Blind Channel Identification with Modulation Induced Cyclostationarity

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Abstract

Recent results have pointed out the importance of inducing cyclostationarity at the transmitter for blind identification and equalization of communication channels. Present paper shows that by modulating the input sequence with a deterministic and periodic sequence, single-input single-output channels can be identified uniquely from cyclic second-order output statistics, irrespective of the location of channel zeros, color of additive stationary noise, or channel order over-estimation errors, provided that the period of modulation induced cyclostationarity is greater than half the channel length. Linear, closed-form, linear inverse, and nonlinear correlation matching approaches are developed for channel estimation and are tested using simulations.

1 Introduction

Blind identification and equalization of wireless communication channels have attracted considerable interest during the last years, since they obviate inefficient bandwidth use incurred with training sequences. One of the main recent results has established that oversampling (or fractionally-sampling) the received waveform induces cyclostationarity, which allows for blind identification and equalization of most non-minimum phase channels, using only second order statistics [7], [10], [5]. It has been shown that the channel can be identified uniquely provided that there are no channel zeros equispaced by $2\pi/P$ angle on a circle, where P denotes the oversampling factor. Also, it has been pointed out that without excess bandwidth, the second-order statistics methods based on fractional sampling may fail [2]. Therefore, it is desirable to develop estimation algorithms which do not depend on the location of channel zeros, the color of additive noise or, other limiting factors such as channel order mismatch, and imperfect synchronization.

Recent works [8], [9], [3], have shown that blind identifiability of any SISO (single-input single-output) FIR (Finite Impulse Response) channel is possible with no restriction on channel zeros, color of additive stationary noise, and channel order over-estimation errors, provided that cyclostationarity is induced at the transmitter (by means of a 'multirate encoder'). However, two drawbacks are associated with these approaches: need for block synchronization and reduction of the information rate. In [1], blind MIMO (multi-input multi-output) channel identification is addressed by allocating different cycles to different inputs (users). In the SISO case, [1] assumes co-prime channels and use of *almost*-cyclostationary inputs. Almost periodicity may cause implementation challenges, and increase in the number of states for the Viterbi decoder.

In the present paper, cyclostationarity is induced at the transmitter by modulating the input symbol stream with a *strictly* periodic sequence. We show that the present channel estimation approaches guarantee identifiability, irrespective of the location of channel zeros, color of additive stationary noise, channel order mismatch, and do not require block synchronization or reduction of information rate.

In our baseband data transmission system the zero-mean, i.i.d. input stream $s(n)$ is modulated with the periodic sequence $p(n)$ with period P; i.e., $w(n) = p(n)s(n)$, with $p(n) = p(n+P)\ \forall\ n$. Sequence $w(n)$ is pulse-shaped by the transmit filter $h_c^{(tr)}(t)$, it propagates through the unknown channel $h_c^{(ch)}(t)$, and at reception it is filtered by the receive (matched) filter $h_c^{(rec)}(t)$. The continuous-time received signal is given by $x_c(t) = \sum_l w(l)h_c(t - lT_s - d) + v_c(t)$, where: $\star$ denotes convolution, and $h_c(t) := (h_c^{(tr)} \star h_c^{(ch)} \star h_c^{(rec)})(t)$; receiver output noise is $v_c(t) := (h_c^{(rec)} \star \nu_c)(t)$; T_s denotes the symbol period, and $d \in (0, T_s)$ is the unknown propagation delay. Sampling at the symbol rate T_s^{-1}, and considering $h(n) := h_c(t-d)|_{t=nT_s}$, $v(n) := v_c(t)|_{t=nT_s}$, we obtain the discrete-time model

$$x(n) := x_c(t)|_{t=nT_s} = \sum_{l=0}^{L} w(l)h(n-l) + v(n)\ , \quad (1)$$

where L stands for the order of the discrete-time filter h. We also assume that $s(n)$ is uncorrelated with the stationary (arbitrary colored) noise $v(n)$. We adopt the equivalent system shown in Fig. 1, with the goal of blind identification of channel $h(n)$, i.e., estimation based on the received data $x(n)$ and knowledge of the period P. Blind identifiability

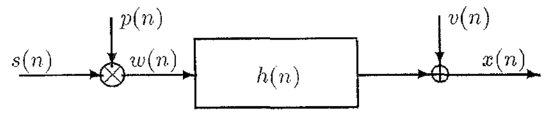

Figure 1. Baseband Discrete-Time Channel

is as usual understood modulo time shift and complex scale ambiguities.

2 Blind Channel Identifiability

Due to the modulation with the periodic sequence $p(n)$, sequence $w(n)$ is cyclostationary. Indeed, the time-varying correlation $c_{ww}(n;\tau) := Ew(n)w^*(n+\tau)$ satisfies $c_{ww}(n;\tau) = c_{ww}(n+lP;\tau), \forall\, l,\tau \in \mathbf{Z}$. The cyclostationarity induced at the input $w(n)$ is, in general, preserved at the output $x(n)$ of the time-invariant channel $h(n)$. Using (1), it can be shown that

$$c_{xx}(n;\tau) = \sigma_s^2 \sum_{m=-\infty}^{\infty} |p(n-m)|^2 h(m)h^*(m+\tau) + r_v(\tau) \quad (2)$$

with $\sigma_s^2 := E|s(n)|^2$. Since $p(n)$ is periodic, we deduce from (2) that $c_{xx}(n;\tau)$ is periodic in n; i.e., $c_{xx}(n;\tau) = c_{xx}(n+P;\tau)$, $\forall\, n,\tau \in \mathbf{Z}$. Being periodic, $c_{xx}(n;\tau)$ accepts a Fourier series expansion over harmonic cycles, with the set of cycles defined as: $A_{xx}^c := \{\alpha_k = 2\pi k/P,\ k = 0,\ldots,P-1\}$; i.e., $c_{xx}(n;\tau)$ and its Fourier coefficients $C_{xx}(\alpha_k;\tau)$, called cyclic correlations, are related by

$$C_{xx}(\alpha_k;\tau) = \frac{1}{P}\sum_{n=0}^{P-1} c_{xx}(n;\tau)e^{-j\alpha_k n} . \quad (3)$$

Upon substituting (2) into (3), the expression of cyclic correlation $C_{xx}(\alpha;\tau)$, at a fixed cycle $\alpha = \alpha_k, 0 \le k \le P-1$, becomes

$$C_{xx}(\alpha;\tau) = \sigma_s^2 P_2(\alpha)\sum_{m=-\infty}^{\infty} h(m)h^*(m+\tau)e^{-j\alpha m} + r_v(\tau)\delta(\alpha), \quad (4)$$

where $P_2(\alpha) := P^{-1}\sum_{n=0}^{P-1}|p(n)|^2 e^{-j\alpha n}$. To remove the contribution of $v(n)$, we assume hereafter that cycle α is chosen such that $\alpha \neq 0$. Since there is at least one cycle α_k, $1 \le k \le P$, such that $P_2(\alpha_k) \neq 0$, we assume also that α satisfies $P_2(\alpha) \neq 0$. Considering the Z-transform of $\{C_{xx}(\alpha;\tau)\}_{\tau=-\infty}^{\infty}$, for $\alpha \neq 0$ we obtain the cyclic spectrum

$$S_{xx}(\alpha;z) = \sigma_s^2 P_2(\alpha)H(e^{j\alpha}z^{-1})H^*(z^*) , \quad (5)$$

where $H(z) := \sum_{n=0}^{L} h(n)z^{-n}$.

It has been pointed out in [3] that the period P of the transmitter induced cyclostationarity can be chosen such that $P > L+1$, while the length of the channel is kept the same. This has to be contrasted with the case when the cyclostationarity is induced by fractionally sampling the receiver output. In this case, we always have $L > P$, which does not allow unique identification of $H(z)$ [3], [7]. The choice $P > L+1$ guarantees identifiability of channel $H(z)$. Indeed, it is sufficient to see that $H(z)$ is the greatest common divisor of the family of polynomials $\{S_{xx}(\alpha_k;z), k = 1,\ldots,P-1\}$. The extraction of $H(z)$ can be performed with a Bezoutian subspace approach, similar to that presented in [4]. Estimation accuracy for cyclic correlations increases by using small values of period P, since more samples can be used in the cyclic correlation estimator. For this reason, we propose here an alternative approach which works even with smaller periods $L \ge P > [L/2]$. In the next section, we show that knowledge of a single cyclic spectrum together with $P > [L/2]$ provides sufficient information for unique identification of $h(n)$.

3 Channel Estimation Algorithms

We first propose an algorithm which provides a closed form expression of $h(n)$ in terms of the cyclic correlations $C_{xx}(\alpha;k)$, computed at a single cycle α.

3.1 Linear and Closed-Form Solutions

Our result is summarized in the following:

Proposition 1. *A sufficient condition for identifiability of $H(z)$ from the cyclic spectrum factorization (5) is $P > [L/2]$ and*

$$e^{j2\alpha l} \neq 1 , \quad \text{for } l = 1,\ldots,[L/2] . \quad (6)$$

Proof. W.l.o.g. we assume $\sigma_s^2\ P_2(\alpha) = 1$ and $h(0) = 1$. We show that channel $H(z)$ is uniquely recoverable from the set of cyclic correlations $C_{xx}(\alpha;L)$, $C_{xx}(\alpha;\pm(L-l))$, $l = 1,\ldots,[L/2]$. Note that from (4)

$$C_{xx}(\alpha;L) = \sigma_s^2 P_2(\alpha)h(0)h^*(L) ,$$

we can uniquely estimate $h(L)$. The system of equations

$$C_{xx}(\alpha;L-1) = h(0)h^*(L-1) + h(1)h^*(L)e^{-j\alpha},$$
$$C_{xx}^*(\alpha;-L+1) = h(0)h^*(L-1)e^{j(L-1)\alpha} + h(1)h^*(L)e^{jL\alpha};$$

allows determination of $h(1)$ and $h(L-1)$, provided that $h(0)h^*(L)(\exp jL\alpha - \exp j(L-2)\alpha) \neq 0$, i.e., $\exp(-j2\alpha) \neq 1$. Similarly, considering $C_{xx}(\alpha;L-l)$ and $C_{xx}^*(\alpha;-(L-l))$, we can determine $h^*(L-l)$ and $h(l)$, for any $l = 1,\ldots,[L/2]$. Thus, if $\exp(j2\alpha l) \neq 1$ for $l = 1,\ldots,[L/2]$, then $h(n)$ can be uniquely identified. □

We note that (6) requires only $P > [L/2]$. When $p(n)$ is chosen such that $p(n) = 0$ for $n \in [M,\ P]$ and $P - M \ge L$, with M an arbitrary integer, $1 \le M \le P$, then a closed-form expression for $h(n)$ can be obtained. It can be easily checked that $h(n) = (\sigma_s\ |p(0)|)^{-2}\ c_{xx}^*(0;n)$. This closed-form expression shows that $h(n)$ is *not* sensitive to order over-estimation errors, and can be easily estimated (perhaps online) from $\{x(n)\}_{n=0}^{N-1}$ using

$$\hat{c}_{xx}(n;\tau) = \frac{1}{[(N-n)/P]}\sum_{k=0}^{[(N-n)/P]-1} x(n+kP)x^*(n+kP+\tau).$$

Note that the minimum period required is $P \geq L + M \geq L + 1$. An alternative linear approach is presented next.

3.2 Linear Inverse Approach

With $'$ denoting transpose, we introduce the definitions:

$$\mathbf{h} := [h(0)\ h(1)\ \ldots\ h(L)]' ,$$
$$\mathbf{h}_r := [h(L)\ h(L-1)\ \ldots\ h(0)]' ,$$
$$\mathbf{E}_\alpha := diag\{e^{jL\alpha},\ e^{j(L-1)\alpha},\ \ldots,\ 1\} ,$$
$$\mathbf{c}_{xx}^{+}(\alpha) := [c_{xx}^{*}(\alpha; L)e^{jL\alpha}\ \ldots\ c_{xx}^{*}(\alpha; 1)e^{j\alpha}\ c_{xx}^{*}(\alpha, 0)]' ,$$
$$\mathbf{c}_{xx}^{-}(\alpha) := [c_{xx}(\alpha; -L)\ \ldots\ c_{xx}(\alpha; -1)\ c_{xx}(\alpha; 0)]' .$$

To an arbitrary vector $\mathbf{x}$ we associate the square and lower triangular Toeplitz matrix $\mathbf{X}$, having as its first column $\mathbf{x}$ and first row $[\mathbf{x}(1)\ 0 \ldots 0]$, where $\mathbf{x}(1)$ is the first entry of $\mathbf{x}$. Using (2), we obtain the following factorizations

$$\mathbf{c}_{xx}^{+}(\alpha) = \sigma_s^2 P_2(\alpha)\mathbf{H}^{*}\ \mathbf{E}_\alpha\ \mathbf{h}_r , \quad (7)$$
$$\mathbf{c}_{xx}^{-}(\alpha) = \sigma_s^2 P_2(\alpha)\mathbf{H}^{*}\ \mathbf{E}_{-\alpha}\ \mathbf{h}_r . \quad (8)$$

Eliminating $\mathbf{h}_r$ from (7) and (8), we arrive at

$$(\mathbf{H}^{*}\ \mathbf{E}_\alpha)^{-1}\ \mathbf{c}_{xx}^{+}(\alpha) = (\mathbf{H}^{*}\ \mathbf{E}_{-\alpha})^{-1}\ \mathbf{c}_{xx}^{-}(\alpha) , \quad (9)$$

where $\mathbf{E}_\alpha^{-1} = \mathbf{E}_{-\alpha}$. The next lemma summarizes known results on Toeplitz matrices:

Lemma 1. *i) The inverse of a lower triangular and Toeplitz matrix is also a lower triangular and Toeplitz matrix. ii) If* $\mathbf{X}$ *and* $\mathbf{Y}$ *are lower triangular and Toeplitz matrices associated, respectively, with two arbitrary vectors* $\mathbf{x}$ *and* $\mathbf{y}$*, then:*

$$\mathbf{Y}\,\mathbf{x} = \mathbf{X}\,\mathbf{y} . \quad (10)$$

Eq. (9) can be rewritten as

$$\mathbf{E}_{-\alpha}\ \mathbf{G}\ \mathbf{c}_{xx}^{+}(\alpha) = \mathbf{E}_\alpha\ \mathbf{G}\ \mathbf{c}_{xx}^{-}(\alpha) , \quad (11)$$

where $\mathbf{G} := (\mathbf{H}^{*})^{-1}$ is a lower triangular and Toeplitz matrix (by i) of the Lemma 1). Let $\mathbf{g}$ be the first column of $\mathbf{G}$. Using (10), we deduce from (11) that

$$\mathbf{E}_{-\alpha}\ \mathbf{C}_{xx}^{+}(\alpha)\ \mathbf{g} = \mathbf{E}_\alpha\ \mathbf{C}_{xx}^{-}(\alpha)\ \mathbf{g} , \quad (12)$$

where matrix $\mathbf{C}_{xx}^{+}(\alpha)$ is the square Toeplitz matrix associated with the vector $\mathbf{c}_{xx}^{+}(\alpha)$. From (12), we arrive at

$$\mathbf{F}\ \mathbf{g} = \mathbf{0} , \quad (13)$$

where $\mathbf{F} := \mathbf{E}_{-\alpha}\mathbf{C}_{xx}^{+}(\alpha) - \mathbf{E}_\alpha\mathbf{C}_{xx}^{-}(\alpha)$. $\mathbf{G} := (\mathbf{H}^{*})^{-1}$ is lower triangular and Toeplitz. It follows that knowledge of $\mathbf{g}$ is sufficient to recover the channel $\mathbf{h}$ (since $\mathbf{h}'^{*}\mathbf{G} = [1\ 0 \ldots 0]$). Also, since $\mathbf{F}$ is lower triangular, it is easy to check that if $\exp(j2\alpha k) \neq 1$, for $k = 1, \ldots, L$, then $\dim\mathcal{N}(\mathbf{F}) = 1$. Thus, uniqueness in recovering $\mathbf{h}$ is guaranteed since $\mathbf{g}$ is uniquely identifiable from (13). Although simple, linear inverse approach is not optimal, since the magnitude of the eigenvalues of $\mathbf{F}$ are of the order of $C_{xx}(\alpha; L)$. Indeed, it can be easily shown that the eigenvalues of $\mathbf{F}$ are $2jC_{xx}^{*}(\alpha; L)\sin(k\alpha)$, where $k = 0, \ldots, L$, and $j = \sqrt{-1}$. At low SNR's and small values of $C_{xx}(\alpha; L)$, the estimates may exhibit large variance, since the unique null eigenvalue of $\mathbf{F}$ in (13) may be confused with a signal eigenvalue of $\mathbf{F}$. In [6], we show an alternative subspace solution which removes this problem. Note that the identifiability condition $\exp(j2\alpha k) \neq 1$, for $k = 1, \ldots, L$, requires $P \geq L + 1$. Next, we propose a nonlinear cyclic correlation approach, which shows robustness to channel order over-estimation errors, requires smaller values for period P, and constitutes the asymptotically best consistent estimator for channel $\mathbf{h}$.

3.3 Nonlinear Cyclic Correlation Matching Approach

We collect all the true channel coefficients $\{h(n)\}_{n=0}^{L}$ into the vector $\mathbf{h} := [h(0)\ h(1)\ \ldots\ h(L)]'$. We have seen that the cyclic correlations $C_{xx}(\alpha, \tau)$, for a fixed α and $\tau = -L, \ldots, L$, contain sufficient information for unique identification of $\mathbf{h}$. We define the cyclic correlation vector $\mathbf{c}_{xx}(\alpha) := [C_{xx}(\alpha; -L)\ C_{xx}(\alpha; -L+1) \ldots C_{xx}(\alpha; L)]'$. Given the observations $\{x(n)\}_{n=0}^{N-1}$, we can consistently estimate the cycle correlation $C_{xx}(\alpha; \tau)$ using the estimate

$$\hat{C}_{xx}^{(N)}(\alpha; \tau) = \frac{1}{P}\sum_{n=0}^{P-1} \hat{c}_{xx}(n; \tau)e^{-j\alpha n} .$$

We collect all these estimates into the vector

$$\hat{\mathbf{c}}_{xx}^{(N)}(\alpha) := [\hat{C}_{xx}^{(N)}(\alpha; -L)\ \ldots\ \hat{C}_{xx}^{(N)}(\alpha; L)] . \quad (14)$$

In the nonlinear cyclic correlation matching estimator, the estimate $\hat{\mathbf{h}}$ (of $\mathbf{h}$) is found by minimizing the distance between vectors $\hat{\mathbf{c}}_{xx}^{(N)}(\alpha)$ and $\mathbf{c}_{xx}(\alpha)$, in a weighted least-squares sense:

$$\hat{\mathbf{h}} := \arg\min_{\mathbf{h}} \mathcal{J}_{\mathbf{Q}}[\hat{\mathbf{c}}_{xx}^{(N)}(\alpha); \mathbf{h}] ,$$

$$\mathcal{J}_{\mathbf{Q}}[\hat{\mathbf{c}}_{xx}^{(N)}(\alpha);\mathbf{h}] := [\hat{\mathbf{c}}_{xx}^{(N)}(\alpha) - \mathbf{c}_{xx}(\alpha)]^{*'}\mathbf{Q}[\hat{\mathbf{c}}_{xx}^{(N)}(\alpha) - \mathbf{c}_{xx}(\alpha)]$$

where $\mathbf{Q}$ is a Hermitian and positive-definite weighting matrix. The nonlinear matching approach has some advantages over the linear approaches: it can attain asymptotically the minimum estimation variance for $\hat{\mathbf{h}}$, within the class of all second order statistics based estimators. In addition, it allows explicit computation of a closed-form expression for its asymptotic estimation variance. These problems are beyond the scope of this paper. Here, we simply consider $\mathbf{Q} = \mathbf{I}$. On the other hand, nonlinear matching approaches present some disadvantages: increased computational effort, need of a good initial estimate in order to speed the convergence to the global minimum. Due to limited space, results concerning performance analysis of linear type and nonlinear matching approaches, together with additional subspace channel estimation approaches will be reported in [6]. Finally, we remark that all the estimation methods developed are consistent, since $\hat{\mathbf{c}}_{xx}^{(N)}(\alpha)$ is a consistent estimate, and channel identifiability is guaranteed.

4 Simulations

We performed a comparative study among several channel estimation algorithms: nonlinear matching approach, linear inverse approach, MDCM [5], and XLTK [10]. We considered the channel $\mathbf{h}=[1, \ -0.812, \ 1.5, \ -1.218]$, with $QPSK$ symbols chosen as $s(n)$. Cyclostationary input $w(n)$ was obtained by modulating $s(n)$ with $p(n) = 2 + \cos(\alpha \ n)$, where $\alpha = \pi/4$ and $P = 8$. Additive noise $v(n)$ was considered white and normally distributed. Oversampling the channel by a factor of 2, yields subchannels with common root $z = -1.5$. Thus, the channel cannot be identified with fractionally spaced algorithms [5], [7], [10]. In all experiments, the nonlinear cyclic correlation matching algorithm was initialized with a channel estimate provided by the linear inverse approach. In Figs. 2a-2b, we plot the average root-mean square (RMSE) and average bias (Avg. Bias) of channel estimates versus no. of samples N, at $SNR = 10\ dB$, and the number of Monte-Carlo runs $MC = 100$. MDCM and XLTK fail to estimate the channel in the mean even for large number of samples ($N = 1000$), while the nonlinear cyclic matching approach provides good channel estimates even for $N = 200$. In Figs. 3a-3b, we considered $N = 1000$ and $MC = 100$, and plotted RMSE/Avg. Bias versus SNR. The nonlinear cyclic matching performs well even at low SNRs. Figs. 4a-4b depict RMSE and Avg. Bias of the channel estimates (obtained using the nonlinear cyclic matching approach) versus over-estimated (channel) order. We over-estimated the channel order with values in the range $\overline{L} = 4 \div 10$, and for each order 100 Monte-Carlo runs were used to compute the RMSE and the Average Bias of channel estimates. We note the robustness of the nonlinear cyclic matching approach to channel order mismatch.

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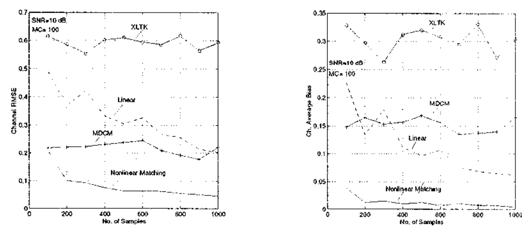

Figure 2. RMSE/Avg. Bias vs. No. of Samples

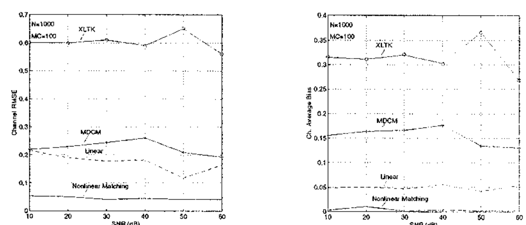

Figure 3. RMSE/Avg. Bias vs. SNR

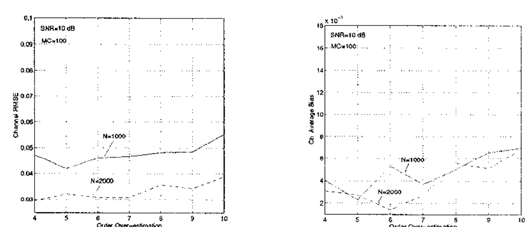

Figure 4. RMSE/Avg. Bias vs. Order Mismatch