Deterministic Approaches for Blind Equalization of Time-Varying Channels with Antenna Arrays

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***Abstract*—In this paper, we study the blind equalization problem of time-varying (TV) systems where the channel variations are too rapid to be tracked with conventional adaptive equalizers. We show that using a finite Fourier basis expansion, a TV antenna array system can be cast into a time-invariant multi-input, multi-output (MIMO) framework. The multiple inputs are related through the bases, thereby allowing blind equalization to be accomplished without the use of higher order statistics. Two deterministic blind equalization approaches are presented: One determines the channels first and then the equalizers, whereas the other estimates the equalizers directly. Related issues such as order determination are addressed briefly. The proposed algorithms are illustrated using simulations.**

I. Introduction

In this paper, we deal with blind identification of single-input, multi-output (SIMO) *time-varying* (TV) channels. Our motivation in the wireless communications context comes from multipath environments that change with time, as the mobiles move. The resulting time-varying channels may exhibit time variations that are too rapid for an adaptive algorithm to track. The need to identify and equalize such TV channels arises in mobile telephony, high-speed modems, and underwater communications [22], [23]. In these scenarios, the mth output of an M-element antenna array system is given by

$$x_m(n) = \sum_{l=0}^{L} f_m(n;l)s(n-l)+v_m(n), \quad m=1,\cdots,M \quad (1)$$

where the inaccessible input (or source) $s(n)$ is allowed to be deterministic or random (white or colored) and independent of the AWGN $v_m(n)$; the sensor's TV impulse response $f_m(n;l)$ depends explicitly on time (n). Modeling $f_m(n;l)$ as stationary stochastic process has been the traditional approach [22], [26], [27], but the focus in this paper is on the deterministic basis expansion models introduced recently in [1]–[3].

For channel identification, finite parameterization is needed on the trajectories of $f_m(n;l)$ in order to obtain a tractable formulation. We adopt the discrete-time linear TV model [1]–[3]

$$x_m(n) = \sum_{l=0}^{L} \underbrace{\left[\sum_{i=1}^{P} c_{mi}(l)e^{j\omega_i n}\right]}_{f_m(n;l)} s(n-l) + v_m(n) \quad (2)$$

where $f_m(n;l)$ is finitely parameterized for each lag l via its expansion coefficients $c_{mi}(l)$ onto known complex exponential bases $\{1, e^{j\omega_2 n}, \cdots, e^{j\omega_P n}\}$, as depicted in Fig. 1. Basis $e^{j\omega_1 n} = 1$ is incorporated here to accommodate the time invariant part of the channel. The linear periodically TV model of (2) was justified in [2] based on the kinematics of mobile communicators and standard (constant velocity) approximations encountered in Doppler radar and sonar processing [4]. Indeed, multipath propagation caused by a small number of distinct reflectors and reception by mobiles moving with constant velocity induces delays that change linearly with time and give rise to Doppler shifts in the carrier frequency [4]. Single path Doppler effects were also included in [23], but the challenge here is on blind mitigation of TV intersymbol interference (ISI) and separation of the multiple paths.

The formulation in (2) allows us to tackle the identification of TV channels $f_m(n;l)$ as a parameter estimation problem of the TI coefficients $c_{mk}(l)$. With input–output data available, the problem of estimating $c_{mk}(l)$ is straightforward and can be solved using regression and prediction error techniques (see [5] and references therein). Using one sensor $(M=1)$ and relying on the whiteness of $s(n)$, the output only (i.e., blind) problem was originally addressed in [1] based on high-order output statistics and more recently in [6] using linear equations and a subspace method based on second-order output statistics. However, insufficient diversity in [6] necessitates strong assumptions on the input (whiteness) and less realistic independence assumptions on the bases to guarantee identifiability.

Exploiting spatio-temporal diversity available with $M>1$ sensors, the objective of this paper is to develop blind TV channel identification methods relying upon the model in (2) and without assuming knowledge of the input statistics. Toward this end, we introduce a representation that transforms the SIMO TV channel into a MIMO TI system (see Fig. 2). The main contributions of the present paper are two *deterministic* blind equalization approaches that are capable of recovering deterministic or even colored input sequences by exploiting the underlying data structures of (2). The first algorithm builds on some existing results on time-invariant channel identification [7]–[9] and estimates the channel coefficients

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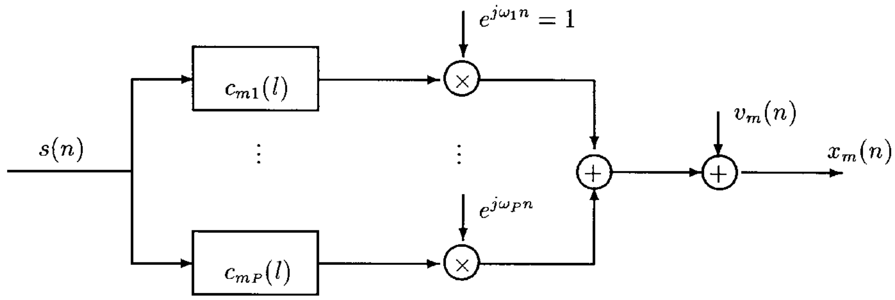

Fig. 1. Fourier expansion of a time-varying system (mth sensor).

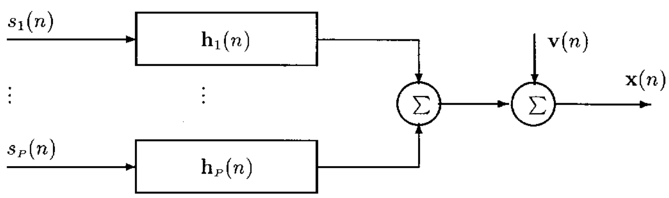

Fig. 2. MIMO model.

utilizing subspace properties of the channel matrix. The second method, on the other hand, estimates the equalizers in one step by taking advantage of the cross-relations of the antenna outputs [18]. Both methods can be regarded as important generalizations of [6], which incorporates deterministic inputs and relaxes existing identifiability conditions. Furthermore, the novel approaches require less data to achieve comparable performance as the method of [6].

Apart from multiple antennas, the multichannel TV model in (2) also arises when the single sensor model of [1], [2] is fractionally sampled. Specifically, it was shown in [25] that oversampling by M offers similar diversity and leads to the same M-channel we set forth in (2), relying on M antennas. When both spatial oversampling and temporal oversampling are used, the effective number of outputs M is the product of the number of antennas and the temporal fractional sampling rate. Note, however, that temporal oversampling may not always produce sufficient diversity due to bandwidth constraints; spatial diversity is more reliable in general.

Throughout the paper, we assume that i) $M > P$ and ii) that the frequencies $\{\omega_i\}_{i=1}^{P}$ in (2) are known. In practice, they can be estimated using tests for cyclostationarity or adaptive maximum-likelihood methods (see [2] for detailed algorithms).

The rest of the paper is organized as follows. In Section II, we cast the TV system under consideration into a MIMO framework and review some important results regarding MIMO blind equalization. Main results of this paper, namely, the two deterministic approaches for blind channel identification and direct equalizer estimation, are derived in Sections III and IV, respectively. Section V discusses the issues of order determination. Examples of computer simulations are provided in Section VI, and the paper is concluded in Section VII.

Notations in this paper are fairly standard. Boldface is used for matrices (in capital letters) and vectors (lower case); symbols $(\cdot)^H, (\cdot)^T$, and $\otimes$ stand for Hermitian, transposition, and convolution, respectively. $\lceil x \rceil$ denotes the smallest integer that is greater than or equal to x. Polynomial $\boldsymbol{a}(z) = \boldsymbol{a}(0) + \boldsymbol{a}(1)z^{-1} + \cdots + \boldsymbol{a}(L)z^{-L}$ is the frequency representation of an (vector) FIR filter. $\hat{\theta}$ of θ denotes the estimate of the parameter, and symbol $\boldsymbol{I}(\boldsymbol{0})$ stands for the identity (zero) matrix or vector.

II. An Equivalent MIMO Framework

Stacking M antenna outputs into a vector, (2) can be represented in a more compact form as

$$\boldsymbol{x}(n) \stackrel{\text{def}}{=} [x_1(n) \cdots x_M(n)]^T = \sum_{l=0}^{L} \left[\sum_{i=1}^{P} \boldsymbol{c}_i(l) e^{j\omega_i n} \right] s(n-l) + \boldsymbol{v}(n) \qquad (3)$$

where $\boldsymbol{c}_i(l) = [c_{1i}(l) \cdots c_{Mi}(l)]^T$, and $\boldsymbol{v}(n) = [v_1(n) \cdots v_M(n)]^T$. The problem of interest herein is the recovery of information-bearing inputs $s(n)$ from the antenna output $\boldsymbol{x}(n)$.

Equation (3) renders a well-defined parameter estimation problem—we can, in principle, obtain channel and noise parameter estimates via maximum likelihood (ML) estimation through multidimensional optimization and then construct equalizers to retrieve the input. Less expensive channel parameter estimation approaches have been proposed in [1] and [6] using second and higher order statistics. Nevertheless, the number of data samples required for satisfactory estimates may still be prohibitive in certain applications. This is mainly attributed to the fact that in a TV system the output statistics are also time-varying and, thus, can only be estimated by an instantaneous approximation. As a result, a large collection of observations is necessary to reduce the estimation variances. To overcome this difficulty, further structure information needs to be incorporated. In the following, we introduce a data model that converts the time-varying system in (3) into a time-invariant MIMO system; based on this, we develop *deterministic* approaches to accomplish blind equalization without relying on statistical knowledge.

Denoting $\zeta_i = e^{j\omega_i}$ for notational simplicity and further defining

$$s_i(n) = s(n)\zeta_i^n, \quad \boldsymbol{h}_i(n) = \boldsymbol{c}_i(n)\zeta_i^n, \quad i = 1, \cdots, P \qquad (4)$$

we have, from (3)

$$\boldsymbol{x}(n) = \sum_{i=1}^{P} \left[\sum_{l=0}^{L} \boldsymbol{c}_i(l) s(n-l) \zeta_i^n \right] + \boldsymbol{v}(n) \qquad (5)$$

$$= \sum_{i=1}^{P} \left[\sum_{l=0}^{L} \underbrace{\boldsymbol{c}_i(l)\zeta_i^l}_{\stackrel{\text{def}}{=} \boldsymbol{h}_i(l)} \underbrace{s(n-l)\zeta_i^{n-l}}_{\stackrel{\text{def}}{=} s_i(n-l)} \right] + \boldsymbol{v}(n) \quad (6)$$

$$= \sum_{i=1}^{P} \boldsymbol{h}_i(n) \otimes s_i(n) + \boldsymbol{v}(n). \quad (7)$$

Surprisingly, we arrive at a MIMO FIR system with *time-invariant* transfer functions $\{\boldsymbol{h}_i(n)\}_{i=1}^P$ and multiple *modulated* inputs $\{s_i(n)\}_{i=1}^P$. As we will show, the above framework enables us to benefit from the recent advances in MIMO blind channel identification and perform blind equalization based solely on the algebraic structure of (7). In the Z domain, (7) can be equivalently expressed as

$$\begin{aligned} \boldsymbol{x}(z) &= \sum_{i=1}^{P} \boldsymbol{h}_i(z)s_i(z) + \boldsymbol{v}(z) \\ &= [\boldsymbol{h}_1(z)\cdots\boldsymbol{h}_P(z)] \begin{bmatrix} s_1(z) \\ \vdots \\ s_P(z) \end{bmatrix} + \boldsymbol{v}(z) \\ &= \boldsymbol{H}(z)\boldsymbol{s}(z) + \boldsymbol{v}(z) \end{aligned} \quad (8)$$

with obvious notations. In the noise-free case ($\boldsymbol{v}(z) = 0$), it is clear from (8) that the $M \times P$ FIR transfer function $\boldsymbol{H}(z)$ has an inverse $\boldsymbol{G}(z) = \boldsymbol{H}^{\dagger}(z)$, provided that $M > P$, and $\boldsymbol{H}(z)$ has full rank (irreducible).

A. Background on MIMO Systems

To facilitate the forthcoming discussion, let us first lay some groundwork by reviewing two important results regarding FIR MIMO systems.

Consider a finite number of noise-free observations, $\{\boldsymbol{x}(n)\}_{n=1}^N$, and define the smoothed data matrix, the channel matrix, and the signal matrix, respectively, as

$$\begin{aligned} \boldsymbol{X}(K) &= \begin{bmatrix} \boldsymbol{x}(1) & \cdots & \boldsymbol{x}(N-K+1) \\ \vdots & \cdots & \vdots \\ \boldsymbol{x}(K) & \cdots & \boldsymbol{x}(N) \end{bmatrix} \\ &= \underbrace{[\mathcal{H}_1(K)\cdots\mathcal{H}_P(K)]}_{\stackrel{\text{def}}{=} \mathcal{H}(K)} \underbrace{\begin{bmatrix} \boldsymbol{S}_1(K+L) \\ \vdots \\ \boldsymbol{S}_P(K+L) \end{bmatrix}}_{\stackrel{\text{def}}{=} \boldsymbol{S}(r), r=K+L} \end{aligned} \quad (9)$$

where

$$\mathcal{H}_i(K) = \underbrace{\begin{bmatrix} \boldsymbol{h}_i(L) & \cdots & \boldsymbol{h}_i(0) & \cdots & \boldsymbol{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \boldsymbol{0} & \cdots & \boldsymbol{h}_i(L) & \cdots & \boldsymbol{h}_i(0) \end{bmatrix}}_{MK \times (K+L)} \quad (10)$$

and

$$\boldsymbol{S}_i(r) = \begin{bmatrix} s_i(K-r+1) & s_i(K-r+2) & \cdots & s_i(N-r+1) \\ s_i(K-r+2) & s_i(K-r+3) & \cdots & s_i(N-r+2) \\ \vdots & \vdots & \cdots & \vdots \\ s_i(K) & s_i(K+1) & \cdots & s_i(N) \end{bmatrix}. \quad (11)$$

As K increases,[1] $\mathcal{H}(K)$ will eventually have more rows than columns if $M > P$. Let $\boldsymbol{H}(l)$ denote the impulse response coefficients of $\boldsymbol{H}(z)$, and assume all subchannels have the same order.

Theorem 1 [7]–[9]: For the M-input, P-output $(M > P)$ FIR system described in (8), the following statements are equivalent.

1) $\boldsymbol{H}(z)$ is irreducible, i.e., rank $[\boldsymbol{H}(z)] = P, \forall z \in \mathcal{C}, z \neq 0$, and rank $[\boldsymbol{H}(0)] = P$ [11].
2) $\mathcal{H}(K)$ in (9) has full column rank when $K \geq ML$.
3) $\boldsymbol{H}(z)$ can be identified up to a $P \times P$ ambiguity matrix from the outputs using second-order statistics.

The above theorem asserts that we can determine what we term the *ambiguous channels* $\tilde{\boldsymbol{H}}(z), \tilde{\boldsymbol{H}}(z)\boldsymbol{T}_{P\times P} = \boldsymbol{H}(z)$ without the use of higher order statistics when the degrees of the columns of $\boldsymbol{H}(z)$ coincide. Additional information, e.g., colored input statistics or finite alphabet inputs, is required to remove the full-rank ambiguity matrix $\boldsymbol{T}$. Throughout this paper, we shall assume the following:

A.1) The channel $\boldsymbol{H}(z)$ is irreducible.
A.2) The input is persistently exciting so that $\boldsymbol{S}(L+K)$ in (9) has full row rank.

From A.1), we have $MK \geq (L+K)P$. The latter is satisfied with a minimum $M_{\min} = P+1$ antennas. In comparison, the TI case requires $M_{\min} = 2$. If $\{\boldsymbol{H}_{pm}(z), p \in [1,\cdots,P], m \in [1,\cdots,M]\}$, denote the TI transfer functions in $\boldsymbol{H}(z)$; A.1) implies that $\boldsymbol{H}_{pm}(z)$ are coprime, i.e., TI channels corresponding to different bases are coprime. Note that $\{\boldsymbol{H}_{p_1 m}(z)\}_{m=1}^M$ may have common zeros, provided that for same $p_2 \neq p_1$, those are not also zeros of $\boldsymbol{H}_{p_2 m}(z)$. To satisfy A.2), the input signal $s(n)$ must have no less than $L+K$ modes [12] and the modes of modulated signals $\{s_i(n)\}$ cannot overlap.

Theorem 2 [13]: If $\boldsymbol{H}(z)$ satisfies A.1), there exists an FIR equalizer $\boldsymbol{G}(z) = [\boldsymbol{g}_1(z)\cdots\boldsymbol{g}_P(z)]$ of length K $(K \geq ML)$ such that

$$\boldsymbol{G}^H(z)\boldsymbol{H}(z) = \begin{bmatrix} \boldsymbol{g}_1^H(z) \\ \vdots \\ \boldsymbol{g}_P^H(z) \end{bmatrix} [\boldsymbol{h}_1(z)\cdots\boldsymbol{h}_P(z)] = z^{-n_o}\boldsymbol{I} \quad (12)$$

where n_o is a delay index.

Matrix $\boldsymbol{G}(z)$ defines a *zero-forcing* equalizer that can perfectly separate and equalize multiple inputs in the absence of noise (see [13] and [14] for more discussion on the zero-forcing conditions and filter design). Theorem 2 reveals another intriguing feature of MIMO FIR systems, namely, that FIR channels can be exactly equalized with FIR filters [13], [14]. Note that as M increases, the minimum equalizer length approaches 1.

B. TI Equalizers of TV Channels

Theorem 2 and (4) suggest that the input of TV systems parameterized as in (2) can be perfectly restored using FIR filters with *fixed* coefficients. More specifically, we may i)

[1] In the array processing literature, K is often referred to as the smoothing factor.

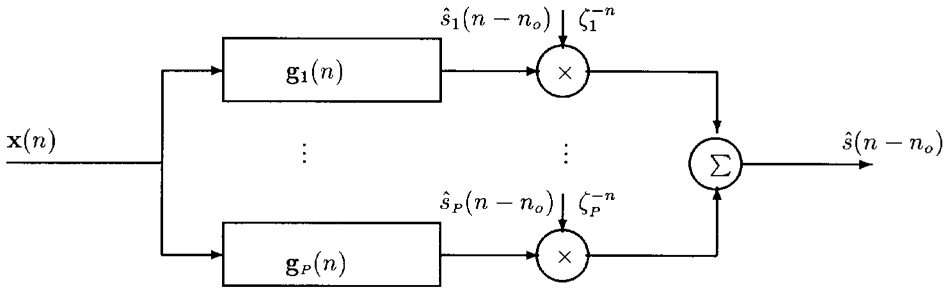

Fig. 3. TV FIR equalizers with TI coefficients.

apply the zero-forcing equalizers of $\boldsymbol{H}(z)$ to separate the modulated inputs $\{s_i(n)\}$ and then ii) compensate for the time variation using the inverse basis functions to recover the input (within a delay n_o) as (see also Fig. 3)

$$\hat{s}(n-n_o) = \sum_{i=1}^{P} \zeta_i^{-n}[\boldsymbol{g}_i(n) \otimes \boldsymbol{x}(n)]. \tag{13}$$

Having established existence and uniqueness of zero-forcing equalizers, the ensuing sections of this paper focus on the blind determination of $\{\boldsymbol{g}_i(n)\}$ from the antenna outputs. For simplicity, we will consider first noise-free data and investigate only the feasibility of blind equalization.

III. Blind TV Channel Equalization

Several approaches have been proposed to blindly identify MIMO channels within a matrix ambiguity [7]–[9], [14]. The most prominent one is the subspace approach proposed in [7], where the subspace of the block Toeplitz channel matrix $\mathcal{H}(K)$ is exploited. For our particular multichannel model, we first choose a smoothing factor K so that $\mathcal{H}(K)$ in (9) has more rows than columns. When $\mathcal{H}(z)$ is irreducible, $\mathcal{H}(K)$ has full column rank [11]. The range space of $\mathcal{H}(K)$ can be obtained by performing an SVD on $\mathcal{X}(K)$

$$\boldsymbol{X}(K) = [\boldsymbol{U}_s \quad \boldsymbol{U}_o]\begin{bmatrix}\Sigma & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\boldsymbol{V}_s^H\\ \boldsymbol{V}_o^H\end{bmatrix} \tag{14}$$

where $\boldsymbol{U}_s$, which is the signal subspace, shares the same range space with $\boldsymbol{H}(K)$ and $\boldsymbol{X}(K)$, whereas $\boldsymbol{U}_o$ is its orthogonal complement. The orthogonality implies that

$$\boldsymbol{U}_o^H \mathcal{H}(K) = \mathbf{0}.$$

Because $\mathcal{H}(K)$ is block Sylvester, it follows that the above equation can be rewritten as

$$\Gamma \begin{bmatrix}\boldsymbol{h}_1(0) & \cdots & \boldsymbol{h}_P(0)\\ \boldsymbol{h}_1(1) & \cdots & \boldsymbol{h}_P(1)\\ \vdots & \vdots & \vdots\\ \boldsymbol{h}_1(L) & \cdots & \boldsymbol{h}_P(L)\end{bmatrix} \stackrel{\text{def}}{=} \Gamma \boldsymbol{H}_{1:P} = \mathbf{0}$$

where Γ is a block Toeplitz matrix constructed from $\boldsymbol{U}_o$ [7]. This shows that the $\mathcal{H}_{1:P}$ is in the null space of Γ. Since for any nonsingular $P \times P$ matrix $\boldsymbol{T}$, we have $\Gamma \boldsymbol{H}_{1:P} = \Gamma \mathcal{H}_{1:P}\boldsymbol{T}^{-1} = \mathbf{0}$, and we can only determine the channel coefficients $\boldsymbol{H}_{1:P}$ from the null vectors of Γ up to a multiplication with a $P \times P$ matrix, i.e., $\tilde{\boldsymbol{H}}_{1:P} = \boldsymbol{H}_{1:P}\boldsymbol{T}^{-1}$ [7].

To determine the remaining ambiguity $\boldsymbol{T}$, some methods invoke higher order statistics [7]–[9], whereas many others require knowledge of the input signals correlation or the finite alphabet property [15], [16]. Since $s_i(n) = s(n)\zeta_i^n$, the multiple inputs in the present problem are strongly related, which suggests that knowledge of the basis functions may be adequate to identify $\boldsymbol{T}$ and fully resolve the system.

In this section, we propose a two-step input recovery scheme. The first step, which is termed as blind multiuser deconvolution, restores the signals from the channel effect or intersymbol interference (ISI), whereas the second step fully separates the modulated inputs utilizing the basis information.

A. Blind MIMO Deconvolution

Since the ambiguous channels $\tilde{\boldsymbol{H}}(z)$ can be determined using one of the existing algebraic approaches, e.g., the subspace method [7], from the antenna outputs, an interesting question to ask is to what extent $\tilde{\boldsymbol{H}}(z)$ can be used to recover the original signals. The following theorem indicates that the ambiguous channels contain sufficient information to deconvolve the antenna outputs.

Theorem 3: Let $\tilde{\boldsymbol{G}}(z)$ be the zero-forcing equalizers for the ambiguous channels $\tilde{\boldsymbol{G}}^H(z)\tilde{\boldsymbol{H}}(z) = z^{-n_o}\boldsymbol{I}$. Matrix $\tilde{\boldsymbol{G}}(z)$ can remove the ISI in $\boldsymbol{x}(n)$ and restore the multiple inputs within a matrix ambiguity $\boldsymbol{T}$. The true zero-forcing equalizer that satisfies (12) is given by $\boldsymbol{G}(z) = \tilde{\boldsymbol{G}}(z)\boldsymbol{T}^{-H}$.

Proof: Applying $\tilde{\boldsymbol{G}}(z)$ to the system output yields [cf., (8) with $\boldsymbol{v}(z) = \mathbf{0}$]

$$\begin{aligned}\tilde{\boldsymbol{G}}^H(z)\boldsymbol{x}(z) &= \tilde{\boldsymbol{G}}^H(z)\sum_{i=1}^{P}\boldsymbol{h}_i(z)s_i(z)\\ &= \tilde{\boldsymbol{G}}^H(z)\boldsymbol{H}(z)\begin{bmatrix}s_1(z)\\ \vdots\\ s_P(z)\end{bmatrix}\\ &= \tilde{\boldsymbol{G}}^H(z)\underbrace{\boldsymbol{H}(z)\boldsymbol{T}^{-1}}_{=\tilde{\boldsymbol{H}}(z)}\boldsymbol{T}\begin{bmatrix}s_1(z)\\ \vdots\\ s_P(z)\end{bmatrix}\\ &= \underbrace{\tilde{\boldsymbol{G}}^T(z)\tilde{\boldsymbol{H}}(z)}_{=z^{-n_o}\boldsymbol{I}}\boldsymbol{T}\begin{bmatrix}s_1(z)\\ \vdots\\ s_P(z)\end{bmatrix}\\ &= z^{-n_o}\boldsymbol{T}\begin{bmatrix}s_1(z)\\ \vdots\\ s_P(z)\end{bmatrix}\end{aligned} \tag{15}$$

which is a linear transformation of the multiple inputs. Multiplying (15) by $\boldsymbol{T}^{-1}$ leads to $\boldsymbol{T}^{-1}\tilde{\boldsymbol{G}}^H(z)\boldsymbol{x}(z) = z^{-n_o}[s_1(z)\cdots s_P(z)]^T$; hence, $\boldsymbol{G}(z) = \tilde{\boldsymbol{G}}(z)\boldsymbol{T}^{-H}$. □

The above theorem verifies the known fact that without the use of higher order statistics, it is possible to obtain ISI-free input sequences within a matrix

$$\tilde{\boldsymbol{s}}(n) = \begin{bmatrix} \tilde{s}_1(n) \\ \vdots \\ \tilde{s}_P(n) \end{bmatrix} = \boldsymbol{T} \begin{bmatrix} s_1(n) \\ \vdots \\ s_P(n) \end{bmatrix} = \boldsymbol{T}\boldsymbol{s}(n). \tag{16}$$

In other words, *blind deconvolution* of a MIMO channel can be accomplished based solely on the algebraic structure of the system. The remaining task is to determine the ambiguity matrix $\boldsymbol{T}$ so that the input sequence can be reconstructed.

B. Ambiguity Matrix Determination

From (16) and the definition in (4)

$$\tilde{\boldsymbol{s}}(n) = \boldsymbol{T} \begin{bmatrix} \zeta_1^n \\ \vdots \\ \zeta_P^n \end{bmatrix} s(n) \stackrel{\text{def}}{=} \boldsymbol{T}\boldsymbol{\zeta}(n)s(n), \quad \forall n \tag{17}$$

where $\boldsymbol{t}_i$ is the ith column of $\boldsymbol{T}$.

Theorem 4: Let $\{\tilde{\boldsymbol{s}}(n)\}$ be given and $\{\zeta_i\}_{i=1}^P$ be distinct. If (17) is satisfied with $s(n) \neq 0$ for at least $2P$ values of n, then columns $\{\boldsymbol{t}_i\}$ and, thus, $\boldsymbol{T}$ can be uniquely determined up to a scalar.

Proof: Without loss of generality, let us assume that $0 = \omega_1 < \omega_2 < \cdots < \omega_P$, and we can always force this by demodulating the received signals with $e^{-j\omega_1 n}$.

Suppose that $\{\hat{\boldsymbol{T}}, \hat{s}(1), \cdots, \hat{s}(N)$: $N \geq 2P, \hat{\boldsymbol{T}}$ is of full rank$\}$ is a solution satisfying (17). We have

$$\boldsymbol{T}\boldsymbol{\zeta}(n)s(n) = \hat{\boldsymbol{T}}\boldsymbol{\zeta}(n)\hat{s}(n), \quad \forall n. \tag{18}$$

On defining $a(n) = \hat{s}(n)/s(n)$ and $\boldsymbol{Q} = [\boldsymbol{q}_1 \cdots \boldsymbol{q}_P] = \boldsymbol{T}\hat{\boldsymbol{T}}^{-1}$, (18) can be rewritten as

$$a(n)\boldsymbol{\zeta}(n) = \boldsymbol{Q}\boldsymbol{\zeta}(n), \quad \forall n.$$

Equating the first element of both sides yields

$$a(n) = \boldsymbol{q}_1(1) + \boldsymbol{q}_2(1)e^{j\omega_2 n} + \cdots + \boldsymbol{q}_P(1)e^{j\omega_P n}, \quad \forall n$$

where $\boldsymbol{q}_i(j)$ denotes the jth element of $\boldsymbol{q}_i$. Similarly, for the last element

$$a(n) = \boldsymbol{q}_1(P)e^{-j\omega_P n} + \boldsymbol{q}_2(P)e^{j(\omega_2-\omega_P)n} + \cdots + \boldsymbol{q}_P(P), \quad \forall n.$$

Combining the above two equations and expressing it in a matrix form, we obtain the equation at the bottom of the page. When $N \geq 2P$, the left-most Vandermonde matrix clearly has only one null vector; we thus conclude that $\boldsymbol{q}_1(1) = \boldsymbol{q}_P(P) = \alpha$, where α is a nonzero constant, and $\boldsymbol{q}_1(2) = \cdots = \boldsymbol{q}_1(P) = \boldsymbol{q}_P(1) = \cdots = \boldsymbol{b}_P(P-1) = 0$. Therefore, $a(1) = \cdots = a(N) = \alpha$, and consequently, $\boldsymbol{Q} = \alpha\boldsymbol{I}$. In other words, $\hat{\boldsymbol{T}} = (1/\alpha)\boldsymbol{T}$, and $\hat{s}(n) = \alpha s(n)$. □

Note that $\boldsymbol{T}$ in Theorem 4 does not have to be a square matrix to be identifiable. As long as its row polynomials are coprime, $\boldsymbol{T}$ can be determined up to a scalar.

Rearranging (17), we obtain

$$s^{-1}(n)\tilde{\boldsymbol{s}}(n) - \sum_{i=1}^{P} \boldsymbol{t}_i\zeta_i^n = \boldsymbol{0}. \tag{19}$$

In (19), $\tilde{\boldsymbol{s}}(n)$ and $\{\zeta_i^n\}$ are available, whereas $\{\boldsymbol{b}_i\}$ and $\{s(n)\}$ are the unknowns. Given a finite collection of $\{\tilde{\boldsymbol{s}}(n)\}_{n=1}^N$, a set of linear equations follows from (19) as in

$$\begin{bmatrix} \zeta_1\boldsymbol{I} & \cdots & \zeta_P\boldsymbol{I} & \tilde{\boldsymbol{s}}(1) & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \zeta_1^2\boldsymbol{I} & \cdots & \zeta_P^2\boldsymbol{I} & \boldsymbol{0} & \tilde{\boldsymbol{s}}(2) & & \vdots \\ \vdots & \vdots & \vdots & \vdots & & \ddots & \boldsymbol{0} \\ \zeta_1^N\boldsymbol{I} & \cdots & \zeta_P^N\boldsymbol{I} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \tilde{\boldsymbol{s}}(N) \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{t}_1 \\ \vdots \\ \boldsymbol{t}_P \\ -s^{-1}(1) \\ \vdots \\ -s^{-1}(N) \end{bmatrix} = \boldsymbol{0}. \tag{20}$$

It is readily seen that there are PN equations and $P^2 + N$ unknowns, which defines an overdetermined estimation problem when $N \geq \lceil P^2/P{-}1 \rceil$. Theorem 4 asserts that matrix $\boldsymbol{T}$ can thus be uniquely determined up to a scalar from (20).

In practice, $\boldsymbol{T}$ needs to be estimated through least-squares fitting of (20). Once $\boldsymbol{T}$ is identified, the modulated inputs $\boldsymbol{s}(n)$ can be recovered by passing $\tilde{\boldsymbol{s}}(n)$ through $\boldsymbol{T}^{-1}$. In effect, the zero-forcing equalizer $\boldsymbol{G}(z)$ is implemented in two steps by concatenating $\tilde{\boldsymbol{G}}(z)$ and $\boldsymbol{T}^{-1}$. The input signal can be reconstructed, subsequently, as in (13).

In summary, blind equalization of basis expansion TV systems can be accomplished with the following steps:

1) Identify the ambiguous channels $\{\tilde{\boldsymbol{h}}_i(l)\}_{i=1}^P$ using any second-order statistics method (e.g., the subspace approach in [7]).
2) Deconvolve the antenna output with zero-forcing equalizers $\tilde{\boldsymbol{G}}(z)$ constructed based on the ambiguous channels $\tilde{\boldsymbol{H}}(z)$.

$$\begin{bmatrix} 1 & e^{j\omega_2} & \cdots & e^{j\omega_P} & e^{-j\omega_P} & e^{j(\omega_2-\omega_P)} & \cdots & 1 \\ 1 & e^{2j\omega_2} & \cdots & e^{2j\omega_P} & e^{-2j\omega_P} & e^{2j(\omega_2-\omega_P)} & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & e^{Nj\omega_2} & \cdots & e^{Nj\omega_P} & e^{-Nj\omega_P} & e^{Nj(\omega_2-\omega_P)} & \cdots & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_1(1) \\ \vdots \\ \boldsymbol{q}_1(P) \\ -\boldsymbol{q}_P(1) \\ \vdots \\ -\boldsymbol{q}_P(P) \end{bmatrix} = \boldsymbol{0}.$$

3) Determine the ambiguity matrix $\boldsymbol{T}$ from the deconvolved outputs by solving the least-squares problem in (20), and separate the inputs by applying $\boldsymbol{T}^{-1}$ to $\{\tilde{s}_i(n)\}$.
4) Compensate the time variations in the modulated inputs using the inverse basis functions. Combine the resulting sequences as the input estimates as depicted in Fig. 3.

The basis expansion TV system can be resolved, provided that conditions in Theorem 1 are satisfied. It is worthwhile to point out that our discussions concern only feasibility of blind estimation. Many factors, e.g., the noise and signal statistics, are attributed to the estimation performance. Optimal weighted solutions is also an important direction for future study.

IV. Direct Blind Equalizers

Most blind equalization approaches, including the method proposed in the previous section, are *indirect* in the sense that equalizer design is accomplished through channel estimation. Since calculating an equalizer based on channel characteristics involves matrix inversion and is thus expensive, it would be of great interest to find methods that yield equalizer estimates directly from data observations [10], [17], [18]. Another drawback of the indirect method developed in the previous section is that it cannot handle channels with different orders. In this section, we study the feasibility of more flexible *direct* blind equalization bypassing channel identification.

Starting from the definitions in (9), we assume that the smoothing factor K is greater than or equal to $\lceil PL/M - P \rceil$ so that the overall channel matrix $\mathcal{H}(K)$ in

$$\boldsymbol{X}_{MK\times N}(K) = \mathcal{H}_{MK\times Pr}(K)\boldsymbol{S}_{Pr\times N}(r), \quad r = L + K \tag{21}$$

has full column rank. The dependence of $\mathcal{H}(K)$ $(\boldsymbol{S}(r))$ on K (r) will be dropped in the following derivation for notational convenience.

Note that $\boldsymbol{X}$ is generally rank deficient since $\mathcal{H}_{MK\times Pr}$ has more rows than columns. As will become clear, the direct equalization algorithm proposed here requires the data matrix $\boldsymbol{X}$ to have full row rank. Toward this end, we may either simply pick Pr rows in $\boldsymbol{X}$ and denote them as $\boldsymbol{Y}$ or condense $\boldsymbol{X}$ by calculating its column signal subspace $\boldsymbol{U}_s$ in (14) and letting

$$\boldsymbol{Y} = \boldsymbol{U}_s^H \boldsymbol{X}. \tag{22}$$

Either way, $\boldsymbol{Y}$ will be full rank and have the same row span as $\boldsymbol{S}$, i.e.,

$$\boldsymbol{S} = \boldsymbol{W}\boldsymbol{Y} \tag{23}$$

where $\boldsymbol{W}$ is a $Pr \times Pr$ full-rank matrix. The blind equalization is accomplished if we can determine $\boldsymbol{W}$.

The algebraic approach presented below is reminiscent of the least-squares approaches for TI channel equalization [10], [19], which exploits the Hankel structure of the signal matrix $\boldsymbol{S}$.

A. The Cross Relations

Notice that $\boldsymbol{S}$ in (9) and (21) has P blocks, each of which has r rows. The key here is to take advantage of the Hankel structure of $\{\boldsymbol{S}_i\}_{i=1}^P$ to establish certain relations among the rows of $\boldsymbol{W}$. Toward this end, denote $\boldsymbol{w}_{i,l}^H$, corresponding to the ith basis, as the $(ir + l)$th row of $\boldsymbol{W}$. From (23) and (11), we have

$$\begin{aligned}\boldsymbol{w}_{i,l}^H\boldsymbol{Y} &= [s_i(K+l-r)s_i(K+l-r+1)\cdots s_i(N+l-r)]\\ &= [s(K+l-r)s(K+l-r+1)\cdots s(N+l-r)]\\ &\quad\times \operatorname{diag}(1, \zeta_i, \cdots, \zeta_i^{N-K+1})\zeta_i^{K+l-r}.\end{aligned}$$

Notice that due to the shift-invariant structure of $\{\boldsymbol{S}_i\}$, there is a common vector in all $\{\boldsymbol{w}_{i,l}^H\boldsymbol{Y}\}$. In particular, if we define a *companion* matrix of $\boldsymbol{w}_{i,l}^H, \boldsymbol{Y}_{i,l}$, as the submatrix comprised of the $(r-l+1)$th through the $(N-r-l+2)$th columns of

$$\boldsymbol{Y}\operatorname{diag}(1, \zeta_i^{-1}, \cdots, \zeta_i^{K-N-1})\zeta_i^{r-K-l}$$

it is straightforward to show that

$$\begin{aligned}\boldsymbol{w}_{i,l}^H\boldsymbol{Y}_{i,l} &= \boldsymbol{w}_{j,m}^H\boldsymbol{Y}_{j,m}\\ &= [s(r)\cdots s(N-r+1)] \begin{cases} i,j = 1,\cdots,P\\ l,m = 1,\cdots,r.\end{cases}\end{aligned} \tag{24}$$

The above equation relates any two rows of $\boldsymbol{W}$ with a linear equation

$$[\boldsymbol{w}_{i,l}^H \quad \boldsymbol{w}_{j,m}^H]\begin{bmatrix}\boldsymbol{Y}_{i,l}\\ -\boldsymbol{Y}_{j,m}\end{bmatrix} = \boldsymbol{0}.$$

To determine all Pr rows in $\boldsymbol{W}$ in a unified fashion, re-order $\{\boldsymbol{w}_{i,l}^H\}$ and $\{\boldsymbol{Y}_{i,l}\}$, respectively, into single-index vectors (matrices) $\boldsymbol{w}_{i,l}^H = \boldsymbol{w}_{(i-1)r+l}^H$ and $\boldsymbol{Y}_{i,l} = \boldsymbol{Y}_{(i-1)r+l}$. We can combine all possible equations simultaneously to formulate a linear estimator for a super vector $\boldsymbol{w} = [\boldsymbol{w}_1^H \cdots \boldsymbol{w}_{Pr}^H]^H$

$$\boldsymbol{w}^H\boldsymbol{D}_{Pr} = \boldsymbol{0} \tag{25}$$

where $\boldsymbol{D}_{Pr}$ is constructed using a so-called data selection transform of $\{\boldsymbol{Y}_i\}_{l=1}^{Pr}$

$$\boldsymbol{D}_2 = \begin{bmatrix}\boldsymbol{Y}_2\\ -\boldsymbol{Y}_1\end{bmatrix}$$

$$\vdots \qquad \vdots$$

$$\boldsymbol{D}_{Pr} = \left[\begin{array}{c|ccc} \boldsymbol{D}_{Pr-1} & \boldsymbol{Y}_{Pr} & & \\ & & \ddots & \\ & & & \boldsymbol{Y}_{Pr}\\ \hline \boldsymbol{0} & -\boldsymbol{Y}_1 & \cdots & -\boldsymbol{Y}_{Pr-1}\end{array}\right]. \tag{26}$$

Theorem 5: Under assumptions A.1) and A.2), $\boldsymbol{w}$ can be determined uniquely as the nontrivial solution of (25), provided that $K \geq ML$ and $N \geq Pr + K - 1$.

Proof: The conditions in the above theorem guarantee that $\boldsymbol{S}$ has full row rank and shares the same span with $\boldsymbol{Y}$.

Let $\hat{\boldsymbol{w}}$ be the nontrivial solution of (25); we therefore have

$$\hat{\boldsymbol{w}}_{i,l}^H\boldsymbol{Y}_{i,l} = \hat{\boldsymbol{w}}_{j,m}^H\boldsymbol{Y}_{j,m} = [\hat{s}(r)\cdots\hat{s}(N-r+1)] \tag{27}$$

where $[\hat{s}(r)\cdots\hat{s}(N-r+1)]$ is nonzero since $\boldsymbol{Y}$ has full row rank.

Substituting the definition of $\{\boldsymbol{Y}_{i,l}\}$ into the above equation suggests that there is an $\hat{\boldsymbol{S}}$ matrix that possesses the same block Hankel structure as $\boldsymbol{S}$. In addition, $\hat{\boldsymbol{S}} = \hat{\boldsymbol{W}}\boldsymbol{Y} = \hat{\boldsymbol{W}}\boldsymbol{W}^{-1}\boldsymbol{S}$.

TABLE I
CHANNEL COEFFICIENTS

i	$c_{i,1}(1)$	$c_{i,1}(2)$	$c_{i,1}(3)$	$c_{i,2}(1)$	$c_{i,2}(2)$	$c_{i,2}(3)$
1	$.541-.585j$	$-2.349-2.312j$	$-.086-1.155j$	$-.261+.199j$	$.394+.729j$	$-.118-.081j$
2	$.140-.741j$	$-3.574-.490j$	$-.359-.626j$	$.090-.822j$	$.137-1.626j$	$-.205-.041j$
3	$.020-.512j$	$1.016-2.263j$	$-.256+.315j$	$.568-.064j$	$.771-.767j$	$-.268+.070j$
4	$.120+.890j$	$1.842-.333j$	$-.031-.162j$	$-.372+.450j$	$-1.680+1.316j$	$.224-.142j$
5	$-.034-.312j$	$-2.078-3.437j$	$.501+.265j$	$.052-.622j$	$-.551-2.672j$	$-.186-.349j$
6	$.188-.337j$	$-2.022+.810j$	$.422+.443j$	$-.196-.435j$	$-.826+.027j$	$-.218-.016j$
7	$-.788+.297j$	$-1.867+3.299j$	$-.815-.007j$	$.147-.038j$	$.447-.749j$	$.259-.254j$
8	$.341-.033j$	$3.636-3.971j$	$.136+.168j$	$.465-.013j$	$2.157-.785j$	$.118+.032j$

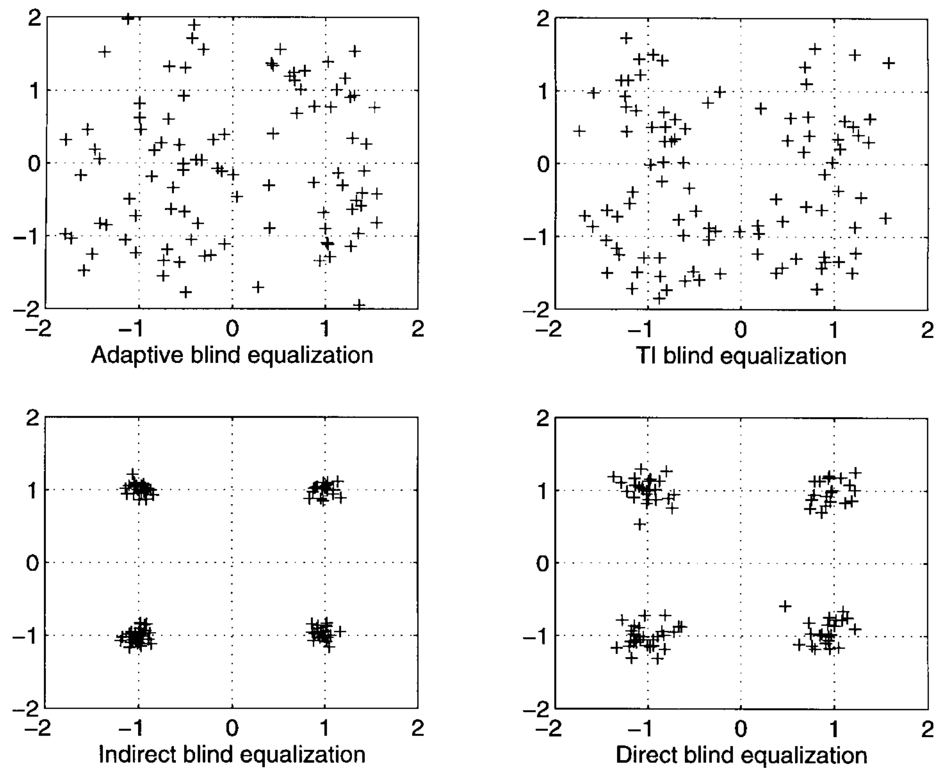

Fig. 4. Received and equalized symbols.

In [14], it is shown that because of the block Hankel structure of $\boldsymbol{S}$ and $\hat{\boldsymbol{S}}$, the symbol sequences $\{\hat{s}_i(n)\}_{i=1}^{P}$ in $\hat{\boldsymbol{S}}$ must be a linear combination of the symbol sequences $\{s_i(n)\}_{i=1}^{P}$ in $\boldsymbol{S}$. In other words, $\tilde{\boldsymbol{s}}(n)=\boldsymbol{T}\boldsymbol{s}(n)$, as in (17). Then, from Theorem 4, we conclude that $\hat{s}(n)=\rho s(n)$, and $\hat{\boldsymbol{W}}=(1/\rho)\boldsymbol{W}$, where ρ is a nonzero scalar. □

In short, (25) yields (within a scalar) the exact estimate of $\boldsymbol{b}_i$ with finite data samples in the absence of noise. Since practical data is always noise corrupted, $\boldsymbol{w}_i$ is estimated using the minimum norm solution

$$\hat{\boldsymbol{w}}=\arg\min_{||\boldsymbol{w}||=1}\boldsymbol{w}^H(\boldsymbol{D}_{Pr}^H\boldsymbol{D}_{Pr})\boldsymbol{w} \tag{28}$$

which, as shown in [10], minimizes the noise power at the equalizer output.

To summarize, direct blind estimation of the equalizer coefficients can be accomplished with the following steps.

1) Calculate the $\boldsymbol{Y}$ matrix by applying $\boldsymbol{U}^H$ to $\boldsymbol{X}$ or picking Pr linearly independent rows from $\boldsymbol{X}$.
2) Construct the companion matrices $\{\boldsymbol{Y}_{i,l}\}_{l=1}^{r}$ and $\boldsymbol{D}_{Pr}$ in (26).
3) Estimate the rows of $\boldsymbol{W}$ from (28).

Once $\boldsymbol{W}$ is determined, the modulated signals can be separated. Because of the structure of $\boldsymbol{S}$, the input with delay n_o can be recovered by coherently combining signals corresponding to different basis functions

$$\hat{s}(n-n_o)=\sum_{i=0}^{P}\boldsymbol{b}_{i,r-n_o}\underbrace{\boldsymbol{U}_s^H[\boldsymbol{x}^H(n-K)\cdots\boldsymbol{x}^H(n)]^H}_{\text{first column of }\boldsymbol{Y}}\cdot\zeta_i^{n_o-n},\qquad n_o=0,\cdots,r.$$

Equating the above equation with (13), it is readily seen that the equalizer in Fig. 3 is given by

$$[\boldsymbol{g}_i^H(K-1)\cdots\boldsymbol{g}_i^H(0)]=\boldsymbol{w}_{i,r-n_o}\boldsymbol{U}_s^H. \tag{29}$$

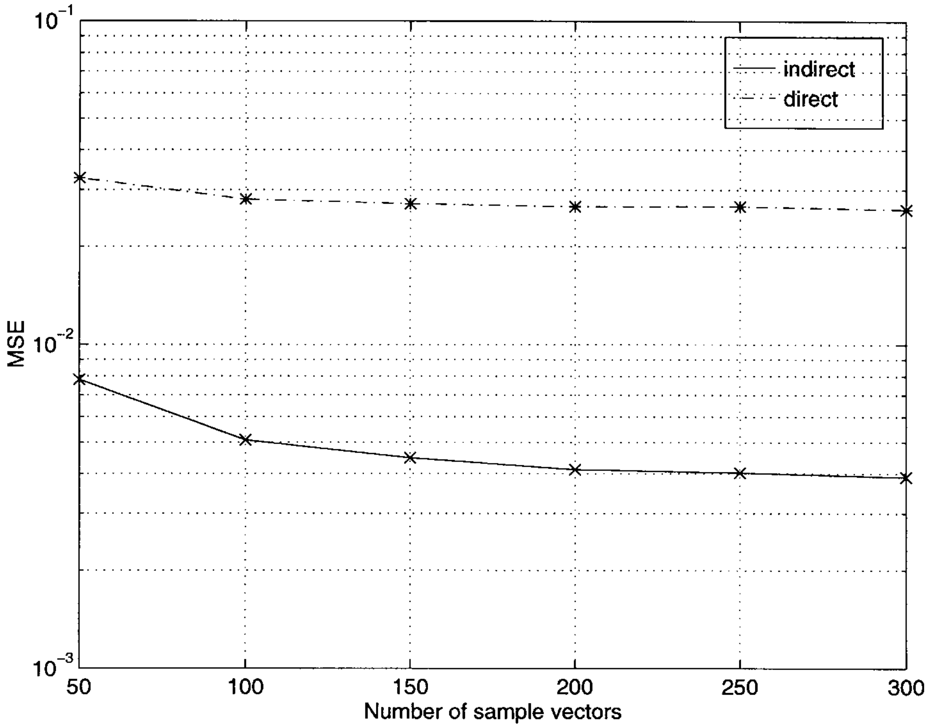

Fig. 5. Number of samples versus MSE.

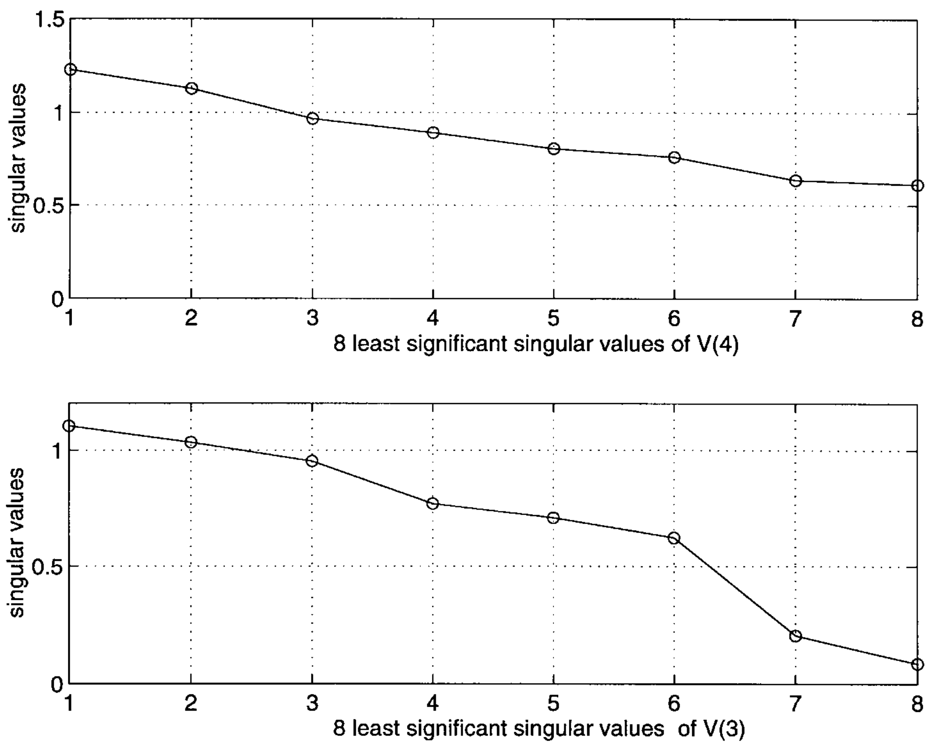

Fig. 6. Distributions of singular values.

V. Order Determination

Among many other important issues, order determination might be the most critical one to parametric estimation methods such as the proposed two blind equalization approaches. It has been recognized that the first step in indirect equalization, i.e., channel identification within a matrix ambiguity, hinges on knowledge of the maximum channel order L. We shall show next that the direct algorithm derived in Section IV relies on the minimum channel order $L_{\min}$. First, however, we introduce two propositions that determine the number of bases P and the channel orders $\{L_i\}$ from rank conditions of data matrices (see also [14] and [18]).

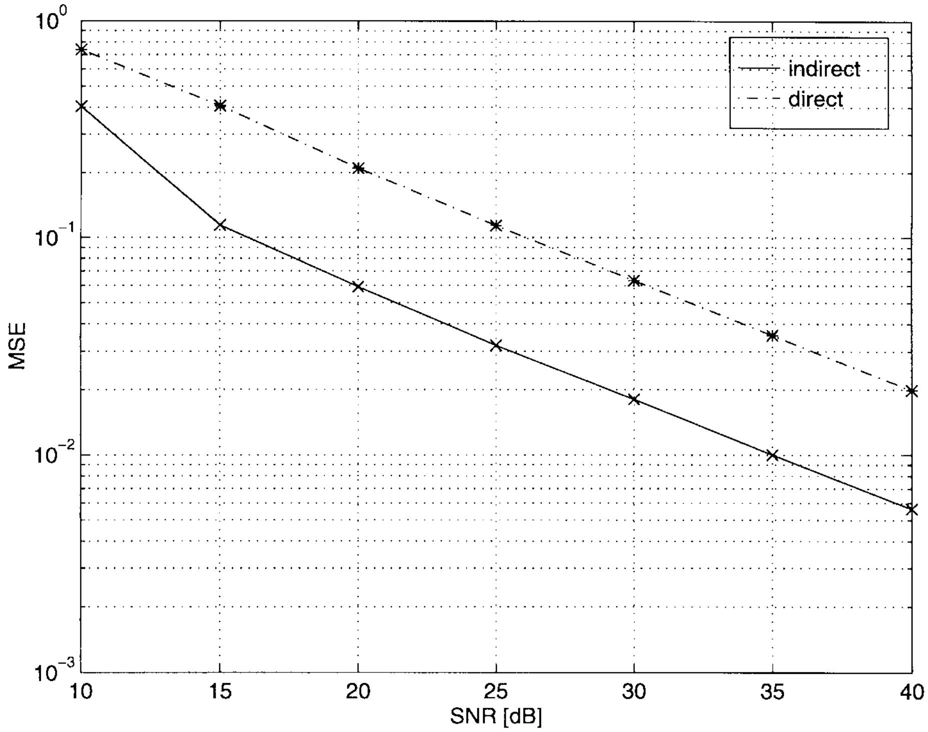

Fig. 7. MSE versus SNR.

Proposition 1: Let $K_{\max}$ denote an upper bound of K that satisfies A.2). Then, under A.1), the number of basis is related to the ranks of $\boldsymbol{X}(K_1)$ and $\boldsymbol{X}(K_2)$ in (9) by

$$P = \frac{\text{rank}\,[\boldsymbol{X}(K_1)] - \text{rank}\,[\boldsymbol{X}(K_2)]}{K_1 - K_2} \quad \forall K_1 \neq K_2, K_1, K_2 \leq K_{\max}. \tag{30}$$

Proof: Denoting L_l as the order of $\boldsymbol{h}_l(n)$ corresponding to the lth basis, Proposition 1 can be easily verified by replacing K by K_i in (9) to obtain

$$\boldsymbol{X}(K_i) = \mathcal{H}(K_i)\begin{bmatrix}\boldsymbol{S}_1(K_i+L_1)\\ \vdots \\ \boldsymbol{S}_P(K_i+L_P)\end{bmatrix}, \qquad i = 1, 2. \tag{31}$$

Matrix $\boldsymbol{X}(K_i)$ in (31) has full rank $\Sigma_{i=1}^{P}(K+L_i) = KP + \Sigma_{i=1}^{P} L_i$ under A.1) and A.2); thus, (30) follows easily by subtraction. □

The following proposition was presented in [14] to determine the orders of $\{\boldsymbol{h}_i(n)\}_{i=1}^{P}$ by examining the rank condition of a so-called deconvolution matrix.

Proposition 2: Let $\boldsymbol{V}_o(K)$ be the row null space of matrices $\boldsymbol{X}(K)$ and $\boldsymbol{Y}(K)$, defined as in (21) and (22), and define matrix $\boldsymbol{V}(K+l)$ as

$$\boldsymbol{V}(K+l) = \underbrace{\begin{bmatrix}\boldsymbol{V}_o(K) & \cdots & \cdots & \mathbf{0}\\ \mathbf{0} & \boldsymbol{V}_o(K) & \cdots & \vdots\\ \vdots & \cdots & \ddots & \mathbf{0}\\ \mathbf{0} & \cdots & \cdots & \boldsymbol{V}_o(K)\end{bmatrix}}_{K+l \text{ blocks}}$$

$$\mathbf{0} = \begin{bmatrix}0\\ \vdots\\ 0\end{bmatrix}. \tag{32}$$

If d_l denotes the number of channels with order l, then $\{d_l\}$ can be determined iteratively from equations

$$\text{Nullity}\,\{\boldsymbol{V}(K+l)\} = \begin{cases}\sum_{i=l}^{L}(i-l+1)d_i & l = 0, \cdots, L\\ 0 & l > L.\end{cases} \tag{33}$$

The above proposition provides the basic idea behind order determination for the proposed blind approaches. In practice, where only noisy data is available, the eigenvalue distribution of the data matrices may not provide consistent estimate of the channel orders. More sophisticated tests such as the MDL [20] and the AIC [21] need to be developed.

When the channel orders of $\{\boldsymbol{h}_i(n)\}$ are different, as in most practical systems, the direct equalization approach requires minor modification, whereas the indirect method can handle the situation as long as L is known.

Similar to (22)

$$\boldsymbol{W}\boldsymbol{Y} = [\boldsymbol{w}_{1,1}\cdots\boldsymbol{w}_{1,K+L_1}, \cdots, \boldsymbol{w}_{P,1}\cdots\boldsymbol{w}_{P,K+L_P}]^H\boldsymbol{Y} = \begin{bmatrix}\boldsymbol{S}_1(K+L_1)\\ \vdots\\ \boldsymbol{S}_P(K+L_P)\end{bmatrix}.$$

Each submatrix $\boldsymbol{S}_i$ now has a different number of rows. However, from the Hankel structure of $\boldsymbol{S}(K+L_i)$, it is not difficult to examine that

$$[s(K+L_{\min})\cdots s(N-K-L_{\min}+1)]$$

is the *unique* common vector embedded in all rows. Therefore, by exploiting the cross-relations in the same fashion as in Section IV, $\{\boldsymbol{w}_{i,l}\}_{i=1,l=1}^{P,K+L_{\min}}$ can be identified from $\boldsymbol{D}_{P(K+L_{\min})}$ constructed from their companion matrices $\{\boldsymbol{Y}_{i,l}\}_{i=1,l=1}^{P,K+L_{\min}}$. The input is then recovered as

$$[\boldsymbol{w}_{1,1}\cdots\boldsymbol{w}_{1,K+L_{\min}},\cdots,\boldsymbol{w}_{P,1}\cdots\boldsymbol{w}_{P,K+L_{\min}}]^H\boldsymbol{Y} = \begin{bmatrix}\boldsymbol{S}_1(K+L_{\min})\\ \vdots \\ \boldsymbol{S}_P(K+L_{\min})\end{bmatrix}.$$

Thus, Step 2 in the direct equalization procedure to

"2. Construct the companion submatrices $\{\boldsymbol{Y}_i, l\}_{l=1}^{K+L_{\min}}$ and subsequently $\boldsymbol{D}_{P(K+L_{\min})}$;"

direct blind equalizer estimation can be achieved for general systems.

VI. Simulation Results

Computer simulations were conducted to validate the efficacy of the proposed algorithms. All examples involved an eight-element array and QPSK source signals. The time-varying channels were generated using two basis functions 1 and $e^{j2\pi n/50}$, and time-invariant coefficients are given in Table I. All identifiability conditions are satisfied under this setup. The TI component was chosen twice as strong as the TV component so that the system is only partially time varying.

Under 20 dB SNR, we first supplied $N = 100$ antenna output vectors to the frequency estimation algorithm in [2] to obtain $\{\hat{\omega}_i\}_{i=1}^P$. The resulting frequencies were then used in the proposed algorithms for blind equalization. The recovered symbols were compared with the equalized symbols using the blind equalizer [19] proposed for time-invariant (TI) channels and that of the standard adaptive Godard algorithm. Fig. 4 illustrates the results. Clearly, the variations of the channels are too rapid for the adaptive algorithm to track and too strong for the TI equalizer to ignore. In contrast, both the proposed direct and indirect blind equalization approaches successfully recover the input sequences. Furthermore, it is observed that the indirect method outperformed the direct algorithm.

The data efficiency of the approaches is demonstrated in Fig. 5. Note that [6] cannot work with the chosen basis because of the linear independence required is not satisfied. Without incorporating statistical information, however, the deterministic methods are not asymptotically efficient in general. This is manifested in Fig. 5 by the lack of performance improvement with a large number of observations.

Fig. 6 illustrates how channel orders can be determined from the singular value distribution of $\{\boldsymbol{V}(K+l)$ defined in (32). K was set to 1 in this particular example. It is clearly seen from the top plot that nullity $\boldsymbol{V}(4) = 0$, which indicates that there is no channel with length >3. The two distinctively smaller singular values in the bottom plot suggest that $d_2 = 2$, i.e., there are two channels with order = 2 (or length = 3). Since $P = 2$, this implies that both $\boldsymbol{h}_1(n)$ and $\boldsymbol{h}_2(n)$ are of order 2.

The performance of the proposed methods versus SNR is studied in Fig. 7. Again, estimated frequencies rather than the true values were used in the proposed algorithms. As expected, the MSE of input estimates decreases as the SNR increases. Although the direct approach, which involves only least-squares minimization, is more computationally attractive, the noticeable gap in MSE's indicates that its performance is inferior to that of the indirect approach.

VII. Conclusion

Blind equalization of time-varying channels has been studied in this paper using basis expansion methods. By converting a SIMO TV channel into a MIMO TI system, we have shown that algebraic approaches can be developed to uniquely recover the input by exploiting the underlying data structure. Apart from a persistence-of-excitation condition, the two approaches proposed in this paper place no constraints on the input data sequence and can accomplish blind estimation with high data efficiency. The order determination problem has been addressed briefly.

Future work is required on performance analysis, basis estimation, and, more importantly, extensions to adaptive algorithms. In particular, the direct equalization algorithm that accomplishes equalization in a least-squares fashion has the potential to be implemented adaptively.

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