BLIND CHANNEL IDENTIFICATION FOR MULTIRATE PRECODING AND OFDM SYSTEMS

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ABSTRACT

Algorithms are proposed for blind estimation of frequency selective channels in a multirate precoding communications system. These methods exploit the cyclostationarity induced by a filter-bank structure at the transmitter to identify FIR nonminimum phase channels without higher-order statistics or fractional-sampling. As an example, OFDM systems are considered with and without the cyclic prefix. Some redundancy, *independent* of the channel memory, is shown to be necessary for OFDM and other orthonormal transformation systems for blind channel identification. Methods of blind channel estimation for more general transform modulations are also supported in this framework. Monte Carlo simulations are used to provide examples of performance.

1. INTRODUCTION

A variety of techniques are available for blind channel identification to facilitate blind equalization of frequency selective communication channels. These techniques employ high order stationary statistics, or, second-order cyclostationary statistics induced by fractional sampling at the receiver [8]. Methods for inducing cyclostationarity at the receiver have also been reported including [9] which entails a block repetition and [4] which proposes a more general multirate precoding structure. The advantages of precoding the input sequence are: blind identification of minimum phase channels is possible using symbol rate sampling, low complexity is required since only the second-order statistics are estimated, and no restrictions on the locations of the zeros of the channel are imposed.

In this paper we derive algorithms for blind channel identification which exploit the cyclostationarity induced by a multirate precoding structure. Unlike [4], we develop linear methods for the general precoding structure using only the cyclic correlation from a single cycle. As an example, these algorithms are applied to OFDM systems, one of many transform modulation schemes which can be viewed as a special case of the multirate precoder presented in this paper.

OFDM is of particular interest because it is being considered in mobile communication systems, [2], and terrestrial television systems [7], applications where multipath interference or impulsive noise degrades system performance. For equalization purposes in OFDM, a channel estimate is found from training data or unmodulated carriers, reducing the overall information rate [1], or through estimating the power of each carrier. In many systems this channel estimate is conveyed back to the transmitter for preemphasis. Implementation of a blind channel estimation algorithm, while increasing receiver complexity, would obviate the need for timing, reduce the amount of training data required in the system, and can allow for more general MMSE or DF equalization structures [10]. A blind algorithm for equalization for OFDM systems was considered in [3] without exploiting the whiteness of the input or the cyclostationarity of the transmitted symbols.

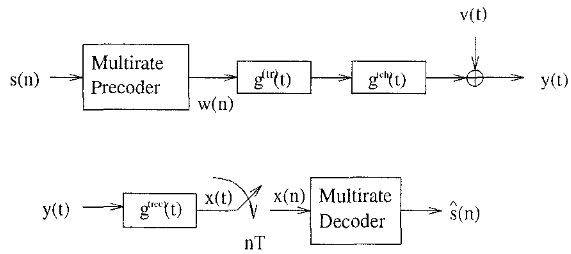

Figure 1. Multirate encoding system

Consider the communications system in Fig. 1. Let $w(n)$ denote a wide-sense cyclostationary process with period P formed from precoding the white information sequence $s(n)$ with a multirate precoder (to be specified in Section 2). Sequence $w(n)$ is pulse-shaped with $g^{(tr)}(t)$, propagates through an unknown frequency selective channel $g^{(ch)}(t)$, is degraded by additive white Gaussian noise $\nu(t)$, and is filtered by $g^{(rec)}(t)$ upon its reception. If we let $g(t) = g^{(tr)}(t) \star g^{(ch)}(t) \star g^{(rec)}(t)$ be the composite channel and $v(t) = \nu(t) \star g^{(rec)}(t)$ the colored noise process at the receiver, then the baseband output of the receive filter is: $x(t) = \sum_{l=-\infty}^{\infty} w(l)g(t-l-\epsilon) + v(t)$ where $\epsilon \in [0, T)$ is an unknown timing offset. Sampling at T, the symbol period of the encoded sequence $w(n)$, we obtain

$$x(n) \quad := \quad x(t)|_{t=nT} \quad = \quad \sum_{l=-\infty}^{\infty} w(l)h(n-l) + v(n), \quad (1)$$

where $h(n) := g(nT - \epsilon)$ and $v(n) := v(nT)$.

Choosing the period $P > L_h$, the memory of the channel, and assuming P is known to the receiver, the goal is to estimate the coefficients of $h(l)$ $l \in [0, L_h]$ *blindly*, without knowledge of the input symbols $s(n)$. As in all blind algorithms, the channel estimate will have a scale and phase ambiguity which can be corrected if necessary with gain control and differential encoding. The coefficients of the channel can then be employed in the multirate pre- or post-equalizer to obtain an estimate of the input sequence.

2. GENERAL MULTIRATE PRECODING

Suppose the multirate precoding structure in Fig. 2 is used as the precoder in Fig. 1. Such an encoder separates the M polyphase components of $s(n)$ using the serial-to-parallel

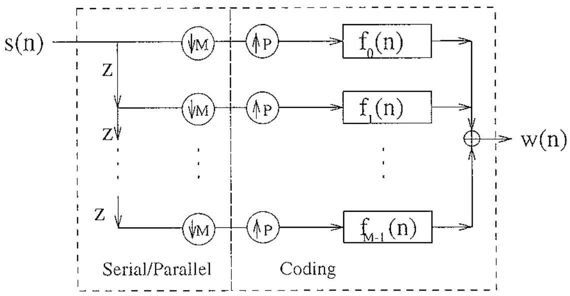

Figure 2. General multirate encoder

converter, upsamples each component by P, and then convolves the upsampled sequences with the order L_f filters $f_m(n)$ for $m \in [0, M-1]$. When $M = P$ the system is maximally decimated; when $M < P$ the system is nonmaximally decimated; when $M > P$ the system is oversampled. In what follows we will consider only $M \leq P$, leaving $M > P$ for further study.

Examining the precoder in Fig. 2, it turns out that $s(n)$ is related to the encoded sequence $w(n)$ as [4],

$$w(n) = \sum_{m=0}^{M-1} \sum_{i=-\infty}^{\infty} s(iM+m) f_m(n-iP). \qquad (2)$$

At every time $n = kP + r$, we obtain in the k^{th} block at index $r \in [0, P-1]$, a sum of the input sequence $s(kP), \ldots, s(kP+M-1)$ weighted by the coefficients of the filter $f_m(l)$. When L_f is greater than $P-1$, such as in discrete wavelet multitone modulation [6], the sum of weighted coefficients may overlap two or more blocks.

Incorporating (2) in (1) we write the input to the decoder (see Fig. 1) as

$$x(n) = \sum_{m=0}^{M-1} \sum_{i=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} s(iM+m) f_m(l-iP) \times h(n-l) + v(n). \qquad (3)$$

Given that $s(n)$ is a zero-mean white stationary process, we are interested in characterizing $x(n)$. We look toward the time-varying correlation of $x(n)$ since $x(n)$ is zero mean from our assumptions about $s(n)$ and $\nu(n)$. The time-varying correlation of $x(n)$, defined as $c_{xx}(n;\tau) := E\{x(n)x^*(n+m)\}$, is

$$c_{xx}(n;\tau) = \sigma_s^2 \sum_{l_1=-\infty}^{\infty} \sum_{l_2=-\infty}^{\infty} h(n-l_1)h^*(n+\tau-l_2) \times \sum_{m=0}^{M-1} \sum_{i=-\infty}^{\infty} f_m(l_1-iP) f_m^*(l_2-iP) + c_{vv}(n;\tau). \qquad (4)$$

Rewriting (4) with $l = l_1$ and $\rho = l_2 - l_1$, and using the fact that the filters $f_m(n)$ have maximum order L_f, we observe that (4) can be rewritten as

$$c_{xx}(n;\tau) = \sigma_s^2 \sum_{l=-\infty}^{\infty} \sum_{\rho=-L_f}^{L_f} h(n-l)h^*(n+\tau-\rho-l) \times c_{ff}(l;\rho) + c_{vv}(n;\tau), \qquad (5)$$

where we have defined:

$$c_{ff}(l;\rho) := \sum_{m=0}^{M-1} \sum_{i=-\infty}^{\infty} f_m(l-iP) f_m^*(l+\rho-iP). \qquad (6)$$

Using (4) and (5) we arrive at

$$c_{xx}(n;\tau) = \sigma_s^2 \sum_{l=0}^{L_h} \sum_{\rho=-L_f}^{L_f} h(l)h^*(l+\tau-\rho) c_{ff}(n-l;\rho) + c_{vv}(n;\tau). \qquad (7)$$

Because $c_{ff}(n;\rho)$ is generally periodic in n with period P, we can infer the same about $c_{xx}(n;\tau)$ from (7). Since $x(n)$ is zero mean with a periodic time-varying correlation, we conclude that it is wide-sense cyclostationary.

Cyclostationarity of $x(n)$ might be expected because of the presence of upsampling in the encoder; however, the output is not cyclostationary for any choice $\{f_m(l)\}_{m=0}^{M-1}$, P, and M; e.g., if $P = M$ and $f_m(l) = \delta(l-m)$, $m \in [0, P-1]$, then $s(n) = w(n)$. Since our blind identification algorithm relies on the periodicity of $c_{xx}(n;\tau)$, we must place constraints on M, P, and $\{f_m(l)\}_{m=0}^{M-1}$ so that $c_{ff}(l;\rho)$, and thus $c_{xx}(n;\tau)$, is periodic. Further, there may be other requirements such as perfect reconstruction in the absence of noise, or minimal redundancy. The latter suggests choosing M as near to P as possible.

Additionally, we are interested in decoding without aliasing (interference among different polyphase components) in the absence of a channel. Further, we would like to eliminate the insertion of redundancy by choosing $M = P$. An initial attempt is to choose $\{f_m(l)\}_{m=0}^{M-1}$, $M = P$ such that

$$\sum_{M=0}^{P-1} \sum_{N=0}^{P-1} \sum_{i} f_m(l-iP) f_n^*(l+\rho-iP) = \lambda\delta(\rho). \qquad (8)$$

The f_m filters in (8) form a complete orthonormal set, scaled by a factor λ. Unfortunately, though desirable for decoding, the functions in (8) do not yield a periodic $c_{ff}(l;\rho)$ because there is no dependence on n (c.f. (6)). Two options become available: consider $M < P$ (overcomplete basis), or $P = M$ and λ a function of m (basis functions orthogonal with different scale coefficients). The choice $M < P$ corresponds to an overcomplete basis expansion and thus introduces redundancy while $\lambda(m)$ gives certain polyphase components a scale different from others. We focus on $M < P$, leaving $\lambda(m)$ for future study.

Our blind channel estimation algorithms rely upon $c_{xx}(n;\tau)$ of (7) for arbitrary cyclostationarity inducing filters f_m. Next, we motivate OFDM systems and demonstrate how the OFDM transmitter can be viewed as a special of Fig. 2 for a particular choice of f_m filters.

3. OFDM SYSTEM MODELING

In a modern orthogonal frequency division multiplexing (OFDM) system (e.g., [5]), multicarrier modulation is performed using an N-point inverse Discrete Fourier Transform (IDFT) and a single modulator instead of employing N different analog modulators. Fig. 3 shows the structure of the multirate encoder and decoder for a simple OFDM system assuming perfect synchronization and ignoring nonlinearities in the transmitter amplifier. In summary, an OFDM system works as follows. An OFDM transmitter modulates the N polyphase components of input sequence $s(n)$ onto N different carriers via the IDFT operation. Before transmission, a cyclic prefix with length equal to the memory of

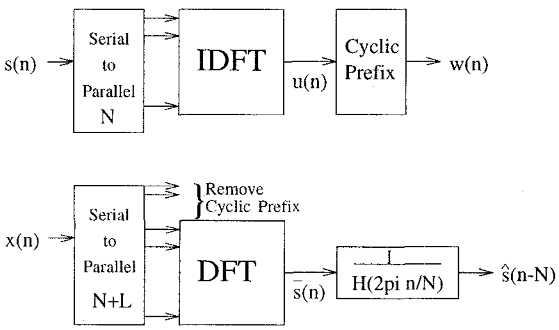

Figure 3. Multirate encoder & decoder for OFDM

the composite channel L_h is often introduced to take advantage of the circular convolution property of the DFT. At the receiver, the data corresponding to the cyclic prefix are removed and the circular convolution is exploited to allow carrier-by-carrier decoding based on gain and phase correction [1].

The relationship between the precoder in Fig. 2 and the OFDM system is interesting and lends insight into the OFDM transmission system. First, the combination of the serial-to-parallel converter and the IDFT is the same as the encoder in Fig. 2 if we choose $M = N$, $P = N$, and

$$\begin{aligned} f_k(l) &= e^{j\frac{2\pi}{N}kl}, \quad l \in [0, N-1] \\ &= 0 \ \text{otherwise}, \end{aligned} \tag{9}$$

i.e., if we choose the basis functions of the IDFT for the precoder filters. Using (9) and (2) we can write the output $u(n)$ of the IDFT as

$$u(n) = \sum_{k=0}^{N-1} \sum_{i=-\infty}^{\infty} s(iN+k) \sum_{l=0}^{L_f} e^{-j\frac{2\pi}{N}kl} \delta(n - iN - l). \tag{10}$$

Now letting $n = qN + r$, (10) becomes

$$u(qN+r) = \sum_{k=0}^{N} s(qN+k) e^{j\frac{2\pi}{N}kr}, \ r \in [0, N-1] \tag{11}$$

which is just the r^{th} coefficient of the IDFT of the block $[s(qN), \ldots, s(qN+N-1)]$ as we would expect.

The cyclic prefix operation takes the last L_h elements of the q^{th} block and appends them to the beginning of the q^{th} block to be transmitted. Thus the pulse-shaping filter sees the vector $[u(qN+N-L), \ldots, u(qN+N-1); u(qN), \ldots, u(qN+N-1)]$. In view of the periodicity of the exponential, we can incorporate the cyclic prefix into the precoder in Fig. 2 by choosing $M = N$, $P = N + L_h$, and

$$\begin{aligned} f_k(l) &= e^{j\frac{2\pi}{N}k(l-L_h)}, \quad n \in [0, N+L_h-1] \\ &= 0 \ \text{otherwise}. \end{aligned} \tag{12}$$

For OFDM without the cyclic prefix, the orthogonality of the exponentials causes $c_{ff}(l;\rho) = \delta(\rho)$ and the CS property is lost (c.f. (8)). On the other hand, with a cyclic prefix, $c_{ff}(l;\rho) = \sum_{m=0}^{M-1} \sum_{k_1} \sum_{k_2} \exp(j2\pi k_1/N) \exp(-j2\pi k_2/N)\ \delta(l - k_1)\ \delta(l + \rho - k_2)$ for $\rho \in [-N - L_h, N + L_h]$ preserving the cyclostationarity.

Avoiding the loss of cyclostationarity for complete sets of orthonormal basis functions motivates us to consider an OFDM with $M < P$ and order $L_h = P - 1$ filters $f_k(n)$. This is equivalent to inserting $P - M$ zeros into every block of M input symbols. Then $c_{ff}(l;\rho) = \sum_{m=0}^{M-1} \exp(-j\pi\rho m/P)$ and cyclostationarity of $x(n)$ is guaranteed. The effect of such a choice of sampling rates implies that M of P possible carriers are used for data transmission. Exploiting the CS property allows us to view the OFDM transmitter as a precoder which induces periodicity into the statistics of the transmitted sequence $w(n)$.

Use of OFDM allows for some interesting equalization options at the receiver. Assuming block synchronization, we discard the last L_h symbols corresponding to the cyclic prefix and take the DFT to obtain a delayed estimate of the input, $\bar{s}(n) = H(2\pi n/N)s(n-N)$, [1], attenuated by $H(2\pi n/N)$, the frequency response of the composite multipath channel $h(n)$. Simple estimation of the received symbols using training data is thus possible and, when the channel is known, equalization can be performed by gain and phase correction of each output symbol.

Alternatively, we can transmit the channel estimates back to the transmitter and perform pre-emphasis on the transmitted sequence in conjunction with differential encoding in the polyphase branches. In either approach, if the attenuation at a certain frequency $2\pi n/N$ is near zero however, symbols are lost as in severe flat fading. Further, by neglecting L_h samples at the receiver, we lose information with every block thereby reducing the transmission rate. A solution is to employ estimates of the channel contained into MMSE, or DFE equalizers [10] using the cyclostationarity induced either from the cyclic prefix or from the choice of $M < P$ in Fig. 2. Note that in the latter, the redundancy added to each block is independent of the channel memory. Regardless of which option is selected, the algorithms in the next section for blind channel estimation can be employed to obtain an estimate of the channel.

4. BLIND CHANNEL IDENTIFICATION

Since the time-varying correlation $c_{xx}(n;\tau)$ in (7) is periodic in n with period P it accepts a Fourier series expansion. The kth-order Fourier Series coefficient (also known as the cyclic correlation) is

$$\begin{aligned} C_{xx}(k;\tau) &:= \frac{1}{P} \sum_{n=0}^{P-1} c_{xx}(n;\tau) e^{-j\frac{2\pi}{P}kn} \\ &= \sigma_s^2 \sum_{l=0}^{L_h} \sum_{\rho=-L_f}^{L_f} h(l) h^*(l+\tau-\rho) \\ &\times \quad C_{ff}(k;\rho) e^{-j\frac{2\pi}{P}kl} + C_{vv}(k;\tau), \end{aligned} \tag{13}$$

where $C_{ff}(k;\rho) := (1/P)\sum_{m=0}^{M-1}\sum_{n=0}^{P-1} f_m(n) f^*(n + \rho)\exp(-j2\pi kn/P)$. We can use any two distinct cycles k_1 and k_2 to solve a set of linear equations for the unknown channel coefficients $\{h(l)\}_{l=0}^{L_h}$ by considering (c.f. (13))

$$\begin{aligned} \sum_{l=0}^{L_h} \sum_{\rho=-L_f}^{L_f} \Big[& C_{xx}(k_1; l+\tau-\rho) C_{ff}(k_2;\rho) e^{-j\frac{2\pi}{P}k_2 l} - \\ & e^{-j\frac{2\pi}{P}k_1 l} C_{xx}(k_2; l+\tau-\rho) C_{ff}(k_1;\rho) \Big] h(l) = 0, \end{aligned} \tag{14}$$

for $\tau \in [-L_f - L_h, L_f + L_h]$. Consistent estimation of $C_{xx}(k;m)$ in (14) is performed using the sample cyclic correlation estimator

$$\hat{C}_{xx}(k;m) \quad = \quad \frac{1}{T}\sum_{n=0}^{T-1} x(n)x^*(n+m)e^{-j\frac{2\pi}{P}kn}. \quad (15)$$

By choosing $k_1 = -k_2 \neq 0$ we can perform estimation using only one nonzero cycle. Only positive cycles need to be computed since (by definition)

$$C_{xx}(-k;\tau) \quad = \quad e^{-j\frac{2\pi}{P}k\tau}C^*_{xx}(k;-\tau). \quad (16)$$

The approach in (14) yields the channel estimate $\hat{h}(n)$ as a solution to a set of linear equations. We can also take a nonlinear matching approach by choosing

$$\hat{h}(n) = \arg\min_{\{h(l)\}_{l=0}^{L_h}} \sum_{\tau=-L_f-L_h}^{L_f+L_h} |C_{xx}(k;\tau) - \hat{C}_{xx}(k;\tau)|^2, \quad (17)$$

over one or all available cycles k. Identifiability of the channel from (17) when all the cyclic correlations are available is proved in [4]. Identifiability with one cycle, in either the linear or the matching solution, will be reported elsewhere. Since the noise in (3) is stationary, its cyclic correlation should be zero for nonzero cycles. Consequently, either approach based on the cyclic correlation should be robust to this interference. Further, as $x(n)$ and $x(m)$ are uncorrelated for $n - m > 2L_f + 2L_h + 1$, this approach should also be resistant to channel overestimation.

The algorithms given by (14) or (17), when applied to an OFDM system with a cyclic prefix, or with $M < P$, enable us to perform blind channel identification from output only data. This eliminates the need for data aided channel estimation. Once the channel is known, it can be used in equalizers such as the MMSE or the DF [10], or can be employed in the Viterbi decoder, to obtain the MLSE solution.

5. SIMULATIONS

To test the algorithm presented in Section 4 we considered a multirate precoding system with $P = 8$, $M = 7$, and the IDFT basis in (9). Symbols from a 32 QPSK alphabet were modulated by this system and passed through a composite channel with impulse response $\mathbf{h} = [1, -0.812, 1.5, -1.218]'$. White noise with a given signal-to-noise ratio (SNR) was then added to the resulting sequence. Channel identification using the linear approach (14) for the $k = 2$ and the $k = -1$ cycles was then performed and the results were averaged over 100 Monte Carlo simulations. As performance measures we used the root mean square channel error (RMSCE) defined as

$$RMSCE \quad := \quad \sqrt{\frac{\sum_k ||\hat{\mathbf{h}}_k - \mathbf{h}||^2}{R(L_h+1)}}, \quad (18)$$

where R is the number of Monte Carlo simulations and $\hat{\mathbf{h}}_k$ is the channel estimate for the k^{th} Monte Carlo simulation, and the average bias defined as

$$ACB \quad := \quad \frac{\sum_k |\hat{\mathbf{h}}_k - \mathbf{h}|}{R(L_h+1)}. \quad (19)$$

Fig. 4 shows the RMSCE and the average bias as the SNR (in dB) is varied given there are $N = 1000$ symbols available while Fig. 5 shows the RMSCE and the average bias as the number of symbols is varied under a given SNR of $20dB$. As we would expect, when more symbols are available or when there is less noise, the channel estimator performs better.

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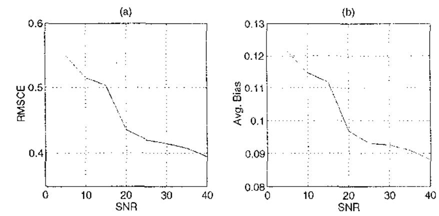

Figure 4. RMSCE-Avg. Bias vs. SNR, 1000 Symbols, 100 Monte Carlo Simulations

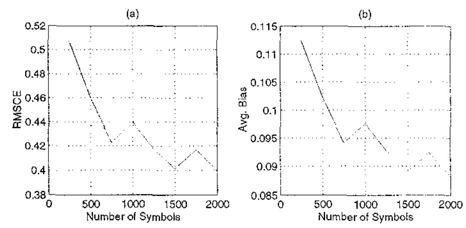

Figure 5. RMSCE-Avg. Bias vs. Number of Symbols, 20 dB noise, 100 Monte Carlo Simulations

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