Space-Time FSK for Slow Frequency-Hopping Multiple Access*

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Abstract — **Frequency shift-keying (FSK) is a popular modulation scheme in applications that do not rely on a high spectral efficiency (e.g., military). This paper introduces space-time FSK (ST-FSK), which does not require any channel state information (CSI) at the transmitter and the receiver, as in conventional FSK. ST-FSK can be viewed as a special unitary ST design. However, ST-FSK has a number of advantages over existing unitary ST designs. One of these advantages is that ST-FSK merges very naturally with slow frequency-hopping multiple-access (FHMA). Simulation results show improved performance of ST-FSK over conventional FSK in a realistic slow FHMA system.**

I. Introduction

Despite the accompanying spectral inefficiency, frequency shift-keying (FSK) and other orthogonal modulation schemes are of interest, especially in systems that rely on frequency-hopping (FH). As a multiple access technique, frequency-hopping combats frequency-selective fading, reduces interference from other sources, and induces some level of security. Global System for Mobile Communications (GSM) and the Bluetooth technology are two well-known commercial examples, where FH is employed in conjunction with FSK-like modems that also maintain phase continuity. In applications where the spectral demands are not as strict (e.g., military), FSK can be used without any additional effort and complexity to address the discontinuous phase issue.

In this paper, we introduce space-time FSK (ST-FSK), which does not require any channel state information (CSI) at the transmitter and the receiver, as in conventional FSK. The ST-FSK modulator transmits FSK waveforms that are structured according to the real orthogonal designs of [7]. It is similar in spirit to unitary ST modulation introduced in [2]. We actually show that ST-FSK can be viewed as a special unitary ST design. However, ST-FSK has a number of advantages over the unitary ST designs of [3]:

- ST-FSK is very simple, whereas the unitary ST designs of [3] require complex numerical search procedures.
- The coding and decoding operations related to ST-FSK can be carried out not only in the digital, but also in the analog domain, as in conventional FSK.
- ST-FSK achieves full diversity and assures perfect symbol recovery in the absence of noise, irrespective of the channel realization. The unitary ST designs of [3], on the other hand, do not guarantee these properties.
- Because FSK waveforms are transmitted, ST-FSK merges very naturally with slow frequency-hopping multiple-access (FHMA), which is employed in ad hoc networks for instance.

The advantages listed above come at an expense. For a fixed bandwidth, the unitary ST designs of [3] may achieve a higher rate, or similarly, for a fixed rate, they may occupy a smaller bandwidth. This is expected since FSK is known to be energy-efficient, but not spectrally efficient.

To illustrate that ST-FSK blends well with slow FHMA, we propose a new slow FHMA system based on ST-FSK that outperforms the conventional slow FHMA system based on FSK. Low-rate RS encoding combined with error-only decoding (when the receiver has no collision knowledge), and high-rate RS encoding combined with error-and-erasure decoding (when the receiver has collision knowledge) is used to recover from collisions, and lower the error floor that results from them.

Notations: Upper (lower) bold face letters denote matrices (column vectors); $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian, respectively; $\otimes$ is used for the Kronecker product; $\delta_{i,j}$ represents the Kronecker delta; $\|\cdot\|$ represents the Frobenius norm; $[\mathbf{A}]_{m,n}$ denotes the (m,n)th entry of the matrix $\mathbf{A}$, and $[\mathbf{a}]_n$ denotes the nth entry of the column vector $\mathbf{a}$; $\mathrm{E}(\cdot)$ is reserved for statistical average; $\mathbf{I}_N$ denotes the $N \times N$ identity matrix; $\mathbf{0}_{M\times N}$ denotes the $M \times N$ all-zero matrix; and finally $|\mathcal{A}|$ is used to denote the cardinality of the set $\mathcal{A}$.

II. Space-Time FSK

We adopt here a similar data model as in [2]. Consider a communication link from M transmit antennas to N receive antennas that operates over a flat fading channel that is constant over T symbol periods. Denoting the $T \times 1$ vector transmitted at the mth transmit antenna as $\mathbf{x}_m$, the $T \times 1$ vector $\mathbf{y}_n$ received at the nth receive antenna can be written as

$$\mathbf{y}_n = \sum_{m=1}^{M} \mathbf{x}_m h_{m,n} + \mathbf{e}_n,$$

where $h_{m,n}$ is the flat fading channel coefficient from the mth transmit antenna to the nth receive antenna, and $\mathbf{e}_n$ is the noise vector received at the nth receive antenna. Denoting the $T \times M$ matrix obtained by stacking the M transmitted vectors as $\mathbf{X} := [\mathbf{x}_1, \ldots, \mathbf{x}_M]$, the $T \times N$ matrix $\mathbf{Y} := [\mathbf{y}_1, \ldots, \mathbf{y}_N]$

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obtained by stacking the N received vectors can be expressed as

$$\mathbf{Y} = \mathbf{X}\mathbf{H} + \mathbf{E}, \tag{1}$$

where $\mathbf{H}$, defined by $[\mathbf{H}]_{m,n} := h_{m,n}$, is the $M \times N$ flat fading channel matrix obtained by stacking the MN flat fading channel coefficients, and $\mathbf{E} := [\mathbf{e}_1, \ldots, \mathbf{e}_N]$ is the $T \times N$ noise matrix obtained by stacking the N noise vectors. We assume that the entries of $\mathbf{H}$ and $\mathbf{E}$ are independent and identically distributed (i.i.d.) zero-mean complex Gaussian variables with variance σ_h^2 and σ_e^2, respectively. Constraining the transmitted matrix $\mathbf{X}$ to an average total energy of MT (i.e., $\mathrm{E}(\|\mathbf{X}\|^2) = MT$), we can define the signal-to-noise ratio (SNR) at each receive antenna as $\rho = M\sigma_h^2/\sigma_e^2$. In general, the transmitted matrix $\mathbf{X}$ is drawn from a constellation $\mathcal{A}_{\mathbf{X}}$.

From [2], we know that it is advantageous from a noncoherent capacity point of view to consider a constellation $\mathcal{A}_{\mathbf{X}}$ that satisfies $\mathbf{X}^H\mathbf{X} = T\mathbf{I}_M$, $\forall \mathbf{X} \in \mathcal{A}_{\mathbf{X}}$. Such a signaling scheme is referred to as *unitary space-time (ST) modulation.* In [3], a number of different unitary ST designs are presented. In this work, we consider a much simpler unitary ST design, which we label as *space-time FSK (ST-FSK).*

We first design a set of real $P \times M$ matrices $\{\mathbf{A}_p\}_{p=1}^P$ that satisfies

$$\mathbf{A}_p^T\mathbf{A}_p = \mathbf{I}_M, \tag{2}$$

$$\mathbf{A}_p^T\mathbf{A}_{p'} = -\mathbf{A}_{p'}^T\mathbf{A}_p, \; p \neq p'. \tag{3}$$

For any M, there exist some values of $P \geq M$, for which such a design is possible [7]. For $M = P = 2$, an example of such a design is (see also [1])

$$\mathbf{A}_1 = \mathbf{I}_2 := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{A}_2 = \mathbf{J}_2 := \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

We then choose K, which represents the number of FSK waveforms that we want to include in our design. Defining $\mathcal{A}_k = \{0, \ldots, K-1\}$, the corresponding set of FSK waveforms is $\{\mathbf{f}_k | k \in \mathcal{A}_k\}$, where

$$\mathbf{f}_k := [1 \; e^{j2\pi k/K} \; \ldots \; e^{j2\pi k(K-1)/K}]^T.$$

We then construct the constellation $\mathcal{A}_{\mathbf{X}} = \{\mathbf{X}_{\mathbf{k}} | \mathbf{k} \in \mathcal{A}_k^{P\times 1}\}$, where

$$\mathbf{X}_{\mathbf{k}} = \sum_{p=1}^{P} \mathbf{A}_p \otimes \mathbf{f}_{[\mathbf{k}]_p}. \tag{4}$$

Note that the block size of ST-FSK is $T = PK$. Moreover, the rate of ST-FSK is the same as that of conventional FSK using the same set of FSK waveforms. Defining R as the rate expressed in bits per symbol, we obtain

$$R = \log_2(|\mathcal{A}_k^{P\times 1}|)/T = \log_2(K^P)/(KP) = \log_2(K)/K.$$

From (4), we have that $\mathbf{X}_{\mathbf{k}}^H\mathbf{X}_{\mathbf{k}} = T\mathbf{I}_M$, $\forall \mathbf{k} \in \mathcal{A}_k^{P\times 1}$, and hence we can view ST-FSK as a special unitary ST design. This allows us to use all the results that were presented in [2].

III. ST-FSK Detector

The noncoherent maximum likelihood (ML) detector can be expressed as [2]

$$\hat{\mathbf{k}} = \arg \max_{\mathbf{k} \in \mathcal{A}_k^{P\times 1}} \|\mathbf{X}_{\mathbf{k}}^H\mathbf{Y}\|^2.$$

Using (4), we can rewrite $\mathbf{X}_{\mathbf{k}}^H\mathbf{Y}$ as

$$\begin{aligned} \mathbf{X}_{\mathbf{k}}^H\mathbf{Y} &= \sum_{p=1}^{P} (\mathbf{A}_p^T \otimes \mathbf{f}_{[\mathbf{k}]_p}^H)\mathbf{Y} \\ &= \sum_{p=1}^{P} \mathbf{A}_p^T (\mathbf{I}_M \otimes \mathbf{f}_{[\mathbf{k}]_p}^H)\mathbf{Y} \\ &= \sum_{p=1}^{P} \mathbf{A}_p^T \mathbf{Z}_{[\mathbf{k}]_p}, \end{aligned}$$

where $\mathbf{Z}_k := (\mathbf{I}_M \otimes \mathbf{f}_k^H)\mathbf{Y}$ can be viewed as the matched filter output corresponding to the FSK waveform $\mathbf{f}_k$. Hence, the noncoherent ML detector can be simplified by first computing the K matched filter outputs $\{\mathbf{Z}_k | k \in \mathcal{A}_k\}$, which yield a sufficient statistic, and subsequently determining

$$\hat{\mathbf{k}} = \arg \max_{\mathbf{k} \in \mathcal{A}_k^{P\times 1}} \Big\| \sum_{p=1}^{P} \mathbf{A}_p^T \mathbf{Z}_{[\mathbf{k}]_p} \Big\|^2. \tag{5}$$

In the next section, we will analyze the performance of this noncoherent ML detector.

IV. Performance Analysis

We can upper bound the block error probability P_e through the union bound as [2]

$$P_e \leq \frac{1}{K^P} \sum_{\mathbf{k},\mathbf{k}' \in \mathcal{A}_k^{P\times 1}, \mathbf{k} \neq \mathbf{k}'} P_{\mathbf{k},\mathbf{k}'},$$

where $P_{\mathbf{k},\mathbf{k}'}$ is the pairwise error probability of mistaking $\mathbf{k}$ for $\mathbf{k}'$ or vice versa. Furthermore, we can upper bound this pairwise error probability through the Chernoff bound as [2]

$$P_{\mathbf{k},\mathbf{k}'} \leq \frac{1}{2} \prod_{m=1}^{M} \left(1 + \frac{(\rho T/M)^2 (1 - d_{\mathbf{k},\mathbf{k}',m}^2)}{4(1 + \rho T/M)} \right)^{-N},$$

where $1 \geq d_{\mathbf{k},\mathbf{k}',1} \geq \cdots \geq d_{\mathbf{k},\mathbf{k}',M} \geq 0$ are the singular values of $\mathbf{X}_{\mathbf{k}}^H\mathbf{X}_{\mathbf{k}'}/T$. Note in this context that we can rewrite $\mathbf{X}_{\mathbf{k}}^H\mathbf{X}_{\mathbf{k}'}/T$ as

$$\begin{aligned} \mathbf{X}_{\mathbf{k}}^H\mathbf{X}_{\mathbf{k}'}/T &= \frac{1}{T} \sum_{p=1}^{P} (\mathbf{A}_p^T \otimes \mathbf{f}_{[\mathbf{k}]_p}^H) \sum_{p'=1}^{P} (\mathbf{A}_{p'} \otimes \mathbf{f}_{[\mathbf{k}']_{p'}}) \\ &= \frac{1}{T} \sum_{p=1}^{P} \sum_{p'=1}^{P} (\mathbf{A}_p^T\mathbf{A}_{p'}) \otimes (\mathbf{f}_{[\mathbf{k}]_p}^H \mathbf{f}_{[\mathbf{k}']_{p'}}) \\ &= \frac{1}{P} \sum_{p=1}^{P} \sum_{p'=1}^{P} (\mathbf{A}_p^T\mathbf{A}_{p'}) \otimes \delta_{[\mathbf{k}]_p, [\mathbf{k}']_{p'}}. \end{aligned} \tag{6}$$

For a sufficiently large ρ, the Chernoff bound on the pairwise error probability $P_{\mathbf{k},\mathbf{k}'}$ depends dominantly on [4]

$$\zeta_{\mathbf{k},\mathbf{k}'} = \left(\prod_{m=1}^{M} (1 - d_{\mathbf{k},\mathbf{k}',m}^2) \right)^{1/2M},$$

which can be interpreted as the geometric mean of the sinusoids of the M principle angles between the subspaces spanned by the columns of $\mathbf{X}_{\mathbf{k}}$ and $\mathbf{X}_{\mathbf{k}'}$ [4]. In short, we can interpret $\zeta_{\mathbf{k},\mathbf{k}'}$ as the *distance* between $\mathbf{X}_{\mathbf{k}}$ and $\mathbf{X}_{\mathbf{k}'}$. Defining the *diversity product* as

$$\zeta = \min_{\mathbf{k},\mathbf{k}' \in \mathcal{A}_k^{P\times 1}, \mathbf{k} \neq \mathbf{k}'} \zeta_{\mathbf{k},\mathbf{k}'},$$

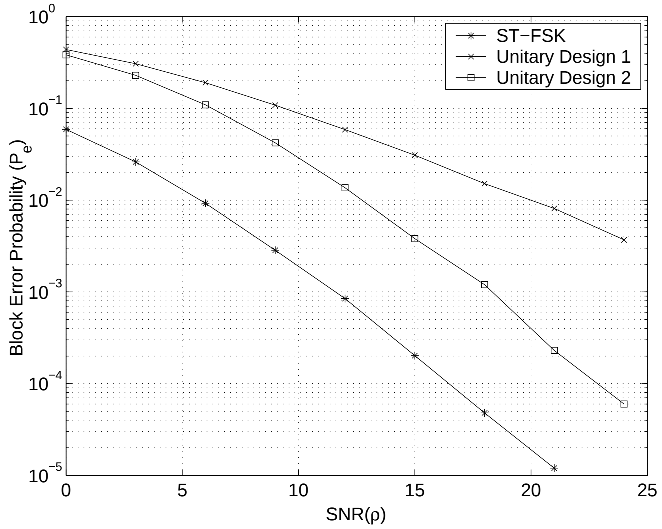

Figure 1: Performance comparison between ST-FSK and existing unitary ST designs. We consider $M = 2$ transmit antennas, $N = 1$ receive antenna, and rate $R = 1/2$ transmission.

full diversity is achieved if the diversity product ζ is nonzero [4].

The unitary ST designs of [3] do not maximize the diversity product[1]. Hence, they do not necessarily have a nonzero diversity product. Consequently, the unitary ST designs of [3] do not guarantee to achieve full diversity and to assure perfect symbol recovery in the absence of noise, irrespective of the channel realization. ST-FSK modulation, on the other hand, has a nonzero diversity product, achieves full diversity, and assures perfect symbol recovery in any noiseless channel. For simplicity, we will illustrate this for $M = P = 2$. More general cases will be reported elsewhere.

When $M = P = 2$, the most general design for the set of real 2×2 matrices $\{\mathbf{A}_p\}_{p=1}^2$ that satisfies (2) and (3) is given by

$$\mathbf{A}_1 = \mathbf{\Phi}, \quad \mathbf{A}_2 = \mathbf{J}_2\mathbf{\Phi},$$

where $\mathbf{\Phi}$ is an arbitrary orthogonal 2×2 matrix. Evaluating (6) for all possible combinations of $\mathbf{k}$ and $\mathbf{k}'$, the product $\mathbf{X}_{\mathbf{k}}^H \mathbf{X}_{\mathbf{k}'}/T$ can be expressed as

$$\frac{\mathbf{X}_{\mathbf{k}}^H \mathbf{X}_{\mathbf{k}'}}{T} = \begin{cases} \mathbf{I}_2, & k_1 = k_1', k_2 = k_2' \\ (\mathbf{I}_2 - \mathbf{J}_2)/2, & k_1 \neq k_2 = k_1' = k_2' \\ & k_1 = k_2 = k_1' \neq k_2' \\ (\mathbf{I}_2 + \mathbf{J}_2)/2, & k_1' = k_2' = k_1 \neq k_2 \\ & k_1' \neq k_2' = k_1 = k_2 \\ \mathbf{I}_2/2, & k_1 = k_1' \neq k_2', k_2 \neq k_1', k_2 \neq k_2' \\ & k_2 = k_2' \neq k_1', k_1 \neq k_1', k_1 \neq k_2' \\ \mathbf{J}_2/2, & k_1 = k_2' \neq k_1', k_2 \neq k_1', k_2 \neq k_2' \\ -\mathbf{J}_2/2, & k_2 = k_1' \neq k_2', k_1 \neq k_1', k_1 \neq k_2' \\ \mathbf{0}_{2\times 2}, & \text{otherwise} \end{cases}, \quad (7)$$

[1]The unitary ST designs of [4] do maximize the diversity product. However, these designs only apply to the case where $T = M$, because they are used in a *differential* unitary ST modulation context.

where we have used $k_p := [\mathbf{k}]_p$ and $k_p' := [\mathbf{k}']_p$. We see that in all cases the two singular values $\{d_{\mathbf{k},\mathbf{k}',m}\}_{m=1}^2$ are equal, i.e, $d_{\mathbf{k},\mathbf{k}',m} = d_{\mathbf{k},\mathbf{k}'}$, $m = 1, 2$. Hence, $\zeta_{\mathbf{k},\mathbf{k}'}$ can be expressed as

$$\zeta_{\mathbf{k},\mathbf{k}'} = \sqrt{1 - d_{\mathbf{k},\mathbf{k}'}^2}.$$

We further observe that the maximum singular value $d_{\mathbf{k},\mathbf{k}'}$ for $\mathbf{k} \neq \mathbf{k}'$ is obtained in cases 2 and 3 of (7) and equals $1/\sqrt{2}$. Hence, the diversity product ζ can be expressed as

$$\zeta = \min_{\mathbf{k},\mathbf{k}' \in \mathcal{A}_k^{2\times 1}, \mathbf{k}\neq\mathbf{k}'} \zeta_{\mathbf{k},\mathbf{k}'} = 1/\sqrt{2}.$$

We can thus conclude that ST-FSK with $M = P = 2$ has a nonzero diversity product and thus achieves full diversity.

In relationship to the above result, we now show that ST-FSK with $M = P = 2$ assures perfect symbol recovery in the absence of noise, irrespective of the channel realization. Assuming that $\mathbf{X}_{\mathbf{k}'}$ was the actually transmitted matrix, let us compute $\|\mathbf{X}_{\mathbf{k}}^H \mathbf{Y}\|^2$ in the absence of noise; i.e., let us compute $\|\mathbf{X}_{\mathbf{k}}^H \mathbf{X}_{\mathbf{k}'}\mathbf{H}\|^2$ (see (1)). Using (7), it is clear that $\|\mathbf{X}_{\mathbf{k}}^H \mathbf{X}_{\mathbf{k}'}\mathbf{H}\|^2$ can be expressed as

$$\|\mathbf{X}_{\mathbf{k}}^H \mathbf{X}_{\mathbf{k}'}\mathbf{H}\|^2 = \begin{cases} 4K^2\|\mathbf{H}\|^2, & k_1 = k_1', k_2 = k_2' \\ 2K^2\|\mathbf{H}\|^2, & k_1 \neq k_2 = k_1' = k_2' \\ & k_1 = k_2 = k_1' \neq k_2' \\ & k_1' = k_2' = k_1 \neq k_2 \\ & k_1' \neq k_2' = k_1 = k_2 \\ K^2\|\mathbf{H}\|^2, & k_1 = k_1' \neq k_2', k_2 \neq k_1', k_2 \neq k_2' \\ & k_2 = k_2' \neq k_1', k_1 \neq k_1', k_1 \neq k_2' \\ & k_1 = k_2' \neq k_1', k_2 \neq k_1', k_2 \neq k_2' \\ & k_2 = k_1' \neq k_2', k_1 \neq k_1', k_1 \neq k_2' \\ 0, & \text{otherwise.} \end{cases},$$

where we again have used $k_p := [\mathbf{k}]_p$ and $k_p' := [\mathbf{k}']_p$. Clearly $\|\mathbf{X}_{\mathbf{k}}^H \mathbf{X}_{\mathbf{k}'}\mathbf{H}\|^2$ attains a unique maximum at $\mathbf{k} = \mathbf{k}'$, regardless of the particular fading channel coefficients. We can thus conclude that ST-FSK with $M = P = 2$ is capable of recovering the symbols perfectly in any noiseless channel.

Example: Let us now compare the performance of ST-FSK for $M = P = 2$ and $K = 2$ (hence, $T = 4$), which has rate $R = 1/2$, with the unitary ST design of [3] for $M = 2$, $T = 4$, and rate $R = 1/2$. We consider only one receive antenna ($N = 1$). First of all, the criterion used in [3] allows for many solutions, some of which do not achieve the full diversity order of $MN = 2$. We therefore look at two possible solutions: one that looses diversity and only achieves diversity order 1 (design 1) and one that maximizes the diversity product and thus achieves full diversity (design 2). Performance results are shown in Figure 1. As expected, ST-FSK outperforms the unitary ST design 1. Furthermore, although the diversity product of the unitary ST design 2 is the same as the diversity product of ST-FSK ($\zeta = 1/\sqrt{2}$), ST-FSK also outperforms the unitary ST design 2. This is due to the fact that ST-FSK has a larger average distance between two different matrices from the constellation $\mathcal{A}_{\mathbf{X}}$ than the unitary ST design 2 (although the minimum distance is the same).

V. Application to slow FHMA

In this section, we show how ST-FSK can be merged with slow FHMA. In FHMA, every user is assigned a distinct hopping sequence and hops from one frequency band to another according to its hopping sequence. This MA technique

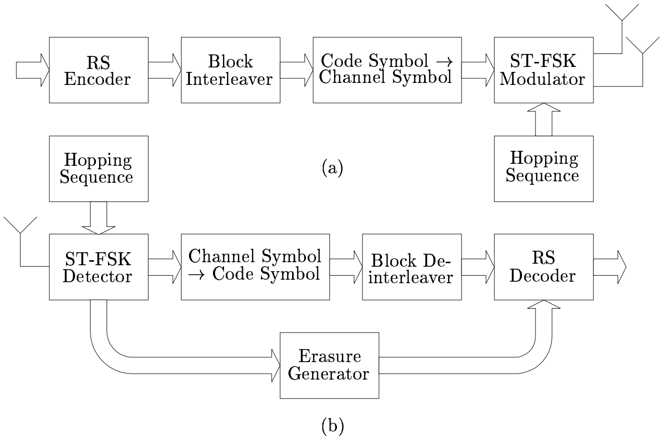

Figure 2: Block diagram of the proposed slow FHMA system based on ST-FSK.

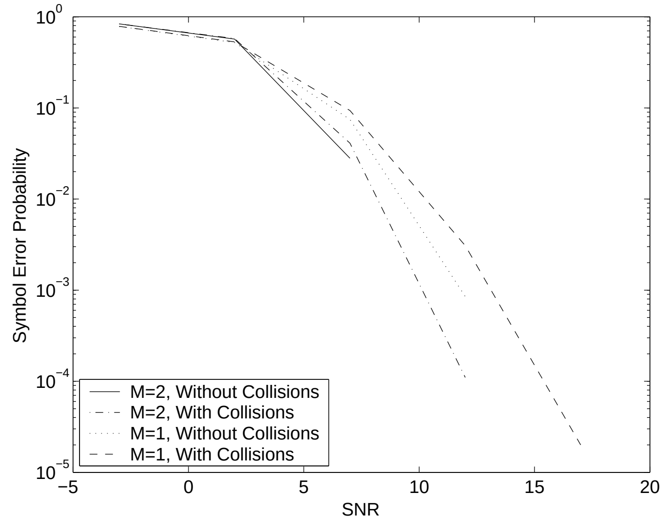

Figure 3: Performance results for a (15,5) RS code and $K = 16$ (16-FSK). No collision knowledge is assumed. Error-only decoding is used.

combats frequency-selective fading, reduces interference from other sources, and induces some level of security. Slow FHMA refers to the case where the period between successive hops, also known as the dwell period, is larger than the symbol period.

We assume here that the dwell period is a multiple of T symbol periods, and consider a communication link from M transmit antennas to N receive antennas that within each frequency band operates over a flat fading channel that is constant over the dwell period. In this case, the data model (1) still holds, but $\mathbf{H}$ and $\mathbf{E}$ now correspond respectively to the flat fading channel matrix and the noise/interference matrix related to the frequency band occupied during the considered dwell period. Hence, ST-FSK can again be used. We can still assume that the entries of $\mathbf{H}$ are i.i.d. zero-mean complex Gaussian variables. However, we can no longer assume that the entries of $\mathbf{E}$ are i.i.d. zero-mean complex Gaussian variables. Due to the nonorthogonality of the hopping sequences, different sources may occupy the same frequency band during the considered dwell period, and thus $\mathbf{E}$ may contain multiple access interference (MAI). As a consequence, the detector given by (5) is no longer the noncoherent ML detector. However, for the sake of simplicity, we will still use this detector. In order to recover from deep fades and collisions, we resort to channel coding as done in [5, 6]. The block diagram of the proposed slow FHMA system based on ST-FSK is depicted in Figure 2. Next, we explain this system in more detail and show some simulation results.

The transmitter consists of a (μ, ν) Reed-Solomon encoder, an interleaver, a code symbol-to-channel symbol converter, a hopping sequence generator, and an ST-FSK modulator (see Figure 2(a)). A (μ, ν) RS code has code word length μ, and each code word entails ν information symbols. A block of μ code words is then interleaved by a square block interleaver, resulting in a stream of μ^2 code symbols. This stream of μ^2 code symbols is grouped into μ blocks of μ code symbols. These μ blocks of μ code symbols are then mapped to μ blocks of channel symbols. Note that the code symbols are drawn from $\mathrm{GF}(\mu + 1)$, whereas the channel symbols are drawn from $\mathcal{A}_k$. Hence, one code symbol may be mapped to several channel symbols. The μ blocks of channel symbols are then transmitted in μ different dwell intervals, after ST-FSK modulation.

The receiver performs the reverse operations and comprises an ST-FSK detector, a channel symbol-to-code symbol converter, a deinterleaver, a Reed-Solomon decoder with erasure decoding capability, and an erasure generator (see Figure 2(b)). Based on the matched filter output (see Section III), which conveys information about the flat fading channel and the noise/interference, the erasure generator produces reliability information for each detected channel symbol, which is used to decide whether to erase a code symbol or not.

Figure 3 shows performance results for the case the receiver has no collision knowledge, and error-only decoding has to be employed (decoding without erasures). To recover from collisions we then need a low-rate RS code. In Figure 3, we consider a (15,5) RS code and $K = 16$ (16-FSK). Hence, the total rate is $R_{\mathrm{tot}} = (1/3)(1/4) = 1/12$. Figure 4 shows performance results for the case the receiver has collision knowledge, and error-and-erasure decoding can be employed (decoding with erasures). This type of decoding allows us to use a high-rate RS code. In Figure 4, we consider a (15,9) RS code and $K = 2$ (2-FSK). Hence, the total rate is $R_{\mathrm{tot}} = (3/5)(1/2) = 3/10$. As performance measure we consider the symbol error probability, i.e., the error probability related to the information symbols at the input of the RS encoder. For both figures, we consider 16 frequency bands and model the frequency hopping sequence as a series of i.i.d. uniformly distributed random variables over 16 system frequencies. The collision probability is 1/16. For comparison means, both figures also show the case no collisions occur (i.e., when the hopping sequences are orthogonal). From these figures we may conclude that it pays off to use two transmit antennas instead of one. Note that regardless of the channel conditions and the coding rate, there is always an error floor. However, the low-rate RS code combined with error-only decoding used when no collision knowledge is present, and the high-rate RS code combined with error-and-erasure decoding used when collision knowledge is present, push this error floor down (not shown in figures).

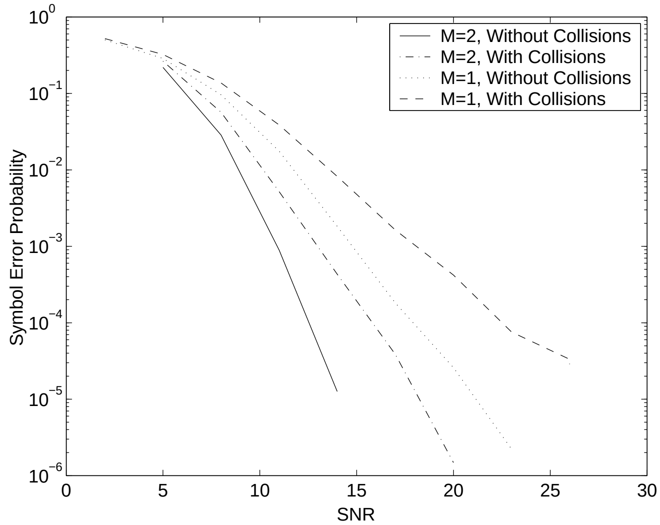

Figure 4: Performance results for a (15,9) RS code and $K = 2$ (2-FSK). Collision knowledge is assumed. Error-and-erasure decoding is used.

VI. Conclusions

In this paper, we have introduced ST-FSK, which can be viewed as a special unitary ST design. However, ST-FSK has a number of advantages over existing unitary ST designs. One of them is that ST-FSK merges very naturally with slow FHMA. To illustrate this, we have proposed a new slow FHMA system based on ST-FSK that outperforms the conventional slow FHMA system based on FSK. Low-rate RS encoding combined with error-only decoding (without collision knowledge), and high-rate RS encoding combined with error-and-erasure decoding (with collision knowledge) is used to recover from collisions, and lower the error floor that results from them.

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