [h^{(\nu,\mu)}(0), \ldots, h^{(\nu,\mu)}(L)]^T, \text{with } L := \left\lfloor \frac{\tau_{\max}}{T_s} \right\rfloor, \quad (2)$$

where $\tau_{\max}$ is the maximum delay among all paths (delay spread), T_s is the symbol sampling period, and L denotes the maximum order of all (ν, μ) channels.

Each receive-antenna output comprises a noisy superposition of the multi-antenna transmissions through the fading channels. We assume ideal carrier synchronization, timing and symbol-rate sampling. At the νth receive-antenna, the symbol rate sampled sequence $x_\nu(n)$ at the receive-filter output is

$$x_\nu(n) = \sum_{\mu=1}^{N_t} \sum_{l=0}^{L} h^{(\nu,\mu)}(l) v_\mu(n-l) + \zeta_\nu(n), \quad (3)$$

where $v_\mu(n) := [\mathbf{v}_\mu(k)]_n$, and $\zeta_\nu(n)$ is complex additive white Gaussian noise (AWGN) with mean zero and variance $\sigma_\zeta^2 = N_0$.

The symbols $x_\nu(n)$ are serial-to-parallel (S/P) converted to form $P\times 1$ blocks $\mathbf{x}_\nu(k) := [x_\nu(kP), \ldots, x_\nu(kP+P-1)]^T$. The matrix-vector counter part of (3) is

$$\mathbf{x}_\nu(k) = \sum_{\mu=1}^{N_t}\Big(\mathbf{H}^{(\nu,\mu)}\mathbf{v}_\mu(k) + \mathbf{H}_{ibi}^{(\nu,\mu)}\mathbf{v}_\mu(k-1)\Big) + \zeta_\nu(k), \quad (4)$$

where $\mathbf{H}^{(\nu,\mu)}$ is a lower triangular Toeplitz matrix with first column $[h^{(\nu,\mu)}(0), \ldots, h^{(\nu,\mu)}(L), 0, \ldots, 0]^T$, $\mathbf{H}_{ibi}^{(\nu,\mu)}$ is an upper triangular Toeplitz matrix with first row $[0, \ldots, 0, h^{(\nu,\mu)}(L), \ldots, h^{(\nu,\mu)}(1)]$, and $\zeta_\nu(k)$ is the AWGN vector.

In this paper, we develop a linearly coded system which collects the maximum joint space-multipath diversity as well as large coding gains. Since in the following we will work on a block-by-block basis, we will drop the block index k.

III. Design Criteria and Problem Statement

Here we introduce criteria for designing our STM codes. Our derivations are based on the following assumptions:
A1) *Channel taps $\{h^{(\nu,\mu)}(l)\}$ are zero-mean, complex Gaussian random variables;*
A2) *Channel state information (CSI) is available at the receiver, but unknown at the transmitter;*
A3) *High SNR is considered for deriving the STM diversity and coding gains.*
When transmissions experience rich scattering, and no line-of-sight is present, the central limit theorem validates A1). Notice that we allow not only for independent random channel coefficients, but also for correlated ones. A3) is needed for the criteria, but is not required for the system operation.

The optimal performance of multi-antenna systems in frequency-selective channels has been considered in e.g., [5,13,6]. Since our design will allow for correlated channels, we will denote the $N_tN_r(L+1)\times N_tN_r(L+1)$ channel correlation matrix and its rank, respectively, by:

$$\mathbf{R}_h := \mathrm{E}[\mathbf{h}\mathbf{h}^{\mathcal{H}}], \ \text{and}\ r_h := \mathrm{rank}(\mathbf{R}_h) \le N_tN_r(L+1),$$

where the $N_tN_r(L+1)\times 1$ channel vector is $\mathbf{h} := [h^{(1,1)}(0), \ldots, h^{(1,1)}(L), \ldots, h^{(1,N_t)}(L), \ldots, h^{(N_r,N_t)}(L)]^T$. Without a proof, we summarize our performance results for the linearly coded systems as follows (see also Fig. 1, and [7] for the proof):

Proposition 1 *The maximum achievable space-multipath diversity order for linearly coded systems is*

$$G_d^{\max} = r_h \le N_tN_r(L+1). \quad (5)$$

When the channel correlation matrix $\mathbf{R}_h$ has full rank, the maximum coding gain for these linearly coded systems is

$$G_c^{\max} = (\det(\mathbf{R}_h))^{\frac{1}{r_h}}\frac{d_{\min}^2}{N_t}, \quad (6)$$

where $d_{\min}$ is the minimum Euclidean distance of the constellation points in the finite alphabet $\mathcal{A}_s$.

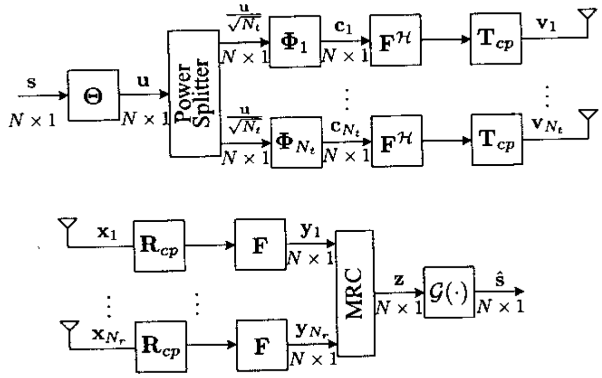

Fig. 2. A discrete-time model of transmitter and receiver for STM designs

In the ensuing section, we will propose an STM design which guarantees $G_d^{\max}$ in (5) and achieves the maximum coding $G_c^{\max}$ in (6) asymptotically as $N\to\infty$. Compared with [5,13], the merit of our STM design is that unlike [5,13] no rate loss is incurred $\forall N_t > 2$.

IV. STM Codec

The design of our STM codec consists of three stages as shown in Fig. 2. The outer codec includes a linear constellation precoding matrix Θ, and the corresponding deprecoder $\mathcal{G}(\cdot)$. The middle codec is our digital phase sweeping (DPS), scheme that includes a power splitter along with a set of matrices $\{\Phi_\mu\}_{\mu=1}^{N_t}$ at the transmitter, and a maximum ratio combiner (MRC) at the receiver. The inner codec performs orthogonal frequency division multiplexing (OFDM). In the following, we will detail these three stages.

A. Inner codec: OFDM

It is well-known that an OFDM module performs an inverse fast Fourier transform (IFFT) operation (via $\mathbf{F}_N^{\mathcal{H}}$), followed by cyclic-prefix (CP) insertion that can be described as a matrix $\mathbf{T}_{cp}$ at the transmitter. At the receiver, two mirror operations take place: the CP is removed (via a matrix $\mathbf{R}_{cp}$), and the FFT is taken. The CP-insertion and removal matrices are given, respectively as:

$$\mathbf{T}_{cp} := \begin{bmatrix}\mathbf{I}_{cp}\\ \mathbf{I}_N\end{bmatrix}, \quad \mathbf{R}_{cp} := \begin{bmatrix}\mathbf{0}_{N\times L_{cp}} & \mathbf{I}_N\end{bmatrix}, \quad (7)$$

where L_{cp} is the CP length, and $\mathbf{I}_{cp}$ denotes the last L_{cp} rows of $\mathbf{I}_N$. Based on these definitions, the input-output relationship from $\mathbf{c}_\mu$ to $\mathbf{y}_\nu$ (see Fig. 2) can be expressed as:

$$\mathbf{y}_\nu = \sum_{\mu=1}^{N_t}\mathbf{F}_N\mathbf{R}_{cp}\mathbf{H}^{(\nu,\mu)}\mathbf{T}_{cp}\mathbf{F}_N^{\mathcal{H}}\mathbf{c}_\mu + \boldsymbol{\xi}_\nu, \quad \forall\nu\in[1,N_r], \quad (8)$$

where the $\boldsymbol{\xi}_\nu$'s are independent identically distributed (i.i.d.) AWGN vectors, and $\mathbf{c}_\mu$ is the output of the middle encoder $\boldsymbol{\Phi}_\mu$. It is well-known that by (inserting) removing the CP and (I)FFT processing, a frequency-selective channel becomes equivalent to a set of flat-fading channels. Mathematically, one can express this property via:

$$\mathbf{F}_N \mathbf{R}_{cp} \mathbf{H}^{(\nu,\mu)} \mathbf{T}_{cp} \mathbf{F}_N^{\mathcal{H}} = \mathbf{D}_H^{(\nu,\mu)}, \ \forall \nu, \mu, \tag{9}$$

where $\mathbf{D}_H^{(\nu,\mu)} := \text{diag}[H^{(\nu,\mu)}(0), \ldots, H^{(\nu,\mu)}(N-1)]$, with $H^{(\nu,\mu)}(n) := \sum_{l=0}^{L} h^{(\nu,\mu)}(l) e^{-j2\pi nl/N}$. Using (9), we can simplify (8) as:

$$\mathbf{y}_\nu = \sum_{\mu=1}^{N_t} \mathbf{D}_H^{(\nu,\mu)} \mathbf{c}_\mu + \boldsymbol{\xi}_\nu, \quad \forall \nu \in [1, N_r]. \tag{10}$$

Notice that the inner codec (OFDM) removes the inter-block interference (IBI), and also diagonalizes the channel matrices.

B. Middle codec: DPS

The analog phase sweeping (a.k.a. intentional frequency offset) idea was introduced in [4]. The two transmit-antenna *analog* implementation, modulates the signal of one antenna with a sweeping frequency f_s in addition to the carrier frequency f_c, that is present in both antennas [4]. This causes bandwidth expansion. In the following, we will propose a *digital* phase sweeping (DPS) encoder. Combined with OFDM, DPS will convert N_t frequency-selective channels, each having $(L+1)$ taps to a single longer frequency-selective channel with $N_t(L+1)$ taps.

We can rewrite the diagonal channel matrix in (9) as:

$$\mathbf{D}^{(\nu,\mu)} = \sum_{l=0}^{L} h^{(\nu,\mu)}(l) \mathbf{D}_l, \ \forall \nu \in [1, N_r], \tag{11}$$

where $\mathbf{D}_l := \text{diag}[1, \exp(-j2\pi l/N), \ldots, \exp(-j2\pi l(N-1)/N)]$. Eq. (11) discloses that different channels may have different channel taps $h^{(\nu,\mu)}(l)$, but they share common delay lags (l) that manifest themselves as common shifts in the FFT domain. Suppose that we shift the $L+1$ taps of each channel corresponding to one of the N_t transmit antennas so that all channel taps become consecutive in their delay lags. Then, we can view the N_t channels to each receive-antenna as one longer frequency-selective channel with $N_t(L+1)$ taps. To realize this intuition, we select the matrices $\{\boldsymbol{\Phi}_\mu\}_{\mu=1}^{N_t}$ as

$$\boldsymbol{\Phi}_\mu = \text{diag}[1, e^{j\phi_\mu}, \ldots, e^{j\phi_\mu(N-1)}], \quad \forall \mu \in [1, N_t], \tag{12}$$

where $\phi_\mu = -2\pi(\mu-1)(L+1)/N$. Based on (11) and (12), we have that

$$\mathbf{D}_l \boldsymbol{\Phi}_\mu = \mathbf{D}_{l+(\mu-1)(L+1)}, \ \forall l \in [0, L], \mu \in [1, N_t]. \tag{13}$$

Define the equivalent long channel vector corresponding to the νth receive-antenna as:

$$\mathbf{h}^{(\nu)} = [(\mathbf{h}^{(\nu,1)})^T, \ldots, (\mathbf{h}^{(\nu,N_t)})^T]^T \tag{14}$$

with the lth entry of $\mathbf{h}^{(\nu)}$ given by: $h^{(\nu)}(l) = h^{(\nu,\lfloor l/(L+1) \rfloor+1)}$ $(l \bmod (L+1))$. According to (13), we define

$$\mathbf{D}_H^{(\nu)} := \sum_{\mu=1}^{N_t} \mathbf{D}_H^{(\nu,\mu)} \boldsymbol{\Phi}_\mu = \sum_{l=0}^{N_t(L+1)-1} h^{(\nu)}(l) \mathbf{D}_l. \tag{15}$$

Since $\mathbf{h}^{(\nu)}$ has length $N_t(L+1)$, we can view it as coming from a single frequency-selective channel. In essence, the DPS matrix $\boldsymbol{\Phi}_\mu$shifts the delay lags of the μth channel (c.f. (13)) from $[0, L]$ to $[(\mu-1)(L+1), \mu(L+1)-1]$. For example, when $\mu = 1$, $\boldsymbol{\Phi}_1 = \mathbf{I}_N$ and then $\mathbf{D}^{(\nu,1)} \boldsymbol{\Phi}_1 = \text{diag}(\sqrt{N} \mathbf{F}_{0:L} \mathbf{h}^{(\nu,1)})$, where $\mathbf{F}_{0:L}$ denotes the first $L+1$ columns of $\mathbf{F}_N$. When $\mu = 2$, $\mathbf{D}^{(\nu,2)} \boldsymbol{\Phi}_2 = \text{diag}(\sqrt{N} \mathbf{F}_{(L+1):(2L+1)} \mathbf{h}^{(\nu,2)})$, where $\mathbf{F}_{(L+1):(2L+1)}$ denotes the $(L+1)$st up to $(2L+1)$st columns of $\mathbf{F}_N$. Proceeding likewise with all N_t DPS matrices, we can also obtain (15). We summarize this observation in the following:

Property 1: *DPS converts the N_t transmit-antenna system, where each frequency-selective channel has $L+1$ taps, to a single transmit-antenna system, where the equivalent channel has $N_t(L+1)$ taps.*

Eq. (15) reveals a similarity between our DPS codec and the delay diversity schemes of [3,8]. However, our DPS scheme does not incur extra bandwidth expansion because it operates in the digital domain instead of the analog domain.

Remark 1 To avoid overlapping the shifted bases, we should make sure that $N > N_t(L+1)$. From the definition of $L := \lfloor \tau_{\max}/T_s \rfloor$, we have that for fixed $\tau_{\max}$ and N, we can adjust the sampling period T_s to satisfy this condition. As for each receive-antenna we have $N_t(L+1)$ unknown channel taps corresponding to N_t channels every N symbols, this condition guarantees that the number of unknowns is less than the number equations. Therefore, even from a channel estimation point of view, this condition is justifiable.

After the DPS encoder, since $\mathbf{c}_\mu = \boldsymbol{\Phi}_\mu \mathbf{u}/\sqrt{N_t}$, the input-output relationship can be rewritten as [c.f. (15)]:

$$\mathbf{y}_\nu = \frac{1}{\sqrt{N_t}} \mathbf{D}_H^{(\nu)} \mathbf{u} + \boldsymbol{\xi}_\nu, \quad \forall \nu \in [1, N_r]. \tag{16}$$

To collect the full diversity and large coding gains, we not only need to design the transmitter properly, but we must also select a proper decoder at the receiver. Since the received blocks $\mathbf{y}_\nu$ from all N_r receive-antennas contain the information block $\mathbf{s}$, we need to combine the information from all received blocks to decode $\mathbf{s}$. To retain decoding optimality, we perform the maximum ratio combining (MRC). The MRC amounts to combining $\{\mathbf{y}_\nu\}$ in (16) to form $\mathbf{z} = \mathbf{G}\mathbf{y}$, where the matrix $\mathbf{G}$ is given by

$$\mathbf{G} = \left(\sum_{\nu=1}^{N_r} \mathbf{D}_H^{(\nu)} \left(\mathbf{D}_H^{(\nu)} \right)^* \right)^{-\frac{1}{2}} \left[(\mathbf{D}_H^{(1)})^* \cdots (\mathbf{D}_H^{(N_r)})^* \right], \tag{17}$$

and $\mathbf{y} = [\mathbf{y}_1^T, \ldots, \mathbf{y}_{N_t}^T]^T$. Existence of the inverse in (17), requires (only for the DPS design) the channels $\mathbf{D}_H^{(\nu)}$ to satisfy the coprimeness condition:

A4) $\det\left(\sum_{\nu=1}^{N_r} \mathbf{D}_H^{(\nu)}(\mathbf{D}_H^{(\nu)})^*\right) \neq 0$.

Assumption A4) is more technical rather than restrictive, since it requires that the equivalent channels do not have common channel nulls. For random channels, A4) excludes an event with probability measure zero.

With the MRC of (17), the vector $\mathbf{z}$ is given by:

$$\mathbf{z} = \frac{1}{\sqrt{N_t}}\left(\sum_{\nu=1}^{N_r}\mathbf{D}_H^{(\nu)}\left(\mathbf{D}_H^{(\nu)}\right)^*\right)^{\frac{1}{2}}\mathbf{u} + \boldsymbol{\eta}, \qquad (18)$$

where $\boldsymbol{\eta} := \mathbf{G}[\boldsymbol{\zeta}_1^T, \ldots, \boldsymbol{\zeta}_{N_r}^T]^T$. Under A4), it can be verified that $\mathbf{G}$ satisfies $\mathbf{G}\mathbf{G}^{\mathcal{H}} = \mathbf{I}$. Since the $\boldsymbol{\zeta}_\nu$'s are uncorrelated AWGN blocks, the noise vector $\boldsymbol{\eta}$ retains their whiteness. Note that the middle codec has converted a multi-input multi-output system into a single-input single-output system.

To achieve full diversity, we still need to design the outer codec properly. If there is no precoding; i.e., $\mathbf{u} = \mathbf{s}$, we obtain that at each receiver, the diversity order is one even if maximum likelihood decoding is used. This happens when we lose the $N_t(L+1)$ transmit-diversity. To enable the latter, we need to design the precoder $\boldsymbol{\Theta}$ judiciously.

C. Outer codec: Linear Constellation Precoding

Grouped Linear Constellation Precoded (GLCP) OFDM was introduced in [5]. It provides one with a means of reducing decoding complexity without sacrificing diversity or coding gains. Towards this objective, we select the transmitted block size $N = N_g N_{sub}$, and demultiplex the information vector $\mathbf{s}$ into N_g groups: $\{\mathbf{s}_g\}_{g=0}^{N_g-1}$. Each group has length N_{sub}, and the gth group contains the symbols collected in a vector $\mathbf{s}_g$ as follows:

$$\mathbf{s}_g = \left[\, [\mathbf{s}]_{N_{sub}g}, \ \ldots, \ [\mathbf{s}]_{N_{sub}(g+1)-1}\,\right]^T. \qquad (19)$$

Correspondingly, we define the linearly precoded block of the gth group as:

$$\mathbf{u}_g = \boldsymbol{\Theta}_{sub}\mathbf{s}_g, \quad \forall g \in [0, N_g - 1], \qquad (20)$$

where $\boldsymbol{\Theta}_{sub}$ is an $N_{sub} \times N_{sub}$ matrix. To enable the maximum diversity, we select $\boldsymbol{\Theta}_{sub}$ from the algebraic designs of [12]. The overall transmitted block $\mathbf{u}$ consists of multiplexed sub-blocks $\{\mathbf{u}_g\}_{g=0}^{N_g-1}$ as follows:

$$\mathbf{u} = \left[[\mathbf{u}_0]_0 \cdots [\mathbf{u}_{N_g-1}]_0; \cdots; [\mathbf{u}_0]_{N_{sub}-1} \cdots [\mathbf{u}_{N_g-1}]_{N_{sub}-1}\right]^T.$$

It is not difficult to verify that $\mathbf{u}$ can be obtained from $\{\mathbf{u}_g\}_{g=0}^{N_g-1}$'s via a block interleaver with depth N_{sub}. Equivalently, it turns out that $\mathbf{u}$ can be related to $\mathbf{s}$ as

$$\mathbf{u} = \boldsymbol{\Theta}\mathbf{s}, \ \text{ with } \boldsymbol{\Theta} := \begin{bmatrix} \mathbf{I}_{N_g} \otimes \boldsymbol{\theta}_1^T \\ \vdots \\ \mathbf{I}_{N_g} \otimes \boldsymbol{\theta}_{N_{sub}}^T \end{bmatrix}, \qquad (21)$$

where $\boldsymbol{\theta}_m^T$ is the mth row of $\boldsymbol{\Theta}_{sub}$. Equations (19)–(20), or equivalently (21), summarize our LCP encoder.

To decode LCP, we split $\mathbf{z}$ in (18) into N_g groups:

$$\mathbf{z}_g = \frac{1}{\sqrt{N_t}}\mathbf{D}_{H,g}\boldsymbol{\Theta}_{sub}\mathbf{s}_g + \boldsymbol{\eta}_g, \ \ \forall g \in [0, N_g - 1], \qquad (22)$$

where $\mathbf{z}_g := \left[[\mathbf{z}]_g, [\mathbf{z}]_{N_{sub}+g}, \ldots, [\mathbf{z}]_{N_{sub}(N_g-1)+g}\right]^T$, $\mathbf{D}_{H,g}$ is the corresponding diagonal sub-matrix from $\left(\sum_{\nu=1}^{N_r}\mathbf{D}_H^{(\nu)}\left(\mathbf{D}_H^{(\nu)}\right)^*\right)^{\frac{1}{2}}$ for the gth group; and similarly defined, $\boldsymbol{\eta}_g$ is the corresponding AWGN.

ML decoding of $\mathbf{z}$ can then be implemented by applying the Sphere Decoding (SD) algorithm [11] on sub-blocks $\mathbf{z}_g$ of small size N_{sub}. Compared to the exponentially complex ML decoder, the SD offers near-ML performance at complexity of order $\mathcal{O}(N_{sub}^\alpha)$, with $\alpha \in [3, 6]$. The SD complexity depends on the block size N_{sub}, but unlike ML, it is independent of the constellation size [11].

The performance of our DPS depends on the selection of the sub-block size N_{sub}. When $N_{sub} \geq N_t(L+1)$, the maximum diversity order in (5) is achieved. When $N_{sub} < N_t(L+1)$ and $\mathbf{R}_h$ has full rank, the achieved diversity order is N_{sub}.

We summarize our diversity and coding gain results for our STM in the following proposition:

Proposition 2 *The maximum achievable space-multipath diversity order $G_d^{\max} = r_h$ is guaranteed by our STM design provided that we select $N_{sub} \geq N_t(L+1)$. When the channel correlation matrix $\mathbf{R}_h$ has full rank $r_h = N_r N_t(L+1)$, our STM design achieves (as $N \gg$) the maximum possible coding gain among all linearly coded transmissions that is given in closed form by: $G_c = (\det(\mathbf{R}_h))^{\frac{1}{r_h}} d_{\min}^2 N/(N_t(N + L_{cp}))$. The transmission rate of our design is $N/(N + L_{cp})$ symbols/sec/Hz, $\forall N_t, N_r$.*

In fact, the group size N_{sub} controls the tradeoff between performance and decoding complexity. When $N_{sub} \leq N_t(L+1)$, as N_{sub} decreases, the decoding complexity decreases, while at the same time, the diversity order decreases. By adjusting N_{sub}, we can balance the affordable complexity with the required performance.

V. Simulated Performance

Here we present simulations to confirm the performance of our STM design.

Test case 1: (Effects of multipath diversity) In order to appreciate the importance of multipath diversity, we simulated the performance of our STM design with $N_t = 2$ transmit and $N_r = 1$ receive antennae in the presence of multi-ray channels with different channel orders $L = 0, 1, 2$. The channel taps are i.i.d. Gaussian random variables with zero mean and variance $1/(L+1)$. The CP length is $L_{cp} = L$. QPSK modulation is selected. The sub-block size is $N_{sub} = N_t(L+1)$ and the number of sub-blocks is $N_g = 6$. The information

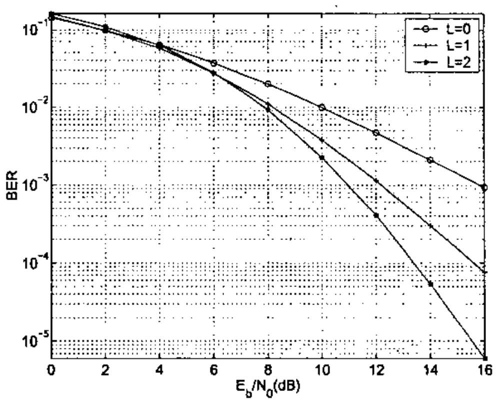

Fig. 3. Variable channel order L $((N_t, N_r) = (2, 1))$

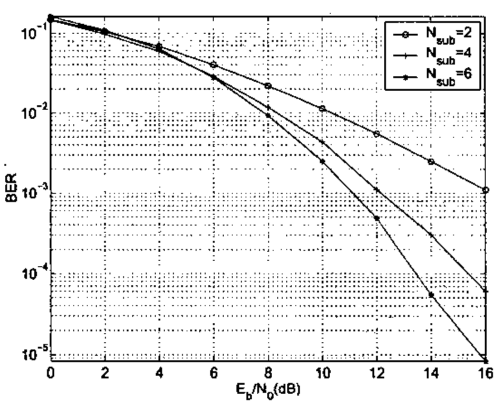

Fig. 4. Variable group size N_{sub} $((N_t, N_r, L) = (2, 1, 2))$

block length is $N = N_{sub}N_g$. Fig. 3 depicts the average bit-error rate versus SNR. We observe that as the channel order L increases, our STM design achieves higher diversity order.

Test case 2: (Tradeoff between diversity and complexity) To tradeoff diversity with complexity, we adjust the group size N_{sub}. The parameters and the channel model are the same as in Test case 1, except that we fix $L = 2$. In this case, $G_d^{\max} = 6$. Fig. 4 confirms that as N_{sub} decreases, the achieved diversity decreases. Since the channel correlation matrix $\mathbf{R}_h$ has full rank, the achieved diversity order is N_{sub}. Comparing the slopes of BER curves in Fig. 3 and Fig. 4 confirms our result. Note that decoding complexity also decreases as N_{sub} decreases. This shows that when the product $N_t L$ is large, we can select N_{sub} small to reduce complexity.

Test case 3: (Comparisons with [5]) In this example, we have $L = 2$, $N_r = 1$ and $N_t = 2, 4$. The channel taps are independent and satisfy an exponentially decaying power profile. When $N_t = 2$, we select QPSK for both STM and STF [5]. From Fig. 5, we infer that STF outperforms STM about 1 dB, while having lower computational complexity. When $N_t = 4$,

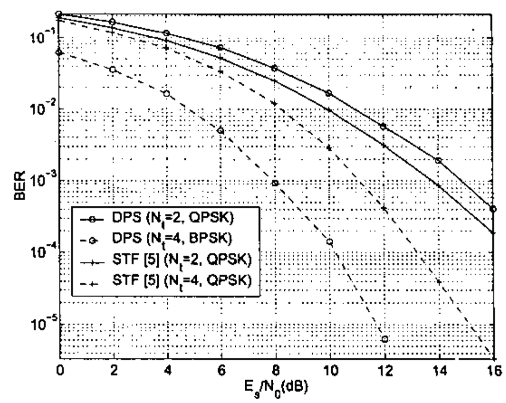

Fig. 5. Comparisons with [5] for variable N_t

to maintain the same transmission rate, we select BPSK for our STM and QPSK for STF, because STF uses the block code of [10] which has rate 1/2 symbols/sec/Hz. From Fig. 5, we observe that our STM outperforms the STF of [5] by about 3 dB.

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