Blind Channel Identification and Equalization with Modulation-Induced Cyclostationarity

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***Abstract*—Recent results have pointed out the importance of inducing cyclostationarity at the transmitter for blind identification and equalization of communication channels. This paper addresses blind channel identification and equalization relying on the modulation induced cyclostationarity, without introducing redundancy at the transmitter. It is shown that single-input single-output channels can be identified uniquely from output second-order cyclic statistics, irrespective of the location of channel zeros, color of additive stationary noise, or channel order overestimation errors, provided that the period of modulation-induced cyclostationarity is greater than half the channel length. Linear, closed-form, nonlinear correlation matching, and subspace-based approaches are developed for channel estimation and are tested using simulations. Necessary and sufficient blind channel identifiability conditions are presented. A Wiener cyclic equalizer is also proposed.**

I. Introduction

Blind identification and equalization of wireless communication channels have attracted considerable interest during the past few years because they avoid training and thus make efficient use of the available bandwidth. It has been shown that the cyclostationarity induced at the *receiver* by oversampling [or fractionally sampling (FS)] the received waveform permits blind identification of most nonminimum phase finite impulse response (FIR) channels, provided that there are no channel zeros equispaced by $2\pi/P$ angle on a circle (where P denotes the oversampling factor) [7], [17], [19], [21], [22], [26]. Such an identifiability condition may fail without excess bandwidth [6]. Moreover, performance of FS-based algorithms degrades when channel zeros are even close to being nonidentifiable [27], and most FS-based algorithms are sensitive to channel order mismatch. Oversampling the received waveform also causes an increase in the channel length. Thus, more parameters have to be estimated, and from an estimation viewpoint, larger data records have to be used in order to obtain reliable channel estimates.

In [9] and [24], a novel approach was pursued by introducing cyclostationarity at the transmitted sequence (as opposed to the received one, as is the case with all FS-based approaches). It was shown that blind channel identification and equalization algorithms can be developed with no restrictions on the channel zeros and color of the additive stationary noise. In these approaches, cyclostationarity is induced at the *transmitter* by means of an encoder: repetition codes [24] or analysis-synthesis (or precoding) filterbanks [9]. In addition, it has been shown in [9] that the loss of information rate originally present in the approach [24] can be alleviated. An alternative viewpoint for inducing cyclostationarity at the transmitter has been proposed by the authors in [18] and independently by Chevreuil and Loubaton in [2] and [3]. In [2], the input symbol stream is modulated with a deterministic and almost periodic sequence. The resulting channel identifiability condition may be restrictive because the modulating sequence entails irrational cyclic frequencies [2]. Relaxed identifiability conditions have been reported independently in [3], [13], and [18] using *strictly* periodic modulating sequences, or, rational cyclic frequencies.

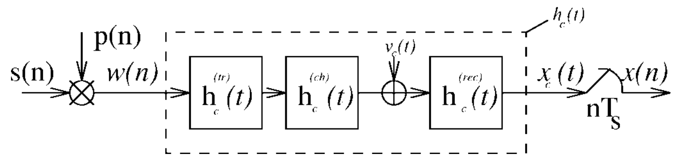

Fig. 1. Baseband transmission channel.

In the present work, we build on our preliminary results in [13] and [18], focusing on strictly periodic modulating sequences. We show that such a precoding guarantees channel identifiability, irrespective of the location of channel zeros, color of additive stationary noise, channel order mismatch, or reduction of information rate, provided that the period of the modulating sequence is greater than half the channel length. Although modulation-induced cyclostationarity (MIC) follows as a particular case of the general filterbank precoder structure of [9] and [24], there are distinct features enjoyed by the MIC framework that motivate separate analysis. MIC has the ability to control easily the modifications induced on the envelope of the transmitted signal, and for certain practical systems, MIC can easily explore some pre-existent forms of transmitter induced cyclostationarity, e.g., for TDMA systems, guard times ("silent bits") are transmitted between any two consecutive frames, and the transmitted power is periodically varied in order to ease hardware design [16]. In addition, MIC does not introduce redundancy at the transmitter and, hence, does not decrease the information rate. The price paid relative to redundant approaches is lack of FIR zero-forcing equalizers for FIR channels.

We consider the baseband transmission system shown in Fig. 1, where the zero-mean, independently and identically

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distributed (i.i.d.) input stream $s(n)$ is modulated by the deterministic and periodic sequence $p(n)$ with period P to obtain $w(n) = p(n)s(n)$ with $p(n) = p(n+P)\ \forall n$. Sequence $w(n)$ is pulse shaped with the transmit filter $h_c^{(\mathrm{tr})}(t)$, propagates through the unknown channel $h_c^{(\mathrm{ch})}(t)$, and, at reception, is filtered by the receive filter $h_c^{(\mathrm{rec})}(t)$. The continuous-time received signal is given by $x_c(t) = \Sigma_l\ w(l)h_c(t - lT_s - d) + v_c(t)$, where $h_c(t) := h_c^{(\mathrm{tr})} * h_c^{(\mathrm{ch})} * h_c^{(\mathrm{rec})}(t)$, $*$ stands for convolution, and $d \in [0, T_s)$ is the unknown propagation delay. The receiver output noise is given by $v_c(t) := (h_c^{(\mathrm{rec})} * \nu_c)(t)$, where T_s denotes the symbol period. Considering a symbol rate T_s^{-1} sampling and defining $h(n) := h_c(t-d)|_{t=nT_s}$, $v(n) := v_c(t)|_{t=nT_s}$, we deduce the discrete-time model

$$x(n) := x_c(t)|_{t=nT_s} = \sum_{l=0}^{L} h(l)w(n-l) + v(n) \tag{1}$$

where L denotes the order of the discrete-time composite channel h. We also assume that input $s(n)$ is independent of the stationary (but arbitrary colored) noise $v(n)$.

We adopt the equivalent system shown in Fig. 2, with the goal of blind identification and equalization of channel $h(n)$, i.e., the recovery of both the channel $h(n)$ and input $s(n)$ from knowledge of the received data $x(n)$ and the periodic sequence $p(n)$. We assume that the receiver is synchronized with the transmitted period—a task that can be accomplished as in [24]. Blind channel identifiability is understood modulo time shift and complex scale ambiguities that are resolved using gain control and differential encoding as usual.

The organization of this paper is the following: Section II establishes a channel identifiability result from second order cyclic statistics of the output, provided that the period of $p(n)$ is greater than the channel length. Various channel estimation algorithms are presented in Section III. A subspace-based channel estimation algorithm reminiscent of that developed in [22] for FS channels is described, which requires the period P to be greater than the channel length. Some guidelines for selecting the periodic sequence $p(n)$ are described in Section IV. A minimum mean-square error (MMSE) cyclic Wiener equalizer is proposed in Section V. Simulation results are described in Section VI, followed by conclusions and research directions.

II. Modulation Inducing Cyclostationarity

The time-varying correlation at time n and lag τ of sequence $w(n)$ is given by $c_{ww}(n;\tau) := Ew(n)\ w^*(n+\tau)$ and satisfies $c_{ww}(n;\tau) = |p(n)|^2\sigma_s^2\delta(\tau) = c_{ww}(n+lP;\tau)$, $\forall l,\tau \in \boldsymbol{Z}$, provided that the sequence $|p(n)|$ is also periodic with fundamental period P, where $*$ stands for conjugation, $|\cdot|$ denotes absolute value, and $\sigma_s^2 := E|s(n)|^2$. Thus, sequence $w(n)$ is cyclostationary, provided that sequence $|p(n)|$ is periodic. In what follows, we assume that the sequence $|p(n)|$ is periodic ($|p(n)| = |p(n+P)|, \forall n$). We refer to the cyclostationarity induced in $w(n)$ as modulation-induced cyclostationarity. The cyclostationarity of $w(n)$ is preserved at the output $x(n)$ of the linear time-invariant channel $h(n)$. Using (1), the output time-varying correlation $c_{xx}(n;\tau)$ is given by

$$\begin{aligned} c_{xx}(n;\tau) &:= E[x(n)x^*(n+\tau)] \\ &= \sum_{l_1=-\infty}^{\infty}\sum_{l_2=-\infty}^{\infty} h(n-l_1)h^*(n+\tau-l_2) \\ &\quad \cdot p(l_1)p^*(l_2)\sigma_s^2\delta(l_1-l_2) + c_{vv}(\tau) \\ &= \sigma_s^2 \sum_{l=-\infty}^{\infty} |p(l)|^2 h(n-l)h^*(n+\tau-l) \\ &\quad + c_{vv}(\tau). \end{aligned} \tag{2}$$

Changing variables ($l \leftrightarrow n-m$) in (2), we obtain

$$\begin{aligned} c_{xx}(n;\tau) &= \sigma_s^2 \sum_{m=-\infty}^{\infty} |p(n-m)|^2 h(m)h^*(m+\tau) \\ &\quad + c_{vv}(\tau). \end{aligned} \tag{3}$$

Since the sequence $|p(n)|$ is periodic with fundamental period P, it follows from (3) that $c_{xx}(n;\tau)$ is also a periodic function in n since $c_{xx}(n;\tau) = c_{xx}(n+P;\tau)$, for any n,τ. Being periodic, $c_{xx}(n;\tau)$ accepts a Fourier Series expansion over the set of complex exponentials with harmonic cycles, where the set of cycles is defined as $A_{xx}^c := \{\alpha_k = 2\pi k/P, k = 0,\cdots,P-1\}$; hence, $c_{xx}(n;\tau)$ and its Fourier coefficients $C_{xx}(\alpha_k;\tau)$, which are called cyclic correlations, are related by the discrete Fourier Series pair

$$\begin{aligned} c_{xx}(n;\tau) &= \sum_{k=0}^{P-1} C_{xx}(\alpha_k;\tau)e^{j\alpha_k n} \overset{FS}{\leftrightarrow} C_{xx}(\alpha_k;\tau) \\ &= \frac{1}{P}\sum_{n=0}^{P-1} c_{xx}(n;\tau)e^{-j\alpha_k n}. \end{aligned} \tag{4}$$

On substituting (3) into (4), the cyclic correlation $C_{xx}(\alpha_k;\tau)$ at a fixed cycle α_k, $0 \le k \le P-1$, is given by

$$\begin{aligned} C_{xx}(\alpha_k;\tau) &= \sigma_s^2 P_2(\alpha_k)\sum_{m=-\infty}^{\infty} h(m)h^*(m+\tau)e^{-j\alpha_k m} \\ &\quad + c_{vv}(\tau)\delta(\alpha_k) \end{aligned} \tag{5}$$

where

$$P_2(\alpha_k) := \frac{1}{P}\sum_{n=0}^{P-1} |p(n)|^2 e^{-j\alpha_k n}. \tag{6}$$

From now on, we consider a nonzero cycle $\alpha_k \neq 0$ in (5) and (6) such that the contribution of stationary noise $v(n)$ is cancelled out in (5). We also take α_k to be a cycle of the periodic sequence $|p(n)|^2$ satisfying $P_2(\alpha_k) \neq 0$. The $\mathcal{Z}$ transform of the cyclic correlation $\{C_{xx}(\alpha_k;\tau)\}_{\tau=-\infty}^{\infty}$ for a fixed cycle α_k is called the cyclic spectrum and is given by [c.f., (5)]

$$\begin{aligned} S_{xx}(\alpha_k;z) &:= \sum_{\tau=-\infty}^{\infty} C_{xx}(\alpha_k;\tau)z^{-\tau} \\ &= \sigma_s^2 P_2(\alpha_k)\left[\sum_m h(m)e^{-j\alpha_k m}z^m\right] \\ &\quad \cdot \left[\sum_\tau h^*\ (m+\tau)z^{-(m+\tau)}\right] \\ &= \sigma_s^2 P_2(\alpha_k)H(e^{j\alpha_k}z^{-1})H^*(z^*) \end{aligned} \tag{7}$$

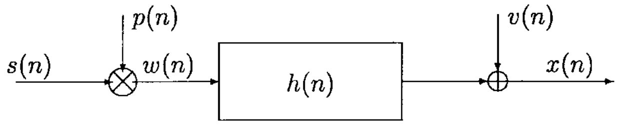


Fig. 2. Baseband discrete-time channel.

where (7) was obtained by changing the order of summation and denoting with $H(z) := \Sigma_{n=0}^{L} h(n)z^{-n}$ the channel's $\mathcal{Z}$ transform. On the "cyclic spectral factorization" (7), a few remarks are now in order.

Remark 1: The period P of $c_{xx}(n;\tau)$ can be chosen such that $P > L+1$, whereas the length of the channel is kept the same [9]. This has to be contrasted with the cyclostationarity induced by oversampling or fractionally sampling the receiver output. In the FS case, we always have $L > P$, which does not allow unique identification of $H(z)$ from the set of cyclic spectra (7) for $k = 1, \cdots, P-1$, [9], [22].

Remark 2: The choice $P > L+1$ guarantees identifiability of $H(z)$ from $L+1$ cyclic spectra based on the cyclic spectral factorization (7). Indeed, it is sufficient to see that $H(z)$ can be found as the greatest common divisor (gcd) of the family of polynomials $\{S_{xx}^{*}(\alpha_k; z),\ k = 1, \cdots, P-1\}$ in (7). The extraction of the gcd $H(z)$ can be performed with the methods presented in [1], [11], and [22]. Using arguments similar to those in [22, Th. 1], we can easily show that the condition $P > L+1$ is both necessary and sufficient for the identifiability of $H(z)$ as the gcd of the family of cyclic spectra $\{S_{xx}(\alpha_k; z)\}_{k=1}^{P-1}$.

In what follows, we develop alternative blind channel identification approaches, which will be shown to relax the aforementioned channel identifiability condition $(P > L+1)$ and are not sensitive to channel order overestimation errors. Furthermore, we will show that knowledge of the cyclic spectrum at a single cycle α_k provides sufficient information for unique identification of $h(n)$.

III. Channel Estimation Algorithms

We first describe algorithms that provide linear and closed-form expressions of the channel $h(n)$ in terms of the cyclic correlations $C_{xx}(\alpha_k; \tau)$ computed at a single cycle α_k.

A. Closed-Form and Linear Solutions

By choosing a sufficiently long period P, simple closed-form expressions for the channel coefficients can be obtained under the whiteness assumption on $v(n)$. Consider that $p(n)$ is chosen such that $p(n) = 0$ for $n \in [M, P)$ and $P - M \geq L$, with M an arbitrary integer $1 \leq M \leq P$. Rewrite (3) for time $n = 0$ as

$$c_{xx}(0;\tau) = \sigma_s^2 \sum_{m=-\infty}^{\infty} |p(-m)|^2 h(m) h^*(m+\tau) + c_{vv}(\tau). \tag{8}$$

Take into account the fact that $p(n) = 0$ for $n \in [M, P)$ to deduce that $c_{xx}(0;\tau) = \sigma_s^2\ |p(0)|^2 h^*(\tau) h(0)$. Solving for $h(\tau)$, we arrive at the closed-form expression

$$h(\tau) = [\sigma_s^2 |p(0)|^2 h(0)]^{-1} c_{xx}^*(0;\tau). \tag{9}$$

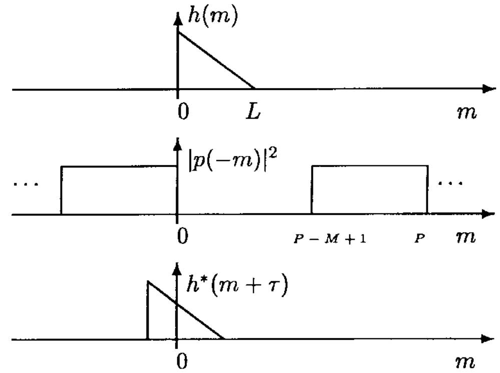


Fig. 3. Exploitation of lag diversity.

Equation (9) can be easily understood also by considering a geometric interpretation of the sum in (8). From Fig. 3, we note that with $P - M \geq L$, the only surviving term in (8) is the one corresponding to $m = 0$ since the terms corresponding to $m < 0$ are cancelled out due to the causality assumption on channel $h(n)$, whereas the terms corresponding to $m > 0$ are cancelled out due to the zeros present in the periodic sequence $p(n)$. The closed-form expression (9) shows that the channel impulse response $h(n)$ is given (modulo a scaling factor) by the sequence of time-varying correlations $c_{xx}^*(0;\tau)$, $\tau = 0, \cdots, L$. We can also see that the channel estimates (9) are *not* sensitive to order overestimation errors and can easily be estimated (perhaps online) from $\{x(n)\}_{n=0}^{N-1}$ using the time-varying correlation estimate, e.g., [10]

$$\hat{c}_{xx}^{(N)}(n;\tau) = \frac{1}{[(N-n)/P]} \sum_{k=0}^{[(N-n)/P]-1} x(n+kP) \cdot x^*(n+kP+\tau). \tag{10}$$

The estimation accuracy of $\hat{c}_{xx}^{(N)}(n;\tau)$ increases by using a smaller period P since more samples are averaged in (10). Recall also that (9) requires $P \geq L + M \geq L+1$. Note that such a modulating sequence eliminates $P - L$ symbols from the input $s(n)$. Thus, it is impossible to recover all the samples unless some sort of redundancy is introduced at the input $s(n)$. As has been noted in [16], many TDMA wireless standards include guard times (or "silent bits") between any consecutive frames, and this feature offers MIC. Thus, for such scenarios, the previous simple expressions for the channel taps may be attractive.

Next, an alternative linear equation approach is shown to require smaller values for the period P without decreasing the information rate. Without loss of generality (w.l.o.g.), we assume that $h(0) = 1$ and select $P_2(\alpha_k)$ such that $\sigma_s^2 P_2(\alpha_k) = 1$. We wish to show that channel $H(z)$ is identifiable from the set of cyclic correlations $C_{xx}(\alpha_k; L)$, $C_{xx}(\alpha_k; \pm(L-l))$, $l = 1, \cdots, [L/2]$, where $[\cdot]$ stands for integer part. With $\tau = L$ and $\alpha_k \neq 0$, we deduce from (5) that

$$C_{xx}(\alpha_k; L) = \sigma_s^2 P_2(\alpha_k) h(0) h^*(L)$$

which enables us to estimate $h(L)$. Next, we consider (5) with $\tau = L-1, -L+1$. The system of equations

$$\begin{aligned} C_{xx}(\alpha_k; L-1) &= h(0)h^*(L-1) + h(1)h^*(L)e^{-j\alpha_k} \\ C^*_{xx}(\alpha_k; -L+1) &= h(0)h^*(L-1)e^{j(L-1)\alpha_k} \\ &\quad + h(1)h^*(L)e^{jL\alpha_k} \end{aligned}$$

can be rewritten in the matrix form as

$$\begin{bmatrix} h(0) & h^*(L)e^{-j\alpha_k} \\ h(0)e^{j(L-1)\alpha_k} & h^*(L)e^{jL\alpha_k} \end{bmatrix} \begin{bmatrix} h^*(L-1) \\ h(1) \end{bmatrix} = \begin{bmatrix} C_{xx}(\alpha_k; L-1) \\ C^*_{xx}(\alpha_k; -L+1) \end{bmatrix}. \tag{11}$$

From (11), we can uniquely estimate $h(1)$ and $h(L-1)$, provided that $h(0)h^*(L)(e^{jL\alpha_k} - e^{j(L-2)\alpha_k}) \neq 0$, i.e., $e^{-j2\alpha_k} \neq 1$. Similarly, considering the expressions of $C_{xx}(\alpha_k; L-l)$ and $C^*_{xx}(\alpha_k; -(L-l))$, we can determine $h^*(L-l)$ and $h(l)$ for any $l = 1, \cdots, [L/2]$. From the expressions of these two cyclic correlations, we form the system of equations

$$\begin{aligned} &\begin{bmatrix} h(0) & h^*(L)e^{-j\alpha_k l} \\ h(0)e^{j\alpha_k(L-l)} & h^*(L)e^{j\alpha_k L} \end{bmatrix} \begin{bmatrix} h^*(L-l) \\ h(l) \end{bmatrix} \\ &= \begin{bmatrix} C_{xx}(\alpha_k, L-l) \\ C^*_{xx}(\alpha_k, -(L-l)) \end{bmatrix} \\ &\quad - \begin{bmatrix} \sum_{m=1}^{l-1} h(m)h^*(m+L-l)e^{-j\alpha_k m} \\ \sum_{m=L-l+1}^{L-1} h^*(m)h(m-L+l)e^{j\alpha_k m} \end{bmatrix}. \end{aligned} \tag{12}$$

Coefficients $h(l)$ and $h(L-l)$ can be uniquely determined from (12), provided that the system of (12) is nonsingular, i.e., $\exp(j2\alpha_k l) \neq 1$ for $l = 1, \cdots, [L/2]$. Thus, $h(n)$ can be uniquely identified by solving (12) iteratively for $l = 1, \cdots, [L/2]$.

Given the observations $\{x(n)\}_{n=0}^{N-1}$, we consistently estimate the cyclic correlation $C_{xx}(\alpha_k; \tau)$ using the estimate $\hat{c}^{(N)}_{xx}(n; \tau)$ from (10) into

$$\hat{C}^{(N)}_{xx}(\alpha_k; \tau) = \frac{1}{P} \sum_{n=0}^{P-1} \hat{c}^{(N)}_{xx}(n; \tau) e^{-j\alpha_k n}. \tag{13}$$

The estimator $\hat{C}^{(N)}_{xx}(\alpha_k; \tau)$ is mean-square (m.s.) consistent and asymptotically normally distributed because our input $s(n)$ has finite moments, and the channel $h(n)$ has finite memory (thus, the so-called mixing conditions in [5] are satisfied). Since the channel identifiability condition is satisfied and the cyclic correlation coefficients are consistently estimated, we conclude that the channel coefficients estimated via (11) and (12) are also consistent. We have thus established the following.

Proposition 1—(Linear Equations: Identifiability Condition): A sufficient condition for identifiability of $H(z)$ from the cyclic spectrum factorization (7) is that the period P (and, hence, the cycle $\alpha_k = 2\pi k/P$ $1 \leq k \leq P-1$) be chosen such that

$$e^{j2\alpha_k l} \neq 1, \quad \text{for } l = 1, \cdots, [L/2]. \tag{14}$$

Under (14), channel estimates based on (12) are m.s. consistent. □

Remark 3: It follows that if the period P of the modulating sequence $p(n)$ is any odd number that satisfies $P > [L/2]$ with $\gcd(k, P) = 1$, then condition (14) is automatically satisfied.

From an estimation viewpoint, the iterative solutions of (12) will be prone to error propagation and sensitive to channel order overestimation errors. Nevertheless, Proposition 1 has value because it establishes identifiability from a single cycle for relatively short periodic sequences with period $P > [L/2]$. In the next subsection, we describe a class of subspace-based channel estimation algorithms that are not sensitive to error propagation or channel order overestimation errors.

B. Subspace-Based Channel Estimation Approaches

We first describe an algorithm that provides a closed-form expression of $h(n)$ in terms of the cyclic correlations $C_{xx}(\alpha_k; \tau)$ computed at a single cycle $\alpha_k \neq 0$.

1) One-Cycle Subspace Approach: Changing variables ($z \leftrightarrow e^{j\alpha_k}/z$ and $z \leftrightarrow e^{j2\alpha_k} z^*$) in (7), we find, respectively, that

$$S_{xx}(\alpha_k; e^{j\alpha_k}/z) = \sigma_s^2 P_2(\alpha_k) H(z) H^*(e^{-j\alpha_k}/z^*) \tag{15}$$

$$S^*_{xx}(\alpha_k; e^{j2\alpha_k} z^*) = \sigma_s^2 P^*_2(\alpha_k) H(e^{-j2\alpha_k} z) H^*(e^{-j\alpha_k}/z^*) \tag{16}$$

and hence

$$\frac{S_{xx}(\alpha_k; e^{j\alpha_k}/z)}{S^*_{xx}(\alpha_k; e^{j2\alpha_k} z^*)} = \frac{P_2(\alpha_k)}{P^*_2(\alpha_k)} \cdot \frac{H(z)}{H(e^{-j2\alpha_k} z)}. \tag{17}$$

Identifying $H(z)$ from (17) amounts to solving for $h(n)$ the system of equations resulting from the inverse $\mathcal{Z}$ transform of

$$\begin{aligned} &H(e^{-j2\alpha_k} z) P^*_2(\alpha_k) z^{-L} S_{xx}(\alpha_k; e^{j\alpha_k}/z) \\ &\quad = H(z) P_2(\alpha_k) z^{-L} S^*_{xx}(\alpha_k; e^{j2\alpha_k} z^*) \end{aligned} \tag{18}$$

where z^{-L} was introduced to make polynomials in both sides causal.[1]

In order to rewrite (18) in a matrix form, we need to introduce some notations. We associate with the vector of coefficients $\boldsymbol{f} := [f(0)\ f(1) \cdots f(L_f)]'$ of an arbitrary polynomial $F(z) = \Sigma_{i=0}^{L_f} f(i) z^{-i}$ the $(\tilde{L} + L_f + 1) \times (\tilde{L} + 1)$ Toeplitz matrix $\mathcal{T}_F(\tilde{L})$ having as first column the vector $[f(0)\ f(1) \cdots f(L_f)\ 0 \cdots 0]'$ and as the first row $[f(0)\ 0 \cdots 0]$ (where prime stands for transpose). Let $\mathcal{T}_{S1}(\tilde{L})$ and $\mathcal{T}_{S2}(\tilde{L})$ denote the $(\tilde{L} + 2L + 1) \times (\tilde{L} + 1)$ Toeplitz matrices associated with the $2L$th-order polynomials $P^*_2(\alpha_k) z^{-L} S_{xx}(\alpha_k; e^{j\alpha_k}/z)$ and $P_2(\alpha_k) z^{-L} S^*_{xx}(\alpha_k; e^{j2\alpha_k} z^*)$, respectively.

Consider also the $(\tilde{L} + 1) \times (\tilde{L} + 1)$ diagonal matrix $\boldsymbol{D}_{\alpha_k}(\tilde{L}) = \text{diag}\{1, e^{j2\alpha_k}, \cdots, e^{j2\tilde{L}\alpha_k}\}$ and the $(L+1) \times 1$ vector $\boldsymbol{h} := [h(0)\ h(1) \cdots h(L)]'$ of the coefficients of $H(z)$. Note that the vector of coefficients $\boldsymbol{h}_{\alpha_k}$ of the polynomial $H(e^{-j2\alpha_k} z)$ is given by

$$\boldsymbol{h}_{\alpha_k} = \boldsymbol{D}_{\alpha_k}(L) \boldsymbol{h}. \tag{19}$$

[1] It is easy to check the equality $S^*_{xx}(\alpha_k; e^{j2\alpha_k} z^*) = S_{xx}(-\alpha_k; e^{j2\alpha_k} z^*)$. However, this equality does not hold for the so-called conjugate cyclospectra (which are defined later in Section IV).

Equating coefficients of both sides of (18), we deduce

$$\mathcal{T}_{S1}(L)\boldsymbol{h}_{\alpha_k} = \mathcal{T}_{S2}(L)\boldsymbol{h} \tag{20}$$

and on substituting (19) into (20), we infer

$$\mathcal{T}_{S1}(L)\boldsymbol{D}_{\alpha_k}(L)\boldsymbol{h} = \mathcal{T}_{S2}(L)\boldsymbol{h} \tag{21}$$

which is equivalent to

$$\boldsymbol{S}_{\alpha_k}(L)\boldsymbol{h} = \boldsymbol{0} \tag{22}$$

$$\boldsymbol{S}_{\alpha_k}(L) := \mathcal{T}_{S1}(L)\boldsymbol{D}_{\alpha_k}(L) - \mathcal{T}_{S2}(L). \tag{23}$$

Similar to (14), we will find it useful to assume that α_k (and, hence, the period P) is chosen to satisfy

$$e^{j2\alpha_k l} \neq 1, \quad \text{for } l = 1, \cdots, L. \tag{24}$$

The vector of channel coefficients $\boldsymbol{h}$ can be uniquely recovered from (22), provided that $\boldsymbol{S}_{\alpha_k}(L)$ in (22) has nullity one. We will show that under (24), the columns of the Toeplitz submatrix[2] $\boldsymbol{S}_{\alpha_k}(:, 1:L)$ are linearly independent. This follows easily since the determinant of the upper triangular submatrix $\boldsymbol{S}_{\alpha_k}(2L+1:3L, 1:L)$ is equal to $[\sigma_s^2|P_2(\alpha_k)|^2 h^*(0)h(L)]^L \Pi_{i=0}^{L-1}(e^{j2i\alpha_k} - e^{j2L\alpha_k})$. If we suppose that $\Pi_{i=0}^{L-1}(e^{j2i\alpha_k} - e^{j2L\alpha_k}) = 0$, then there must exist i, $0 \leq i \leq L-1$ such that

$$e^{j2i\alpha_k} - e^{j2L\alpha_k} = 0 \Rightarrow e^{j2(L-i)\alpha_k} - 1 = 0. \tag{25}$$

However, (25) contradicts assumption (24). Since matrix $\boldsymbol{S}_{\alpha_k}(L)$ is obtained by adding only one column to the linearly independent columns of submatrix $\boldsymbol{S}_{\alpha_k}(:, 1:L)$, it follows that $\dim(\mathcal{N}(\boldsymbol{S}_{\alpha_k}(L))) = 1$. Hence, channel $\boldsymbol{h}$ can be found as the unique null eigenvector of (22). In practice, the entries of $\boldsymbol{S}_{\alpha_k}$ are replaced by the cyclic correlation estimates computed as in (13).

We also remark that knowledge of the order L is not necessary for estimating $\boldsymbol{h}$. Indeed, since the columns of submatrix $\boldsymbol{S}_{\alpha_k}(:, 1:L)$ are linearly independent, it follows that the channel order L can be estimated as the minimal positive integer $\tilde{L}$ for which $\dim(\mathcal{N}(\boldsymbol{S}_{\alpha_k}(\tilde{L}))) = 1$. In other words, condition (24) guarantees that there is no solution of degree less than L for (18). We briefly note that an alternative proof for showing that there is no solution $H(z)$ of degree less than L consists of equating the coefficients of highest degree terms in both sides of (18). Condition (24) implies that the solution of minimum degree occurs for L.

Next, we prove that (24) is also necessary for (18) to yield a unique channel estimate. We show that if there exists l, $1 \leq l \leq L$ such that $\exp(j2\alpha_k l) = 1$, then there are channels $H(z)$ that cannot be retrieved from (18). It suffices to consider a channel $H(z)$ whose, say, the first l roots satisfy $r_i = r_{i+1}\exp(j2\alpha_k)$, $i = 1, \cdots, l-1$, and $r_l = r_1 \exp(j2\alpha_k)$, i.e., there is a common "cycle" among the roots of $H(z)$. Channel $H(z)$ cannot be recovered from (18) since all the roots $\{r_1, \cdots, r_L\} = \{r_1\exp(j2\alpha_k), \cdots, r_L\exp(j2\alpha_k)\}$ cancel out in both sides of (18) [see also (17)]. Thus, we lose all the information concerning the existence of these roots in (18), and the channel $H(z)$ cannot be recovered unless the condition $\exp(j2l\alpha_k) \neq 1$, holds true for any $l = 1, \cdots, L$. Summarizing, we have established the following.

[2] In writing this submatrix, we adopted Matlab's notation.

Theorem 1—Subspace-Based Approach: Identifiability Condition: The necessary and sufficient condition for unique identification of channel $H(z)$ from the cyclic spectra in (18) is that the period P (and hence cycle α_k, $1 \leq k \leq P-1$) of $|p(n)|$ be chosen such that (24) holds true. □

Remark 4: A necessary condition for cycle α_k to satisfy (24) is $P \geq L+1$, i.e., the period P of sequence $|p(n)|$ should be greater than or equal to the channel length $L+1$. Condition (24) can be easily fulfilled by choosing the cycle $\alpha_k = 2\pi k/P$ with $\gcd(2k, P) = 1$ and $P \geq L+1$.

Remark 5: We note that a similar multicycle algorithm has been proposed in [22] for FS systems. Note, though, that in contrast to [22], our symbol rate approach guarantees identifiability irrespective of channel zeros.

Proposition 1 and Theorem 1 essentially assert that the cyclic spectrum $S_{xx}(\alpha_k; z)$ factorizes uniquely in the form (7), provided that the period of cyclostationarity is greater than half or full length of the channel. Next, we address the problem of identifying the channel $h(n)$ when channel order overestimation errors occur. We maintain that if the channel order is overestimated in the range $L \leq \overline{L} < L+P$, then we can still identify the channel, i.e., we show that $\dim\mathcal{N}(\boldsymbol{S}_{\alpha_k}(\overline{L})) = 1$, and the unique null eigenvector of (22) is given (modulo a constant) by the $(\overline{L}+1) \times 1$ vector $[h(0)\ h(1)\cdots h(L)\ 0\cdots 0]'$ for $\forall \overline{L} = L, L+1, \cdots, L+P-1$. For this, we first need to establish the following.

Proposition 2—Order Mismatch: If (24) holds true, then for a given cyclic spectrum $S_{xx}(\alpha_k; z)$, all the solutions $\overline{H}(z)$ of (18), with $\deg[\overline{H}(z)] = \overline{L} \geq L$, are of the form $\overline{H}(z) = H(z)F(z)$, where $F(z)$ is a scalar constant or an arbitrary polynomial that satisfies $F(z) = F(ze^{-j2\alpha_k})$ for any $z \in \boldsymbol{C}$. □

Proof: See the Appendix.

According to Proposition 2, any solution of (18) is of the form $\overline{H}(z) = H(z)F(z)$. Because $F(z) = F(ze^{-j2\alpha_k})$, the highest degree coefficient satisfies $f(L_f) = f(L_f)\exp(j2\alpha_k L_f)$. Thus, $\exp(j2\alpha_k L_f) = 1$, which, according to (24), implies that $L_f \geq P$. Hence, if $\overline{L} > L$, then necessarily, $\overline{L} \geq L+P$. It follows that there is no polynomial $\overline{H}(z)$ that satisfies (18) for $L < \overline{L} < L+P$. This implies that dim $\mathcal{N}(\boldsymbol{S}_{\alpha_k}(\overline{L})) = 1$ for $L \leq \overline{L} < L+P$, and the null eigenvector is unique modulo a constant given by $[h(0)\ h(1)\cdots h(L)\ 0\cdots 0]'$.

This result can be explained alternatively by noting that for $L < \overline{L} < L+P$, matrix $\boldsymbol{S}_{\alpha_k}(\overline{L})$ is obtained by adding a number of $\overline{L} - L$ independent columns to the columns of $\boldsymbol{S}_{\alpha_k}(L)$ (this follows from the Toeplitz structure of $\boldsymbol{S}_{\alpha_k}$). Thus, we do not change the dimension of the null space (dim $\mathcal{N}(\boldsymbol{S}_{\alpha_k}(\overline{L})) = \dim \mathcal{N}(\boldsymbol{S}_{\alpha_k}(L)) = 1$ for $L \leq \overline{L} < L+P$), and moreover, the unique null eigenvector of $\boldsymbol{S}_{\alpha_k}(\overline{L})$ is obtained from the null eigenvector of $\boldsymbol{S}_{\alpha_k}(L)$ by adding $\overline{L} - L$ zeros, i.e., $[h(0)\cdots h(L)\ 0\cdots 0]'$. We have thus proved the following.

Theorem 2—Channel Identifiability Under Order Mismatch: If (24) is satisfied and the Lth-order $H(z)$ is overestimated by $\overline{L}$ with $\overline{L} \in [L, L+P)$, then blind identifiability of the channel vector $\boldsymbol{h}$, which is obtained as the null eigenvector of $\boldsymbol{S}_{\alpha_k}(\overline{L})$ in (22), is guaranteed (modulo a complex scale). □

Theorem 2 is also useful for cases when the channel order is not known exactly but some known upper bound for the channel order is available. For example, if the multipath channels are limited to 20 taps, by choosing P sufficiently large, e.g., $P = 32 \gg L$, the channel vector $\boldsymbol{h}$ can be uniquely estimated from (22), provided that the model order $\overline{L}$ satisfies $L \leq \overline{L} < L + P = L + 32$. Simulations will be presented to corroborate the robustness of this approach to channel order overestimation errors.

Remark 6: As we have seen, the channel vector is found as the unique null eigenvector of a certain matrix (call it $\boldsymbol{S}$) whose entries depend implicitly on the overestimated order $\overline{L}$ (through the estimated cyclic correlations). Although identifiability is guaranteed for any $\overline{L}$ in the interval $L \leq \overline{L} < L + P$, it turns out that when $\overline{L}$ approaches $L + P$, matrix $\boldsymbol{S}$ becomes ill-conditioned numerically. Thus, small perturbations of $\boldsymbol{S}$ will affect its null space. For a given period P, the allowable range for $\overline{L}$ to obtain good channel estimates is often smaller than $L + P - 1$. However, by choosing larger values for P, our simulations show that the order overestimation error is allowed to be large, without modifying significantly the numerical conditioning of matrix $\boldsymbol{S}$.

From an implementation viewpoint, we note that when the channel order is overestimated in the range $L \leq \overline{L} < L + P$, equations similar to (18) and (22) can be used to estimate the channel $\boldsymbol{h}$. Let $\hat{S}_{xx}(\alpha, z)$ denote an estimate of $S_{xx}(\alpha, z)$. The equation

$$[z^{-\overline{L}} P_2^*(\alpha)\hat{S}_{xx}(\alpha; e^{j\alpha}/z)]\overline{H}(e^{-j2\alpha}z) = [z^{-\overline{L}} P_2(\alpha)\hat{S}_{xx}^*(\alpha; e^{j2\alpha}z^*)]\overline{H}(z)$$

can be easily brought to the form of (22).

Finally, we remark that Theorems 1 and 2 apply to the estimation of both the delay (time-shift) d as well as the transfer function $H(z)$ for the channel model $H_1(z) = z^{-d}H(z)$, provided that the conditions $L + d \leq \overline{L} < L + d + P$ and $\exp(j2\alpha_k l) \neq 1$ for $l = 1, \cdots, L + d$ are satisfied. The proof is similar to that of Theorem 1, and it is not repeated here.

2) Two-Cycle Subspace Approach: Here, we briefly develop channel estimation algorithms that make use of more than one cycle frequencies. We have seen that for a single cycle α_k, $1 \leq k \leq P - 1$, channel $H(z)$ can be simply recovered from (22) as the unique null eigenvector. We term such an approach the one-cycle subspace approach (OCSA) since the recovery is based on a single cyclic spectrum $S(\alpha_k; z)$. Note that the use of cycle $-\alpha_k$ does not introduce any additional information since the cyclic spectrum $S(-\alpha_k; z)$ is related to $S(\alpha_k; z)$ via [c.f., (5)]

$$C_{xx}(-\alpha_k; l) = C_{xx}^*(\alpha_k; -l)e^{-j\alpha_k l} \tag{26}$$

for $\forall l = -L, \cdots, L$.

When two cycles α_i, $i = 1, 2$, $(\alpha_1 \neq \pm\alpha_2)$ are used, two equations similar to (22) can be derived $\boldsymbol{S}_{\alpha_i}\boldsymbol{h} = \boldsymbol{0}$, $i = 1, 2$ to estimate the channel $h(n)$. Alternatively, using (7), the following cross-relation between spectra $S_{xx}(\alpha_1; z)$ and $S_{xx}(\alpha_2; z)$ results:

$$\frac{S_{xx}(\alpha_1; z)}{S_{xx}(\alpha_2; z)} = \frac{P_2(\alpha_1)}{P_2(\alpha_2)} \cdot \frac{H(e^{j\alpha_1}z^{-1})}{H(e^{j\alpha_2}z^{-1})}. \tag{27}$$

Relation (27) can be brought to a form similar to (22), i.e., $\boldsymbol{S}_{\alpha_{12}}\boldsymbol{h} = \boldsymbol{0}$. The channel coefficient vector $\boldsymbol{h}$ can then be found as the null eigenvector of $[\boldsymbol{S}'_{\alpha_1}; \boldsymbol{S}'_{\alpha_2}; \boldsymbol{S}'_{\alpha_{12}}]'\boldsymbol{h} = \boldsymbol{0}$. We refer to this approach as the two-cycle subspace approach (TCSA). In the simulation section, we also consider estimating $\boldsymbol{h}$ using only the equation $\boldsymbol{S}_{\alpha_{12}}\boldsymbol{h} = \boldsymbol{0}$, and we will refer to this as modified-TCSA (MTCSA). Following the proof of Theorem 1, it can be shown that the necessary and sufficient condition for identifiability in the MTCSA approach is $\exp(j(\alpha_1 - \alpha_2)l) \neq 1$, for $l = 1, \cdots, L$. Similarly, we may consider more than two cycles, but the problem of selecting the number of cycles and the periodic sequence optimally (from an estimation accuracy viewpoint) is currently under study.

In the next subsection, we develop a nonlinear cyclic correlation matching approach that, at the expense of computational complexity, improves on the linear equation type approach in (12) or the subspace approach of (22).

C. Nonlinear Cyclic Correlation Matching Approach

Proposition 1 shows that the cyclic correlations $C_{xx}(\alpha_k, \tau)$, for a fixed α_k and $\tau = -L, \cdots, L$, contain sufficient information for unique identification of $\boldsymbol{h}$. We define the cyclic correlation vector $\boldsymbol{c}_{xx}(\alpha_k) := [C_{xx}(\alpha_k; -L) \quad C_{xx}(\alpha_k; -L + 1) \cdots C_{xx}(\alpha_k; L)]'$ and collect all the cyclic correlation estimates into the vector $\hat{c}_{xx}^{(N)}(\alpha_k) := [\hat{C}_{xx}^{(N)}(\alpha_k; -L) \cdots \hat{C}_{xx}^{(N)}(\alpha_k; L)]'$.

In the one cycle correlation matching estimator, the estimate $\hat{\boldsymbol{h}}$ is found by minimizing the distance between vectors $\hat{\boldsymbol{c}}_{xx}^{(N)}(\alpha_k)$ and $\boldsymbol{c}_{xx}(\alpha_k)$ in a weighted least-squares sense

$$\hat{\boldsymbol{h}} := \arg\min_{\boldsymbol{h}} \mathcal{J}_{\boldsymbol{Q}}[\hat{c}_{xx}^{(N)}(\alpha_k)\boldsymbol{h}]$$

$$\mathcal{J}_{\boldsymbol{Q}}[\hat{c}_{xx}^{(N)}(\alpha_k); \boldsymbol{h}] := [\hat{\boldsymbol{c}}_{xx}^{(N)}(\alpha_k) - \boldsymbol{c}_{xx}(\alpha_k)]^{*'} \cdot \boldsymbol{Q}[\hat{\boldsymbol{c}}_{xx}^{(N)}(\alpha_k) - \boldsymbol{c}_{xx}(\alpha_k)] \tag{28}$$

where $\boldsymbol{Q}$ is a Hermitian and positive-definite weighting matrix. As with to the previous one-cycle criterion, the channel can be estimated by matching all the cyclic correlations corresponding to a number of cyclic correlations. Due to the symmetry relation (26), it follows that the cyclic correlations $\{C_{xx}(\alpha_k; l)\}_{k=1}^{[P/2]}$, $l = -L, \cdots, L$ provide all the available second-order statistical information about the channel $h(n)$.

The nonlinear matching approach offers some advantages over the linear approaches: It is criterion based and can attain asymptotically the minimum estimation variance for $\hat{\boldsymbol{h}}$ within the class of all second-order statistics-based estimators. Results concerning asymptotic performance analysis of linear type and nonlinear matching approaches will be reported elsewhere. In our simulation section, we tested (28) with $\boldsymbol{Q} = \boldsymbol{I}$. On the other hand, nonlinear matching approaches present increased computational complexity and require a good initial estimate in order to speed convergence to the global minimum and to avoid local minima. Based on simulations, we verified that the nonlinear cyclic correlation matching approach is not sensitive to channel order overestimation errors, provided that the local minima are avoided by means of a good initialization.

IV. Selecting the Modulating Sequence—Discussion

In this section, some guidelines for selecting $p(n)$ are presented. Periodic sequence $p(n)$ must be chosen such that $w(n)$ is cyclostationary, and some additional conditions are fulfilled. It is desirable that $p(n)$ maintains the constant envelope condition of the input in the case of frequency (or phase) modulated signals $s(n)$. This condition is important since, by keeping the transmitted envelope constant, efficient Class C power amplifiers can be used for radio frequency power amplification. However, this constant envelope condition cannot be generally satisfied. For a constant envelope input $s(n)$, we have to use periodic sequences with constant modulus $|p(n)| = 1, \forall n$, in order to maintain constant envelope in $w(n)$. However, for such a choice of $p(n)$, the cyclostationarity of $w(n)$ and $x(n)$ is lost. Note that $|p(n)| = 1$ implies that $w(n)$ and $x(n)$ are stationary $c_{ww}(n;\tau) = c_{ww}(\tau)$, $c_{xx}(n;\tau) = c_{xx}(\tau)$ for $\forall n$ and $P_2(\alpha_k) = 0$ for $\alpha_k \neq 0$. Thus, we cannot recover the channel $h(n)$ from (7).

However, when the input sequence is BPSK (or any real-valued PAM sequence), a solution is possible by modifying the correlation definition. Consider the modulating sequence $p(n) = \exp(j\omega_0 n)$, with $\omega_0 = 2k\pi/P$ and $\gcd(2k, P) = 1$. Instead of (2), we adopt the nonconjugated time-varying correlation $\overline{c}_{xx}(n;\tau) := E[x(n)x(n+\tau)]$, which is given by

$$\overline{c}_{xx}(n;\tau) = \sigma_s^2 \sum_{m=-\infty}^{\infty} p^2(n-m)h(m)h(m+\tau) + \overline{c}_{vv}(\tau). \tag{29}$$

We note that $\overline{c}_{xx}(n;\tau)$ is periodic in n, whereas $c_{xx}(n;\tau)$ in (2) does not exhibit periodicity. Thus, sequence $x(n)$ exhibits the so-called conjugate cyclostationarity and not cyclostationarity. Following the derivation of (4) and (5), we find that the corresponding cyclic correlations $\overline{C}_{xx}(\alpha_k;\tau)$ are now given by

$$\overline{C}_{xx}(\alpha_k;\tau) := \frac{1}{P}\sum_{n=0}^{P-1} \overline{c}_{xx}(n;\tau)e^{-j\alpha_k n} \tag{30}$$

$$= \sigma_s^2\, \overline{P}_2(\alpha_k) \sum_{n=-\infty}^{\infty} h(n)h(n+\tau)e^{-j\alpha_k n} + \overline{c}_{vv}(\tau)\delta(\alpha_k) \tag{31}$$

where $\overline{P}_2(\alpha_k) := (1/P)\,\Sigma_{n=0}^{P-1}\, p^2(n)e^{-j\alpha_k n}$. We denote the $\mathcal{Z}$ transform of $\{\overline{C}_{xx}(\alpha_k;\tau)\}_{\tau=-\infty}^{\infty}$ as the cyclic spectrum $\overline{S}_{xx}(\alpha_k;z)$. Taking into account (31), we deduce

$$\overline{S}_{xx}(\alpha_k;z) := \sum_{\tau=-\infty}^{\infty} \overline{C}_{xx}(\alpha_k;\tau)z^{-\tau} = \sigma_s^2 \overline{P}_2(\alpha_k)H(e^{j\alpha_k}z^{-1})H(z). \tag{32}$$

Relation (32) has similar form with (7). There exists a cycle $\alpha_k \neq 0$ such that $\overline{P}_2(\alpha_k) = (1/P)\,\Sigma_{n=0}^{P-1}\, e^{j2\omega_0 n}e^{-j\alpha_k n} \neq 0$. Indeed, choosing $\alpha_k = 2\omega_0$, we obtain $\overline{P}_2(2\omega_0) = 1$. Blind identification of channel $H(z)$ as the gcd of the family of cyclic spectra $\{\overline{S}_{xx}(\alpha_k;z)\}$, $k = 1, \cdots, P-1$, from (32), cannot be performed since for the modulating sequence $p(n) = \exp(j\omega_0 n)$, there is only one cycle for which $\overline{P}_2(\alpha_k) \neq 0$, i.e., $\alpha_k = 2\omega_0$. We have shown in Proposition 1 that blind identifiability of the channel $h(n)$ is possible from only one cyclic spectrum, namely, from $\overline{S}_{xx}(2\omega_0;z)$, provided that the period $P > [L/2]$. However, as the simulations show, the performance of this algorithm is limited, and better channel estimates can be obtained using a subspace algorithm, which includes at least two cyclic frequencies. The solution consists of modulating alternatively on succesive time windows with two complex exponential periodic sequences of the form $\exp(jn\omega)$, where ω takes two distinct values ω_0 and ω_1. This approach is reminiscent of the frequency-hopping approach (the hopping period being $P > L$) [20, p. 601]. The channel can be estimated by means of the MTCSA (as discussed in Section III-B). Simulation experiments indicate a good performance of this algorithm.

We conclude that blind channel identifiability is possible for BPSK inputs while preserving the constant envelope of the transmitted sequence. Practical implementation of this system can be performed similarly to that of a QAM system.

For other constant modulus constellations, it appears that there is no modulating sequence that preserves both the constant envelope condition and also induces cyclostationarity in $w(n)$. For such constellations, we may consider modulating sequences $p(n)$ for which the variation of $|p(n)|$ within one period is below a certain threshold. Based on our experience with simulations, small variations of $|p(n)|$ within one period introduce a very small "degree of cyclostationarity" in the output sequence $x(n)$ and, consequently, a decrease in the performance of channel estimation algorithms. Thus, in general, there is a tradeoff between channel identifiability (requiring a high "degree" of cyclostationarity) and the constant envelope condition.

From an estimation viewpoint, the condition $P_2(\alpha_k) \neq 0$ must be satisfied. It is useful to know how many cycles α_k's have to be selected and how these cycles should be distributed in the spectral domain for an optimal (large sample) performance of the proposed channel identification algorithms. Such issues are beyond the scope of this paper and have been dealt with in [4].

V. Channel Equalization

Having estimated the channel, equalization can be performed using a maximum likelihood sequence estimator based on Viterbi decoding. From a computational viewpoint, the number of states in the trellis does not increase due to the modulation of $s(n)$ by $p(n)$. The search in the trellis for the Viterbi detector can be implemented by keeping the same states as if there were no modulating sequence $p(n)$ for the input stream $s(n)$ and by viewing the channel as a periodically time-varying channel with known variations. At a given stage, the trellis metric uses the corresponding part of the periodic sequence to update the time-varying channel. However, due to the high computational complexity of Viterbi's algorithm, it is often desirable to perform linear equalization, which is less computationally expensive, especially when adaptive forms are sought for online processing. Because MIC approaches do not introduce redundancy, it is impossible to devise perfect reconstruction (or zero-forcing) linear FIR equalizers for SISO FIR channels. This is the advantage of redundant TIC approaches in [9], [16], and [24], and FS equalizers in [7], [17], [21], [22], and [26].

Here, we propose an MMSE cyclic Wiener equalizer and implement it using an FIR periodically time-varying filter with period P: $g(n;k) = g(n+P;k)$, $\forall n \in \boldsymbol{Z}$ and $k = -L_1, \cdots, L_2$ [8]. Recall that $s(n)$ is i.i.d., and uncorrelated with the additive white noise $v(n)$. At a given time n, the output $x(n)$ is processed by the Wiener filter $g(n \bmod P; k)$. Denote by $\hat{s}(n)$ the output of the Wiener equalizer at time n (see Fig. 4)

$$\hat{s}(n) = \sum_{k=-L_1}^{L_2} g(n;k)x(n-k)$$

and the output mean-square error as

$$\begin{aligned} E|e(n)|^2 &:= E|\hat{s}(n) - s(n)|^2 \\ &= E\left|\sum_{k=-L_1}^{L_2} g(n;k)x(n-k) - s(n)\right|^2. \end{aligned} \tag{33}$$

The equalizer coefficients $g(n;k)$, $k = -L_1, \cdots, L_2$ are obtained using the orthogonality principle, i.e.,

$$E\left[\sum_{k_1=-L_1}^{L_2} g(n;k_1)x(n-k_1) - s(n)\right]x^*(n-k_2) = 0$$

which leads to a system of linear equations for each $n = 0, \cdots, P-1$, and $k_2 = -L_1, \cdots, L_2$

$$\sum_{k_1=-L_1}^{L_2} g(n;k_1)c_{xx}(n-k_1; k_1-k_2) = Es(n)x^*(n-k_2). \tag{34}$$

The system of (34) can be rewritten in the equivalent matrix form

$$\boldsymbol{R}_{xx}(n)\boldsymbol{g}(n) = \boldsymbol{r}_{sx}(n) \tag{35}$$

where

$$\begin{aligned} \boldsymbol{g}(n) &:= [g(n;-L_1) \quad g(n;-L_1+1) \quad \cdots \quad g(n;L_2)]' \\ \boldsymbol{r}_{sx}(n) &:= [Es(n)x^*(n+L_1) \cdots Es(n)x^*(n-L_2)]' \\ \boldsymbol{R}_{xx}(n) &:= E\boldsymbol{x}^*(n)\boldsymbol{x}'(n) \\ \boldsymbol{x}(n) &:= [x(n+L_1) \quad x(n+L_1-1) \quad \cdots \quad x(n-L_2)]'. \end{aligned}$$

We wish to express $\boldsymbol{g}(n)$ in (35) in terms of the channel coefficient vector $\boldsymbol{h}$. Define the $(L_1+L_2+1) \times (L+L_1+L_2+1)$ Toeplitz matrix $\mathcal{H}_p(n)$ with the first row $[h(0)p(n+L_1) \cdots h(L)p(n+L_1-L) \cdots 0]$ and the first column $[h(0)p(n+L_1)\ 0 \cdots 0]'$. In addition, define the vectors

$$\begin{aligned} \boldsymbol{s}(n) &:= [s(n+L_1) \quad s(n+L_1-1) \quad \cdots \quad s(n-L_2-L)]' \\ \boldsymbol{v}(n) &:= [v(n+L_1) \quad v(n+L_1-1) \quad \cdots \quad v(n-L_2)]'. \end{aligned}$$

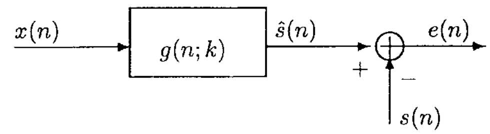


Fig. 4. Cyclic Wiener filtering.

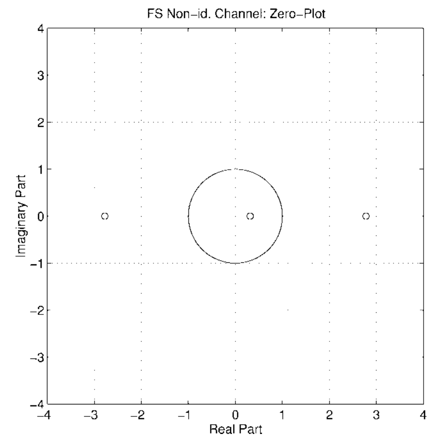


Fig. 5. FS nonidentifiable channel: Zero plot.

From (1), we have $\boldsymbol{x}(n) = \mathcal{H}_p(n)\ \boldsymbol{s}(n) + \boldsymbol{v}(n)$. It follows easily that

$$\boldsymbol{R}_{xx}(n) = \mathcal{H}_p^*(n)E\boldsymbol{s}^*(n)\boldsymbol{s}'(n)\mathcal{H}_p'(n) + \sigma_v^2\boldsymbol{I} \tag{36}$$

$$\begin{aligned} \boldsymbol{r}_{sx}(n) &= Es(n)[\mathcal{H}_p^*(n)\boldsymbol{s}^*(n) + \boldsymbol{v}^*(n)] \\ &= \mathcal{H}_p^*(n)\sigma_s^2\boldsymbol{e}_{L_1+1} \end{aligned} \tag{37}$$

since $s(n)$ is i.i.d., and $E\boldsymbol{s}(n)\ s^*(n) = \sigma_s^2\ \boldsymbol{e}_{L_1+1}$, with $\boldsymbol{e}_{L_1+1}$ the (L_1+1)st column of the $(L+L_1+L_2+1) \times (L+L_1+L_2+1)$ unity matrix. From (35)–(37), we infer that for $n = 0, \cdots, P-1$

$$\boldsymbol{g}(n) = [\mathcal{H}_p^*(n)\mathcal{H}_p'(n) + \sigma_v^2/\sigma_s^2\boldsymbol{I}]^{-1}\mathcal{H}_p^*(n)\boldsymbol{e}_{L_1+1} \tag{38}$$

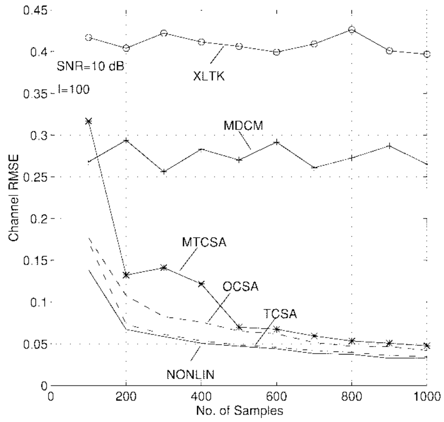
and by substituting (38) into (33), we obtain the equivalent expression for mmse

$$\begin{aligned} E|e(n)|^2 = \sigma_s^2[1 - \boldsymbol{e}_{L_1+1}'\mathcal{H}_p^{*'}(n)(\mathcal{H}_p(n)\mathcal{H}_p^{*'}(n) \\ + \sigma_v^2/\sigma_s^2\boldsymbol{I})^{-1}\mathcal{H}_p^*(n)\boldsymbol{e}_{L_1+1}]. \end{aligned}$$

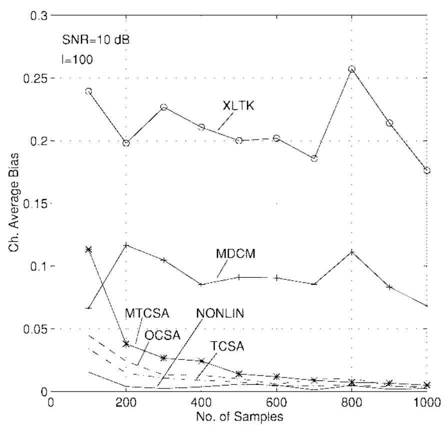
Implementation of the linear periodically varying Wiener equalizer (38) assumes knowledge of the noise power σ_v^2. A consistent estimate for the noise power can be obtained from (2) (considering $n = 0$) or from (5) (considering $\alpha = 0$) once the channel has been consistently estimated. The equalization of the channel can alternatively be performed by making use of the Viterbi decoder or decision-feedback equalizer.

VI. Simulation Results

We have compared the performance of the nonlinear cyclic correlation matching algorithm (NONLIN), one-cycle (OCSA), two-cycle (TCSA), and modified two-cycle (MTCSA) subspace approaches with the FS-based approaches MDCM [17] and XLTK [26] in identifying two different channels. One channel has zeros nonidentifiable by FS approaches, and the other one is identifiable by FS approaches.

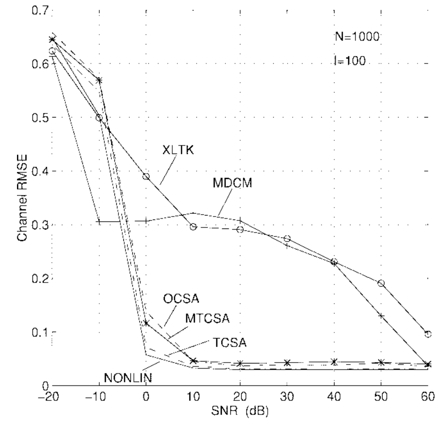


(a)

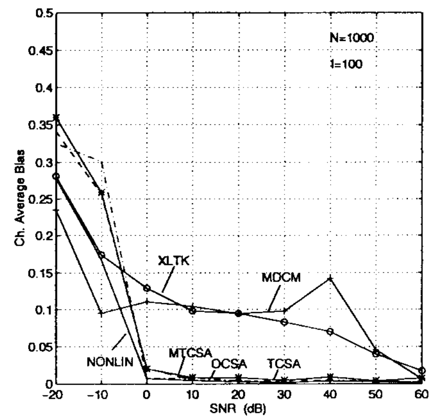


(b)

Fig. 6. RMSE average bias versus number of samples.



(a)



(b)

Fig. 7. RMSE average bias versus SNR.

The nonlinear cyclic correlation matching algorithm has been initialized with an estimate provided by TCSA. We have also compared the performance of the proposed MMSE Wiener equalizer (38) with the MMSE Wiener equalizer derived from the channel estimates of [17], [24], and [26]. In both experiments, input $s(n)$ is an i.i.d. QPSK sequence $(s(n) = \pm 1/\sqrt{2} \pm j1/\sqrt{2})$, and additive noise $v(n)$ is white and normally distributed. As channel estimation performance measures, we have plotted the normalized root-mean-square error (RMSE) and average bias (Avg. Bias) of channel estimates versus signal-to-noise ratio and number of sample data. The normalized channel root-mean-square error is defined through the relationship [21]

$$\text{RMSE} := \frac{1}{\|\boldsymbol{h}\|} \sqrt{\frac{1}{I(L+1)} \sum_{i=1}^{I} \|\hat{\boldsymbol{h}}^{(i)} - \boldsymbol{h}\|^2} \tag{39}$$

where

- I number of Monte Carlo runs;
- $\|\boldsymbol{h}\|$ Euclidean norm of channel vector $\boldsymbol{h}$;
- $\hat{\boldsymbol{h}}^{(i)}$ estimate of the channel obtained in the ith trial.

The average bias is computed via [17]

$$\text{Avg. Bias} := \frac{1}{I(L+1)\|\boldsymbol{h}\|} \sum_{l=0}^{L} \left| \sum_{i=1}^{I} [\hat{h}^{(i)}(l) - h(l)] \right| \tag{40}$$

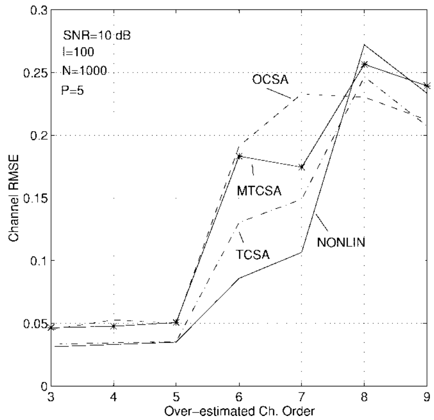
where $\hat{h}^{(i)}(l)$ denotes the lth coefficient estimate in the ith Monte Carlo run. In the overestimated order case, the channel RMSE and average bias are computed similar to (39) and (40) by appending a corresponding number of zeros to the vector $\boldsymbol{h}$. For all simulations, we have considered the signal-to-noise ratio (SNR) defined at the cyclostationary input of the equalizer as

$$\text{SNR} := \sqrt{\frac{\sum_{n=0}^{P-1} E\{|x(n)|^2\}/P}{E\{|v(n)|^2\}}}.$$

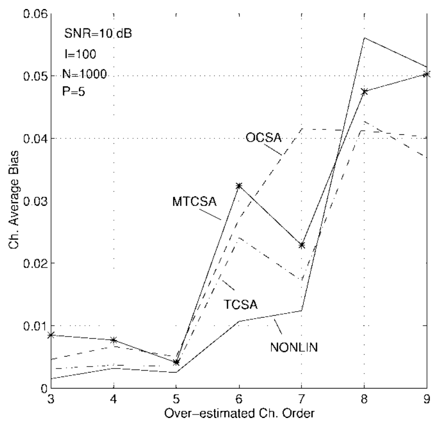
As performance measures for the blind equalizer, the mean-square error after equalization and the probability of symbol error have been considered. The normalized mean-square error (RMSE) after equalization RMSE $:= E\{|\hat{s}(n-d) - s(n)|^2\}/\sigma_s^2$ [25] has been estimated using

$$\text{RMSE} := \frac{1}{\sigma_s^2} \sqrt{\frac{1}{IN} \sum_{i=1}^{I} \sum_{j=1}^{N} |\hat{s}^{(i)}(l) - s(l)|^2}$$

where $\hat{s}^{(i)}(l)$ is the lth component of the recovered input in the ith Monte Carlo run, and N denotes the number of samples used per Monte Carlo run.

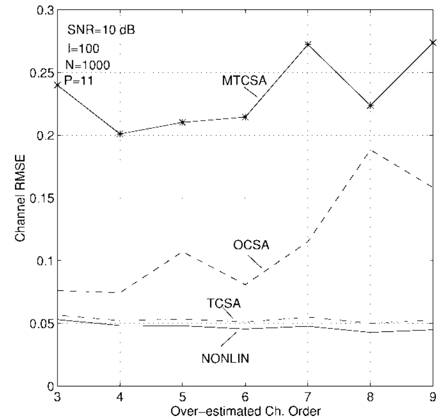


(a)

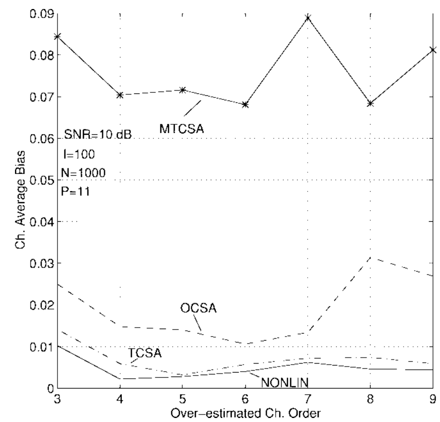


(b)

Fig. 8. RMSE average bias versus order mismatch.



(a)



(b)

Fig. 9. RMSE average bias versus order mismatch.

Experiment 1—FS Nonidentifiable Channel: We have considered the channel $\boldsymbol{h} = [1.000, -3.2000, -0.1296, 0.4147]$, which is not identifiable with FS-based approaches MDCM [17] and XLTK [26], since the two subchannels (resulted by oversampling the output with a factor 2) share a common root. Indeed, from the plot of the zeros of channel $h(n)$ represented in Fig. 5, we note that there are two roots that are located on the same circle and are equispaced by an angle $\pi = 2\pi/2$. The QPSK symbol stream $s(n)$ is modulated with the periodic sequence $p(n) = 0.4(3 + \cos(\alpha_1 n) + \cos(\alpha_2 n))$, where $\alpha_1 = 2\pi/5$, and $\alpha_2 = 4\pi/5$. It is easy to check that the period $P = 5$ and one period is $\{p(n)\}_{n=1}^{5} = \{1, 1, 1, 1, 2\}$. With such a modulating sequence, only a subsequence of the input symbol stream is encoded. We have used the cycles α_1 and α_2 and only α_1 for TCSA and OCSA, respectively. For the nonlinear approach NONLIN, the cyclic correlations corresponding to both cycles α_1 and α_2 have been matched.

In Fig. 6(a) and (b), we plot the average root-mean square (RMSE) and average bias (Avg. Bias) of channel estimates versus N at SNR $=$ 10 dB, using $I = 100$ Monte Carlo runs. MDCM and XLTK fail to estimate the channel in the mean even for large number of samples $(N = 1000)$, whereas OCSA, TCSA, and NONLIN provide good channel estimates even for data records as short as $N = 200$. In Fig. 7(a) and (b), we considered $N = 1000$ and $I = 100$ and plotted RMSE/Avg. Bias versus SNR. OCSA, TCSA, MTCSA, and NONLIN perform well, even at low SNR's. We note also that the performance of these channel estimation algorithms is pretty robust to SNR variations.

Fig. 8(a) and (b) depict RMSE and Avg. Bias of the channel estimates versus overestimated (channel) order. We overestimated the channel order with values in the range $\overline{L} = 3 \div 9$, and for each order, 100 Monte Carlo runs were used to compute the RMSE and the Avg. Bias of channel estimates. For small order overestimation errors, OCSA, TCSA, and NONLIN perform well. We note that for large order overestimation errors $\overline{L} = 8, 9$, large channel estimation errors appear. This is reasonable since consistent channel estimates are guaranteed only for orders in the range $L = 3 \leq \overline{L} \leq L + P - 1 = 3 + 5 - 1 = 7$. From Fig. 8, we note that even for orders smaller than the identifiability limit $L + P - 1 = 7$ (i.e., $\overline{L} = 6, 7$), the performance of the algorithms degrades. This behavior is due to the fact the channel estimate is found as the null eigenvector of a certain matrix [similar to (23)], which becomes ill-conditioned numerically as the overestimated order $\overline{L}$ approaches the identifiability limit $L + P - 1 = 7$. Due to this ill-conditioning, small perturbations of the matrix (23) may cause large variations in the channel vector estimates. However, by increasing the period of modulating sequence $p(n)$ to 11, e.g., considering $p(n) = 3 + \cos(\alpha_1 n) + \cos(\alpha_2 n)$, with $\alpha_1 = 2\pi/11$ and $\alpha_2 = 4\pi/11$, better channel estimates are obtained over a larger order overestimation

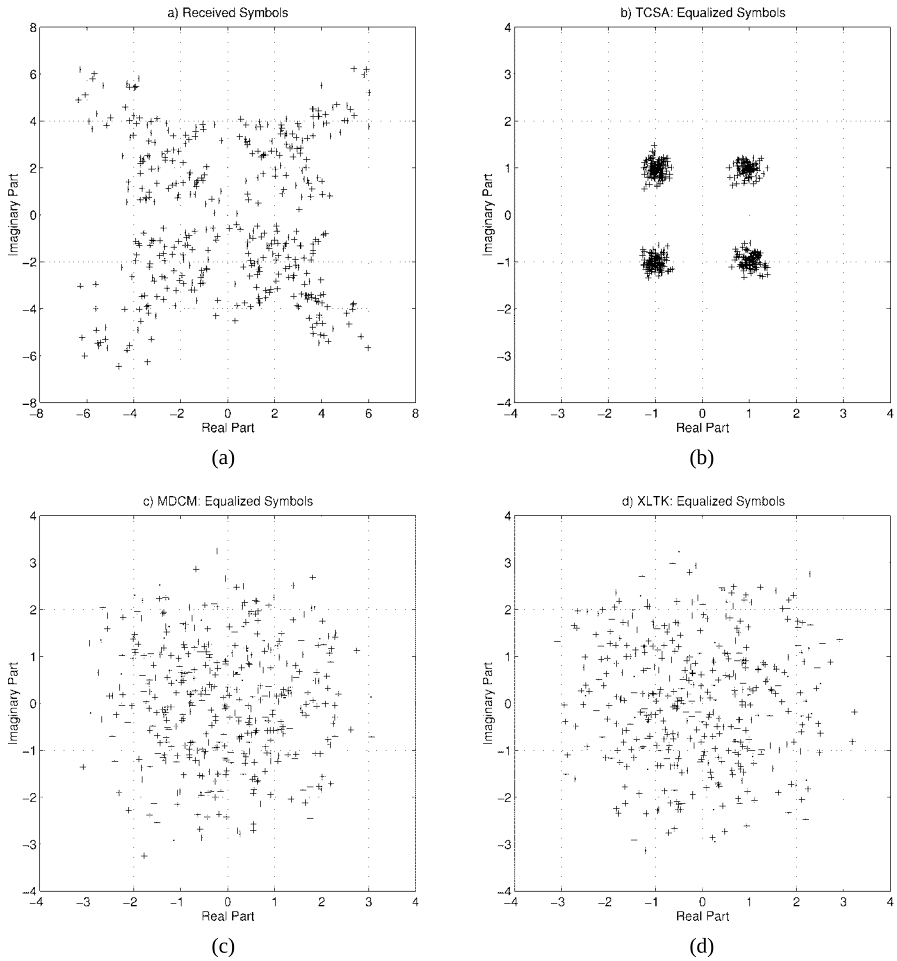


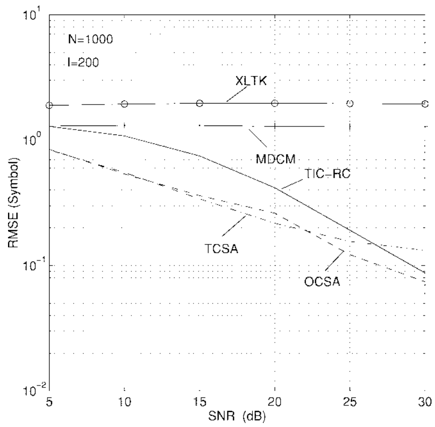
Fig. 10. Received symbols. Before and after equalization.

range, i.e., a larger time span for the overestimated order $\overline{L}$ is allowed for maintaining the well-conditioning of matrix (23). The results for this case $(P = 11)$ are plotted in Fig. 9(a) and (b).

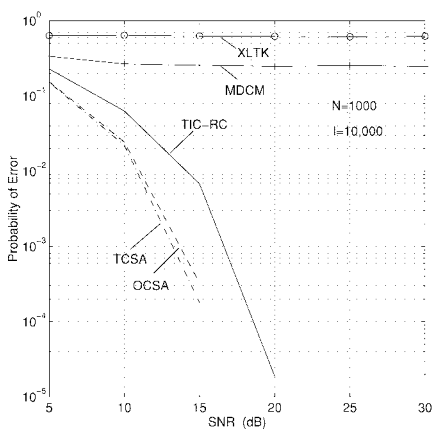
We also tested the performance of MDCM, XLTK, TIC-RC [24], and TCSA-based blind MMSE equalizers with respect to the probability of symbol error and symbol mean-square error (RMSE) after equalization for different SNR levels. We considered, in all cases, a Wiener MMSE equalizer with 17 taps and, for all experiments, symbol spaced equalizers. The TCSA-based MMSE equalizers were designed according to (38). For MDCM and XLTK, the channel estimates are obtained first within the FS-framework, assuming a stationary input, and then, the MMSE equalizers are obtained by plugging the estimated channel into their standard expressions. In Fig. 10(a)–(d), the symbol constellations were plotted before and after MMSE equalization at SNR $= 20$ dB. When the channel is identified using TCSA, the MMSE equalizer does a good job, whereas for FS-based approaches, the MMSE equalizers fail to work properly because the channel is non-identifiable. The symbol mean-square error curves are plotted in Fig. 11(a), where 200 Monte Carlo runs were averaged. The channel was estimated at each Monte Carlo run by using $N = 1000$ samples. The symbol error curves are plotted in Fig. 11(b), where 10 000 Monte Carlo runs were averaged per SNR point. The number of sample data used per Monte Carlo run was $N = 1000$. We note that TCPA performs better than TIC-RC [24], whereas MDCM and XLTK fail to decode the received symbols since the channel is not identifiable by FS methods.

Finally, we tested the properties of the linear system of equations (12) and the MTCSA approaches when the input i.i.d. BPSK symbol stream is modulated by a constant envelope preserving modulating sequence. For the linear system of (12), the modulating sequence was $p(n) = \exp(j2\pi n/3)$, whereas for MTCSA, the input symbol stream was succesively modulated with the periodic sequences $p_1(n) = \exp(j\pi n/5)$ and $p_2(n) = \exp(j2\pi n/5)$. The average RMSE and Avg. Bias of channel estimates versus number of samples N are depicted in Fig. 12(a) and (b). In these experiments, the SNR is fixed at SNR $= 10$ dB, and the number of Monte Carlo runs equals $I = 100$. In Fig. 13(a) and (b), the channel RMSE/Avg. Bias are plotted versus SNR, for $N = 2000$ samples and $I = 100$ Monte Carlo runs. From these plots, improved performance of MTCSA with respect to the linear system of equations approach is observed even for $N = 200$ samples.

Experiment 2—FS-Identifiable Channel: In this experiment, we simulated a FS identifiable channel and compared the performance of the TCSA, MDCM, and XLTK-based channel estimation algorithms. The continuous-time channel

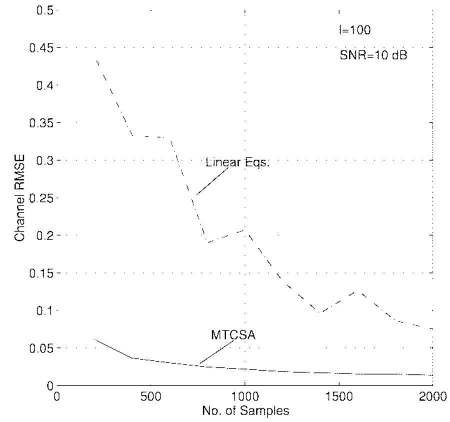


(a)

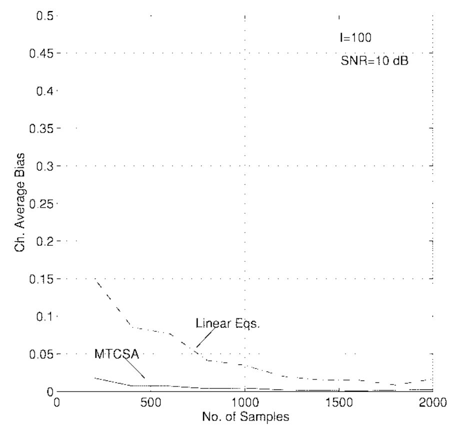


(b)

Fig. 11. Symbol RMSE/SER versus SNR.



(a)



(b)

Fig. 12. RMSE average bias versus number of samples.

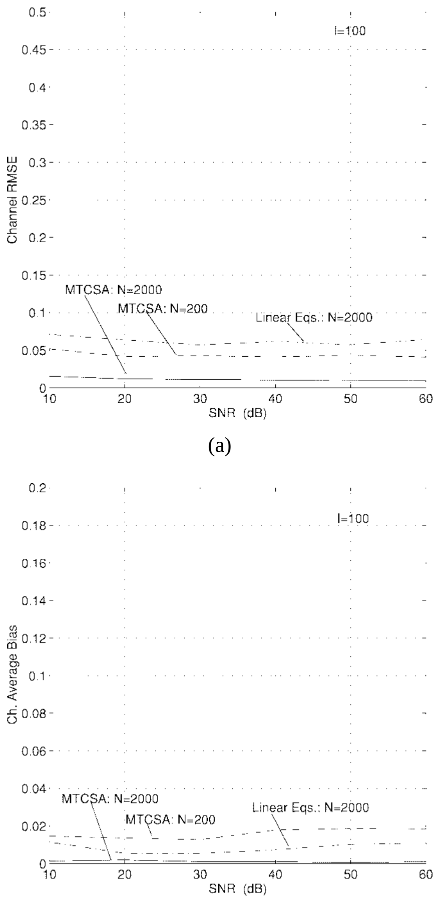
is a two-ray multipath channel, which spans four symbol periods and is given for $t \in [0, 4T)$ by

$$h_c(t) = e^{-j2\pi(0.15)} r_c(t - 0.25T, \beta) + 0.8e^{-j2\pi(0.6)} r_c(t - T, \beta)$$

where $r_c(t)$ is a raised-cosine pulse shape with roll-off factor $\beta = 0.35$. The path delays are equal to $0.25T$ and T, respectively. The discrete-time equivalent channel is found by sampling $h_c(t)$ at a rate of $T/2$ ($h_{FS}(n) := h_c(nT/2)$, $n = 0, 1, \cdots, 11$). The zero plots of the two subchannels resulted by oversampling with a factor 2 are shown in Fig. 16. We note that there is no common zero between the two channels. The zeros of the two subchannels, resulting from FS with a factor 2, are represented by "+" and "o," respectively. Thus, the channel is identifiable with the FS-based approaches: MDCM and XLTK. The coefficients of these two subchannels are presented in Table I. We have also identified the channel by using the present TCSA, i.e., we do not oversample the receiver output, but we maintain the symbol rate T^{-1} and induce the cyclostationarity at the transmitter using the periodic sequence $p(n) = 0.4(3 + \cos(\alpha_1 n) + \cos(\alpha_2 n))$ with $\alpha_1 = 2\pi/5$ and $\alpha_2 = 4\pi/5$. Thus, the channel to be identified by TCSA is $h_{\text{TCSA}}(n) := h_c(nT_s)$, $n = 0, \cdots, 5$. Through this experiment, we want to point out that besides channel identifiability restrictions, FS approaches increase the number of parameters to be estimated. For this experiment, MDCM and XLTK have to estimate twice as many parameters (12 complex coefficients) with respect to TCSA (six complex coefficients). In Fig. 14(a) and (b), the channel RMSE and average bias are plotted versus SNR. The number of samples is fixed to $N = 1000$, and the number of Monte Carlo runs is $I = 100$. The experiment for TCSA has been repeated for a smaller number of samples $N = 500$. It turns out that TCSA outperforms MDCM and XLTK. Thus, even if the channel is identifiable by FS methods, it appears that the present method has merits, especially with short data records. However, asymptotic performance analysis is required before definitive conclusions can be drawn with regard to any performance comparison claims. The symbol error rates have also been plotted in Fig. 15, assuming symbol-spaced MMSE equalizers for all approaches. It turns out that the number of samples ($N = 1000$) used for identification is not enough for a good performance of the MMSE equalizers. Thorough performance analysis is required for a just comparison between the symbol-spaced MMSE equalizers for the MIC framework and the FS-based MMSE equalizers for FS-induced cyclostationarity framework [14]. However, this problem is beyond the scope and the length of this paper.

VII. Conclusions and Future Research Directions

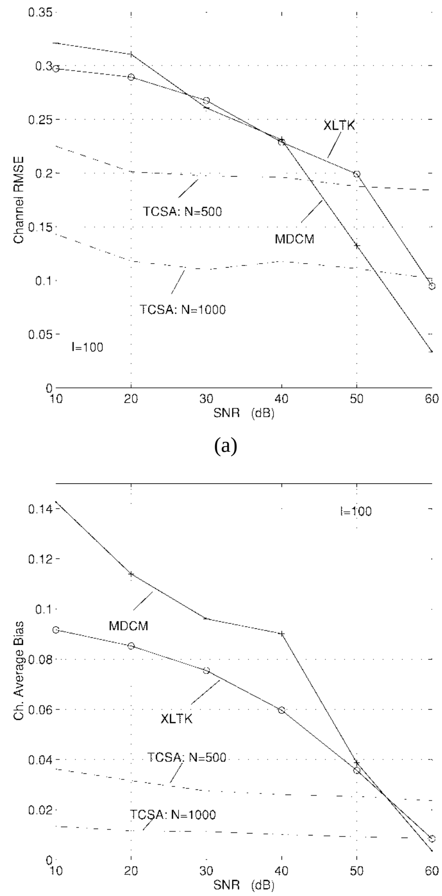
This paper has shown that modulation-induced cyclostationarity allows blind identification and equalization of SISO FIR



(a)

(b)

Fig. 13. RMSE average bias versus SNR.



(a)

(b)

Fig. 14. RMSE average bias versus SNR.

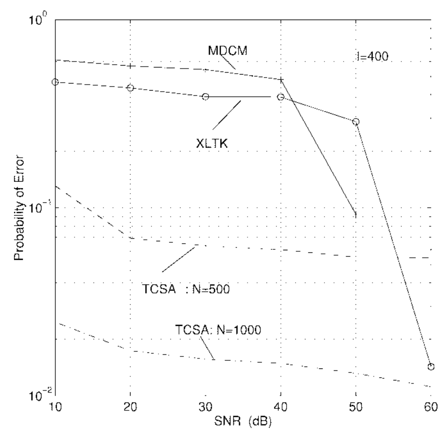


Fig. 15. SER versus SNR.

channels without introducing redundancy and irrespective of location of channel zeros, order overestimation errors, or color of additive stationary noise. The paper has also presented a number of blind channel identification and equalization algorithms. Blind channel identification is accomplished by exploiting cyclostationarity induced at the transmitter when the input is modulated with a strictly periodic sequence. The only condition required is that the period of modulating sequence be greater than half or full length of the channel. Several open problems remain to be solved. Among these, we mention performance analysis, derivation of asymptotically optimal algorithms, selection of optimal periodic sequences $p(n)$, adaptive solutions, and extensions to multiuser systems.

APPENDIX
PROOF OF PROPOSITION 2

According to the Division Theorem, we have

$$\overline{H}(z) = H(z)F(z) + R(z) \tag{41}$$

where remainder $R(z)$ satisfies $\deg[R(z)] < L$. Substituting (41) into both sides of (18), we obtain

$$\begin{aligned}&[H(ze^{-j2\alpha_k})F(ze^{-j2\alpha_k}) + R(ze^{-j2\alpha_k})]\\ &\quad \cdot P_2^*(\alpha_k)z^{-L}S_{xx}(\alpha_k; e^{j\alpha_k}/z)\\ &= [H(z)F(z) + R(z)]P_2(\alpha_k)z^{-L}S_{xx}^*(\alpha_k; e^{j2\alpha_k}z^*)\end{aligned} \tag{42}$$

which can be rewritten as

$$\begin{aligned}&R(ze^{-j2\alpha_k})P_2^*(\alpha_k)z^{-L}S_{xx}(\alpha_k; e^{j\alpha_k}/z)\\ &\quad = P_2(\alpha_k)[H(z)(F(z) - F(ze^{-j2\alpha_k}))\\ &\qquad + R(z)]z^{-L}S_{xx}^*(\alpha_k; e^{j2\alpha_k}z^*).\end{aligned} \tag{43}$$

Since both sides of (43) must have the same degree, and since $\deg[z^{-L}S_{xx}(\alpha_k; e^{j\alpha_k}/z)] = \deg[z^{-L}S_{xx}^*(\alpha_k; e^{j2\alpha_k}z^*)]$ and $\deg[R(z)] < \deg[H(z)]$, (43) implies that $F(z) -$

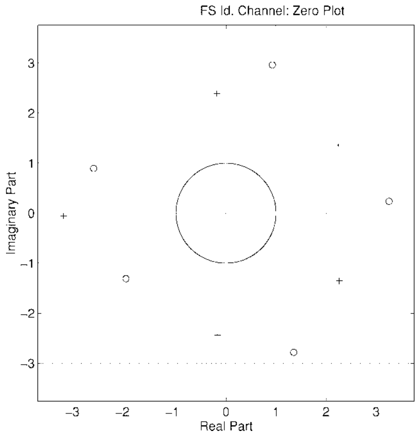


Fig. 16. FS identifiable channel. Zero plots of subchannels.

TABLE I
FS—IDENTIFIABLE CHANNEL

Subchannel Coefficients	
$h_0(n)$	$h_1(n)$
-0.4819 + 0.2427i	-0.4885 + 0.4121i
-0.0523 + 0.0719i	0.1363 - 0.1193i
0.0177 - 0.0243i	-0.0458 + 0.0390i
-0.0033 + 0.0046i	0.0091 - 0.0067i
-0.0014 + 0.0019i	0.0030 - 0.0033i
0.0015 - 0.0021i	-0.0036 + 0.0032i

$F(ze^{-j2\alpha_k}) = 0$. Thus, (43) reduces to

$$R(ze^{-j2\alpha_k})P_2^*(\alpha_k)z^{-L}S_{xx}(\alpha_k; e^{j\alpha_k}/z) = R(z)P_2(\alpha_k)z^{-L}S_{xx}^*(\alpha_k; e^{j2\alpha_k}z^*). \quad (44)$$

Equation (44) has identical form with (18). We showed in the proof of Theorem 1 that there is no solution for (18) with $\overline{L} < L$. Thus, $R(z) = 0$, and $\overline{H}(z) = H(z)F(z)$, with $F(z) = F(ze^{-j2\alpha_k})$, $\forall z$. □

REFERENCES

[1] A. Chevreuil, F. Desbouvries, A. Gorokhov, Ph. Loubaton, and C. Vignat, "Blind equalization in the presence of jammers and unknown noise: Solutions based on second-order cyclostationary statistics," *IEEE Trans. Signal Processing*, vol. 46, pp. 259–263, Jan. 1998.

[2] A. Chevreuil and P. Loubaton, "Blind second-order identification of FIR channels: Forced cyclo-stationarity and structured subspace method," in *Proc. SPAWC*, Paris, La Villette, France, Apr. 16–18, 1997, pp. 121–124.

[3] ——, "Blind second-order identification of FIR channels: Forced cyclo-stationarity and structured subspace method," *IEEE Signal Processing Lett.*, vol. 4, pp. 204–206, July 1997.

[4] A. Chevreuil, E. Serpedin, P. Loubaton, and G. B. Giannakis, "Performance analysis of blind channel estimators based on nonredundant periodic modulation precoders," in *Proc. ICASSP*, Seattle, WA, 1998, vol. VI, pp. 3397–3400.

[5] A. V. Dandawaté and G. B. Giannakis, "Asymptotic theory of mixed time averages and kth order cyclic-moment and cumulant statistics," *IEEE Trans. Inform. Theory*, vol. 41, pp. 216–239, Jan. 1995.

[6] Z. Ding, "Characteristics of band-limited channels unidentifiable from second-order cyclostationary statistics," *IEEE Signal Processing Lett.*, vol. 3, pp. 150–152, May 1996.

[7] Z. Ding and Y. Li, "On Channel identification based on second-order cyclic spectra," *IEEE Trans. Signal Processing*, vol. 42, pp. 1260–1264, May 1994.

[8] W. A. Gardner, "Cyclic Wiener filtering: Theory and method," *IEEE Trans. Commun.*, vol. 41, pp. 151–163, Jan. 1993.

[9] G. B. Giannakis, "Filterbanks for blind channel identification and equalization," *IEEE Signal Processing Lett.*, vol. 4, pp. 184–187, June 1997.

[10] ——, "Cyclostationary signal analysis," chapter in *Digital Signal Processing Handbook*, V. K. Madisetti and D. Williams, Eds. Boca Raton, FL: CRC, 1997.

[11] G. B. Giannakis and M. K. Tsatsanis, "Restoring identifiability of fractionally sampled blind channel estimators using HOS," in *Proc. Int. Conf. Higher Order Statist.*, Barcelona, Spain, June 1995, pp. 409–413.

[12] G. B. Giannakis and S. D. Halford, "Asymptotically optimal blind fractionally-spaced channel estimation and performance analysis," *IEEE Trans. Signal Processing*, vol. 45, pp. 1815–1830, July 1997.

[13] G. B. Giannakis and E. Serpedin, "Blind channel identification with modulation induced cyclostationarity," in *Proc. DSP*, Santorini, Greece, July 1997, vol. I, pp. 111–114.

[14] R. D. Gitlin and S. B. Weinstein, "Fractionally-spaced equalization: An improved digital transversal equalizer," *Bell Syst. Tech. J.*, vol. 60, pp. 275–296, Feb. 1981.

[15] A. Grace, *Optimization Toolbox—For Use with Matlab, User's Guide*. Natick, MA: The Math Works, Aug. 1986.

[16] S. D. Halford and G. B. Giannakis, "Direct blind equalization for transmitter induced cyclostationarity," in *Proc. SPAWC*, Paris, France, Apr. 16–18, 1997, pp. 117–120.

[17] E. Moulines, P. Duhamel, J. Cardoso, and S. Mayrargue, "Subspace methods for the blind identification of multichannel FIR filters," *IEEE Trans. Signal Processing*, vol. 43, pp. 516–525, Feb. 1995.

[18] E. Serpedin and G. B. Giannakis, "Blind channel identification and equalization with modulation induced cyclostationarity," in *Proc. CISS*, Baltimore, MD, Mar. 1997, vol. II, pp. 792–797.

[19] D. Slock, "Blind fractionally-spaced equalization, perfect-reconstruction filter banks and multichannel linear prediction," in *Proc. ICASSP*, Adelaide, Australia, 1994, vol. 4, pp. 585–588.

[20] R. Steele, Ed., *Mobile Radio Communications*. New York: Wiley, 1996, ch. 7, p. 601.

[21] L. Tong, G. Xu, and T. Kailath, "Blind identification and equalization based on second-order statistics: A time domain approach," *IEEE Trans. Inform. Theory*, vol. 40, pp. 340–349, Mar. 1994.

[22] L. Tong, G. Xu, B. Hassibi, and T. Kailath, "Blind channel identification based on second-order statistics: A frequency-domain approach," *IEEE Trans. Inform. Theory*, vol. 41, pp. 329–334, Jan. 1995.

[23] J. R. Treichler, I. Fijalkow, and C. R. Johnson, "Fractionally spaced equalizers: How long should they really be?," *IEEE Signal Processing Mag.*, vol. 13, pp. 65–81, May 1996.

[24] M. K. Tsatsanis and G. B. Giannakis, "Transmitter induced cyclostationarity for blind channel equalization," *IEEE Trans. Signal Processing*, vol. 45, pp. 1785–1794, July 1997.

[25] J. K. Tugnait, "Blind equalization and estimation of FIR communications channels using fractional sampling," *IEEE Trans. Commun.*, vol. 44, pp. 324–336, Mar. 1996.

[26] G. Xu, H. Liu, L. Tong, and T. Kailath, "A least-squares approach to blind channel identification," *IEEE Trans. Signal Processing*, vol. 43, pp. 2982–2993, Dec. 1995.

[27] H. Zeng and L. Tong, "Blind channel estimation using the second-order statistics: Asymptotic performance and limitations," *IEEE Trans. Signal Processing*, vol. 45, pp. 2060–2071, Aug. 1997.

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