Symbol Timing Estimation in Ultra Wideband Communications

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ABSTRACT

This paper takes on a cyclostationary approach to recover timing of ultra-wideband (UWB) transmissions over rich multipath environments. It is demonstrated that timing dependent cyclic statistics exist without resorting to oversampling, due to the symbol repetition pattern inherent to UWB modulation. Based on the received signal's second-order cyclic statistics, non-data-aided time offset estimation algorithms are developed. Timing acquisition relies only on frame rate samples, while low-complexity tracking utilizes pulse rate samples. Both acquisition and tracking schemes are tolerant to UWB channel fading, and additive stationary (possibly colored) noise.

1. Introduction

Ultra wideband (UWB) technology has received increasing attention for its broad applicability to short-range wireless communications and radar applications as well. The basic concept is to transmit, and receive a baseband impulse-like stream of very low power density and ultra-short duration pulses – typically a few tens of pico-seconds to a few nanoseconds. Such transmissions give rise to rich multipath diversity, low probability of detection, enhanced penetration capability, high user-capacity with time hopping (TH) codes, and potential spectrum compatibility with existing narrowband systems [1, 8].

These unique advantages of UWB transmissions are somewhat encumbered by stringent timing requirements. Timing offset estimation (TOE) is more challenging for UWB signals due to the strict power limitation, and the extremely short pulse duration. Conventional synchronization techniques based on pulse-rate sliding correlation are not only sub-optimum in the presence of dense multipath, but also very slow to converge, due to the prohibitively large number of fine bins (chips) to be searched over. There are limited works on rapid acquisition [2], [4], and tracking [6] techniques for UWB.

In this paper, we apply the cyclostationarity-based timing estimation principle to UWB transmissions in the presence of rich multipath. Cyclostationarity (CS) typically arises in narrowband systems when sampling at a fraction of the Nyquist rate [3]. Such a sampling rate however, is infeasible for UWB. We recognize that CS is naturally present in UWB signals due to the inherent pulse repetition across multiple frames comprising each symbol [6]. For acquisition, we derive a sliding cyclic correlation method that relies on frame-rate samples, while for tracking we develop a low-complexity CS-based algorithm that utilizes pulse-rate samples. Both schemes are non-data aided (a.k.a. blind).

The ensuing Section 2 outlines our system model and operating transceiver conditions. Section 3 derives an acquisition method based on the cyclic statistics induced directly from the pulse repetition pattern. Section 4 develops our tracking algorithms. Due to lack of space, the reader is referred to the journal version [7] for detailed proofs.

The following notations are used throughout: $\lfloor x \rfloor$ denotes integer-floor, $[x]_y := x - \lfloor x/y \rfloor$ denotes both integer and real-valued modulo operations with base y.

2. Modeling Preliminaries

In impulse radio multiple access, every information symbol is transmitted by repeating over N_f frames (each of duration T_f) an ultra short pulse $p(t)$ that has duration $T_p \ll T_f$. The pulse (a.k.a. monocycle) can have rectangular, triangular, or, typically Gaussian shape [8]. With T_p at the sub-nanosecond scale, $p(t)$ is UWB with bandwidth $B_s \approx 1/T_p$. The user of interest suppresses multiple access interference (MAI) with a pseudo-random TH code sequence $c(k) \in [0, N_c-1]$ that time-shifts the pulse positions at multiples of the chip duration (T_c) [8]. We will deal with UWB binary pulse amplitude modulation (PAM) [5], while generalizations to pulse position modulation are possible [7].

During the acquisition phase, we consider slow hopping by fixing the TH code for all frames within a symbol, but allowing it to change from symbol to symbol. Slow TH ensures multiple access without inducing excessive spikes in the power spectrum density of the transmitted signal. During the tracking phase however, we allow for fast hopping where the TH changes on a frame-by-frame basis but remains the same from symbol-to-symbol of the same user. Fast TH leads to smoother spectrum and can accommodate more users than slow TH. With information bearing PAM

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symbols $s(k)$ being *i.i.d.* with zero mean and variance σ_s^2, the transmitted pulse stream is:

$$u(t)=\sum_{k=0}^{\infty}s_k p(t-kT_f-c_kT_c), \tag{1}$$

where $s_k:=s(\lfloor k/N_f\rfloor)$, and $c_k:=c(\lfloor k/N_f\rfloor)$.

After multipath propagation, the received waveform is given by $r(t)=\sum_{l=0}^{L}\alpha_l u(t-\tau_l)+w(t)$, where $(L+1)$ is the total number of propagation paths, each with tap α_l, and delay τ_l satisfying $\tau_l<\tau_{l+1}$, $\forall l$. The channel is random and quasi-static, with $\{\alpha_l\}_{l=0}^L$ and $\{\tau_l\}_{l=0}^L$ remaining invariant within one symbol period, but possibly changing independently from symbol to symbol. Taps $\{\alpha_l\}_{l=0}^L$ are assumed zero-mean and uncorrelated for $|l_1-l_2|\gg 0$. Rich multipath is assumed, i.e., $\tau_{l+1}-\tau_l<2T_p, \forall l$, which is well justified for indoor propagation channels. The additive noise $w(t)$ is assumed wide-sense stationary complex process, but not necessarily white and/or Gaussian, as it consists of both ambient noise and MAI. Also, $w(t)$ is assumed independent of $s(k)$, $\{\alpha_l\}_{l=0}^L$ and $\{\tau_l\}_{l=0}^L$.

In TOE, the receiver cannot distinguish two time delays that are separated by multiple symbol durations, e.g., τ_0 and $\tau_0+kN_fT_f$. Thus, we confine our timing offset estimation (TOE) problem to be resolvable only within a symbol duration, and express the first arrival time as $\tau_0=N_\epsilon T_f+\epsilon$, where $N_\epsilon\in[0,N_f-1]$, and $\epsilon\in[0,T_f)$. Accordingly, other path delays can be described by $\tau_{l,0}:=\tau_l-\tau_0$. With these definitions, the received signal can be expressed as

$$r(t)=\sum_{l=0}^{L}\alpha_l u(t-(N_\epsilon T_f+\epsilon)-\tau_{l,0})+w(t). \tag{2}$$

Estimation of N_ϵ accomplishes *frame-level TOE* or *acquisition*, while that of ϵ enables *pulse-level TOE* or *tracking*.

A correlator-based receiver uses a frame-by-frame sliding correlation template $p(t)$ to yield the discrete-time samples $x(n)=\int_{nT_f}^{(n+1)T_f}p(t-nT_f)r(t)dt$. Let the pulse correlation be $R_p(\tau):=\int_0^{T_p}p(t)p(t-\tau)dt$. From (1) and (2), we obtain (neglecting the noise for brevity)

$$x(n)=\sum_{l=0}^{L}\sum_{k=0}^{\infty}\alpha_l s_k R_p((k+N_\epsilon-n)T_f+c_kT_c+\epsilon+\tau_{l,0}). \tag{3}$$

We will simplify (3) based on the fact that $R_p(\tau)$ is nonzero only for $\tau\in(-T_p,T_p)$, and by selecting $T_f-T_p>\tau_{L,0}$. If $k+N_\epsilon-n\geq 1$, then the argument of R_p in (3) exceeds T_p, since $c_kT_c+\epsilon+\tau_{l,0}$ is always positive, and $T_f>T_p$. Likewise, if $k+N_\epsilon-n\leq -3$, then the argument of R_p in (3) goes below $-T_p$, because $\max(c_kT_c+\epsilon+\tau_{l,0})=2T_f+\tau_{L,0}<3T_f-T_p$. Hence, the values of k and l contributing nonzero summands in $x(n)$ must satisfy:

$$k:\quad k+N_\epsilon-n=-q,\qquad q=0,1,2; \tag{4}$$

$$l:\quad -qT_f+(c_kT_c+\epsilon)+\tau_{l,0}\in(-T_p,T_p). \tag{5}$$

Using (4) and (5), we can re-write (3) as

$$x(n)=\sum_{q=0}^{2}g_q(n-N_\epsilon-q)s_{n-N_\epsilon-q}+w(n), \tag{6}$$

where $g_q(k):=\sum_{l=0}^{L}\alpha_l R_p(-qT_f+c_kT_c+\epsilon+\tau_{l,0})$ for $q=0,1,2$ denote the frame-rate equivalent channel taps, and $w(n):=\int_0^{T_p}p(t)w(t+nT_f)dt$ is the sampled noise. Depending on c_k and ϵ, we notice from (5) that for each q, only certain path(s) l contribute nonzero summands to $g_q(k)$. For $q\neq q'$, these paths, picked according to (5), are far apart; i.e., $|l-l'|\gg 0$. This implies that $E\{\alpha_l\alpha_{l'}\}=0$, which in turn proves that the correlation of $g_q(k)$ satisfies: $E\{g_q(k)g_{q'}(k+\nu)\}=R_{g,q}(\nu)\delta(q-q')$.

3. Frame-Level TOE for Acquisition

Let $R_x(n;\nu):=E\{x(n)x(n+\nu)\}$ denote the correlation of $x(n)$, and $R_w(\nu):=E\{w(n)w(n+\nu)\}$ the correlation of the stationary noise $w(n)$ in (6). It then follows by direct substitution from (6) that:

$$R_x(n;\nu)=\sum_{q=0}^{2}R_{g,q}(\nu)R_s(n-N_\epsilon-q;\nu)+R_w(\nu), \tag{7}$$

where $R_s(n;\nu):=E\{s_ns_{n+\nu}\}=\sigma_s^2$, when $\nu\in[0,N_f-1]-[n]_{N_f}$, and 0 otherwise. Because $[n+kN_f]_{N_f}=[n]_{N_f}$ for any integer k, we deduce that $R_s(n;\nu)$ (and thus $R_x(n;\nu)$) is periodic in n with period N_f. This establishes that s_k and $x(n)$ are cyclostationary processes.

Being periodic in n, $R_x(n;\nu)$ accepts a Fourier Series expansion, which gives rise to (the so-termed *cyclic* correlation) coefficients that are given by [c.f. (7)]:

$$\mathcal{R}_x(l;\nu):=\frac{1}{N_f}\sum_{n=0}^{N_f-1}R_x(n;\nu)e^{-j\frac{2\pi}{N_f}ln}\ . \tag{8}$$

Henceforth, we will rely on $\mathcal{R}_x(l;\nu)$ for $l\neq 0$ in order to suppress the stationary noise whose cyclic correlation is $\mathcal{R}_w(l;\nu)=R_w(\nu)\delta(l)$. Substituting (7) into (8), and using the periodicity of $R_x(n;\nu)$ to shift the summation limits in (8), we find that for $l\neq 0$,

$$\begin{aligned}\mathcal{R}_x(l;\nu)\\ &=\sum_{q=0}^{2}R_{g,q}(\nu)\left(\frac{1}{N_f}\sum_{n=N_\epsilon+q}^{N_\epsilon+q+N_f-1}R_s(n-N_\epsilon-q;\nu)e^{-j\frac{2\pi}{N_f}ln}\right)\\ &=\quad e^{-j\frac{2\pi}{N_f}lN_\epsilon}\left(\sum_{q=0}^{2}R_{g,q}(\nu)e^{-j\frac{2\pi}{N_f}lq}\right)\mathcal{R}_s(l;\nu)\ ,\end{aligned} \tag{9}$$

where $\mathcal{R}_s(l;\nu)$ is the cyclic correlation of s_k, defined similar to (8). We can express the latter in closed-form as [7]

$$\mathcal{R}_s(l;\nu)=-\frac{\sigma_s^2}{N_f}\frac{\sin(\pi l|\nu|/N_f)}{\sin(\pi l/N_f)}e^{j\frac{\pi}{N_f}l(\nu+1)}. \tag{10}$$

Based on our *frame-rate samples* $x(n)$, we can also estimate $\mathcal{R}_x(l;\nu)$ consistently via sample averaging:

$$\hat{\mathcal{R}}_x(l;\nu) = \frac{1}{N}\sum_{n=0}^{N-\nu-1} x(n)x(n+\nu)e^{-j\frac{2\pi}{N_f}ln}. \quad (11)$$

Relying on (11) and (10), we will pursue our estimation of N_ϵ using the normalized cyclic correlation $\bar{\mathcal{R}}_x(l;\nu) := \hat{\mathcal{R}}_x(l;\nu)/\mathcal{R}_s(l;\nu)$, which for $l \neq 0$ is given by [c.f. (9)]:

$$\bar{\mathcal{R}}_x(l;\nu) = e^{-j\frac{2\pi}{N_f}lN_\epsilon}\mathcal{R}_g(l;\nu)\ , \quad (12)$$

where $\mathcal{R}_g(l;\nu) := \sum_{q=0}^{2} R_{g,q}(\nu)\exp(-j2\pi lq/N_f)$ contains the unknown equivalent channel taps $\{R_{g,q}(\nu)\}_{q=0}^2$. Let $|\bar{\mathcal{R}}_x(l;\nu)|$ ($\bar{\theta}_x(l;\nu)$) and $|\mathcal{R}_g(l;\nu)|$ ($\theta_g(l;\nu)$) denote the amplitude (phase) of $\bar{\mathcal{R}}_x(l;\nu)$ and $\mathcal{R}_g(l;\nu)$, respectively. For each ν, three amplitude values, $\{|\bar{\mathcal{R}}_x(l;\nu)|\}_{l=1}^3$, suffice to determine one (out of four possible) spectrally equivalent triplet of channel taps $\{\bar{R}_{g,q}(\nu)\}_{q=0}^2$, using a spectral factorization algorithm. For improved resilience to noise, a spectrally equivalent triplet can be found also via nonlinear minimization of a cost function that performs quadratic amplitude matching as follows:

$$\{\bar{R}_{g,q}(\nu)\}_{q=0}^2 = \arg\min_{\{R_{g,q}(\nu)\}_{q=0}^2} \sum_{l=1}^{3}\left[|\bar{\mathcal{R}}_x(l;\nu)| - |\mathcal{R}_g(l;\nu)|\right]^2 \quad (13)$$

To specify the correct triplet $\{R_{g,q}(\nu)\}_{q=0}^2$, and also find N_ϵ, we resort to the phase of $\bar{\mathcal{R}}_x(l;\nu)$, which from (12) is given by:

$$\bar{\theta}_x(l;\nu) = -(2\pi/N_f)lN_\epsilon\ +\ \theta_g(l;\nu)\ . \quad (14)$$

Trying all four spectrally equivalent triplets of channel taps, $\{\bar{R}_{g,q}^{(i)}(\nu)\}_{i=1}^4$, we can estimate N_ϵ as:

$$\hat{N}_\epsilon = \arg\min_{N_\epsilon,\{R_{g,q}^{(i)}(\nu)\}_{i=1}^4} \sum_{l=1}^{4}\left[\bar{\theta}_x(l;\nu) + \frac{2\pi}{N_f}lN_\epsilon - \theta_g^{(i)}(l;\nu)\right]^2 \quad (15)$$

We have proved in [7] that four values of $\bar{\mathcal{R}}_x(l;\nu)$ indeed suffice to identify N_ϵ and $\{R_{g,q}(\nu)\}_{q=0}^2$ uniquely for each ν. This establishes:

Proposition 1 (Acquisition by cyclic correlation) *Timing can be acquired consistently using four nonzero lags of the cyclic correlation of the frame-rate sampled received sequence. An estimate can be obtained from (15) for each* $\nu \in [-(N_f-1), N_f-1]$, *possibly followed by averaging over* ν *to further enhance estimation accuracy.*

It is also possible to bypass estimation of $\{R_{g,q}(\nu)\}_{q=0}^2$, and form coarse N_ϵ estimates without the one-dimensional nonlinear search required by (15). As the resulting algorithms rely on the peaks of $R_x(n;\nu)$ or the phase $\theta_x(l;\nu)$, they are simpler; but they specify $\hat{N}_\epsilon$ with an ambiguity that can be as high as $2T_f$ [9]. On the other hand, the acquisition schemes in [9] collect energy over N_f frames, and thus improve estimation accuracy. Our $\hat{N}_\epsilon$ estimator can enjoy similar benefits, if instead of $x(n)$, we apply the results of this section to the cyclostationary process $y(n) := (1/N_f)\sum_{k=0}^{N_f-1} x(n+k)$. Additional SNR enhancement is possible, if instead of the monocycle template $p(t)$, our frame-rate sliding correlator employs a composite template $\bar{p}(t) := (1/N_p)\sum_{n_p=0}^{N_p-1} p(t-n_pT_p)$, that consists of $N_p := \lfloor T_f/T_p \rfloor$ shifted monocycles per frame.

4. Pulse-Level TOE for Tracking

During the tracking phase, we consider fast hopping by allowing the TH codes to change across frames, but repeating the hopping pattern from symbol to symbol; i.e., by setting $c_k = c([k]_{N_f})$ in (1). Fast TH may induce inter-frame interference within a symbol, but not necessarily inter-symbol interference, provided that we set the TH code of the last monocycle in each transmitted symbol to be zero, so that it does not hop into the next symbol.

Compensating $r(t)$ by $N_\epsilon T_f$ that we acquired in Section 3, we obtain from (2)

$$r_1(t) = r(t+N_\epsilon T_f) = \sum_{l=0}^{L}\alpha_l u(t-\epsilon-\tau_{l,0}) + w(t)$$
$$= \sum_{l=0}^{L}\alpha_l\sum_{k=0}^{\infty} s_k p(t-kT_f-c_kT_c-\epsilon-\tau_{l,0})+w(t). \quad (16)$$

With the acquisition of N_ϵ, the TH code c_k (that is always the same for the k-th frame of each symbol) is known to the receiver. This enables usage of time-hopping templates in the correlator.

Depending on the available N_ϵ and the unknown $\epsilon \in [0, T_f)$, $r_1(t)$ may entail one or two successive symbols. Specifically, if $N_\epsilon = 0$ and ϵ is within the first frame ($t \in nN_fT_f + [0, T_f]$) of the nth symbol, then two symbols, $s(n-1)$ and $s(n)$, contribute to $r_1(t)$. For all other frames of the nth symbol, only $s(n)$ contributes to $r_1(t)$. Applying pulse-by-pulse sliding correlation to $r_1(t)$ in one out of every N_f frames, we obtain the pulse-rate discrete-time samples $x_1(nN_f;m) := \int_{nN_fT_f+mT_p}^{nN_fT_f+(m+1)T_p} p(t-nT_f-c_0T_c-mT_p)r(t)dt$, for $m \in [0, N_p]$, where $N_p := \lfloor T_f/T_p \rfloor$. The correlation template in this interval is affected by $c(0)T_c$, since $c_{nN_f} = c(0), \forall n$. Skipping the noise in (16) for brevity, and using the pulse correlation R_p, we obtain

$$x_1(nN_f;m) = \sum_{l=0}^{L}\sum_{k=0}^{\infty}\alpha_l s_k$$
$$\cdot R_p((k-nN_f)T_f+(c_k-c_0)T_c-mT_p+\epsilon+\tau_{l,0}). \quad (17)$$

Because $(c_k - c_0)T_c \in [-T_f + T_c, T_f - T_c], \forall k$, when $(k-nN_f)T_f \notin (-3T_f, 2T_f)$, all the correlations in (17) are zero for any possible m, ϵ and $\tau_{l,0}$, due to the finite non-zero support of $p(t)$. Hence, the values for k and l that contribute non-zero summands in $x_1(nNf;m)$ must satisfy:

$$k:\ k-nN_f=-q, \qquad q=-1,0,1,2; \tag{18}$$
$$l:\ (c_k-c_0)T_c+\epsilon-mT_p-qT_f+\tau_{l,0}\in(-T_p,T_p). \tag{19}$$

We observe that the contributing l's are determined by c_k, ϵ, q, and the correlating pulse position m. Letting $\Delta_q(m) := (c_k-c_0)T_c+\epsilon-mT_p-qT_f+\tau_{l,0}$, and recalling that c_k is a fast TH code, we deduce from (18) that the c_k associated with $\Delta_q(m)$ is given by $c_k=c_{nN_f-q}=c_{-q}$, of which $c_{-1}=0$. Close examination of (19) reveals that, for a given $m\in[0,N_p]$, there only exist two possible q's that do not violate the constraint (19). In the following two cases, $\Delta_q(m)\notin(-T_p,T_p)$ occurs for the listed q values:

Case I. $\epsilon\geq(m+1)T_p$

q	$(c_{-q}-c_0)T_c$	$(\epsilon-mT_p)$	$-qT_f$	$\tau_{l,0}$	$\Delta_q(m)$
-1	$\geq -T_f+T_c$	$\geq T_p$	$+T_f$	≥ 0	$\geq T_c+T_p$
0	0	$\geq T_p$	0	≥ 0	$\geq T_p$

Case II. $\epsilon\leq mT_p$

q	$(c_{-q}-c_0)T_c$	$(\epsilon-mT_p)$	$-qT_f$	$\tau_{l,0}$	$\Delta_q(m)$
1	≤ 0	≤ 0	$-T_f$	T_f-T_p	$\leq -T_p$
2	$\leq T_f-T_c$	≤ 0	$-2T_f$	$\leq T_f-T_p$	$\leq -T_p-T_c$

Note that $\Delta_q(m)$ does not depend on the corresponding symbol nN_f. Defining $g_{1,q}(m) := \sum_{l=0}^{L}\alpha_l R_p(\Delta_q(m))$ for $q\in[-1,2]$, we can simplify (17) to:

$$\begin{aligned}&\text{If } mT_p<\epsilon, \text{then}\\ &x_1(nN_f;m)=s(n-1)\sum_{q=1}^{2}g_{1,q}(m)=s(n-1)\bar{g}_{-1}(m);\\ &\text{If } mT_p\geq\epsilon, \text{then}\\ &x_1(nN_f;m)=s(n)\textstyle\sum_{q=-1}^{0}g_{1,q}(m)=s(n)\bar{g}_0(m),\end{aligned} \tag{20}$$

where $\bar{g}_i(m) := \sum_{q=-2i-1}^{-2i}g_{1,q}(m)$ for $i=-1,0$. We observe that $\{g_{1,q}(m)\}_{q=-1}^{2}$, and hence $\{\bar{g}_i(m)\}_{i=-1}^{0}$, are fully specified by m and q, for a given TH pattern, and a fixed delay ϵ. Note from (20) that there is a symbol transition in the interval $((m_0-1)T_p, m_0T_p]$, where $m_0 := \lfloor\epsilon/T_p\rfloor$. Moreover, since $N_pT_p>T_f-T_p\geq\epsilon$, the last sample $x_1(nN_f;N_p)$ in each symbol period always includes $s(n)$. Subsequently, we develop several estimators for ϵ, based on this symbol transition property. These estimators are coarse in the sense that they rely on an estimate of m_0, which is quantized with pulse period levels. As a result, these coarse estimates of ϵ entail an ambiguity up to T_p, even when free of noise.

4.1. Tracking by Cross Correlation

Let $R_{2x}(m) := E\{x_1(nN_f;m)x_1(nN_f;N_p)\}$, for $l\in[0,N_p-1]$. From (20), we have:

$$\begin{aligned}&\text{When } m\in[0,m_0-1],\\ &\quad R_{2x}(m)=E\{s(n-1)s(n)\}E\{\bar{g}_{-1}(m)\bar{g}_0(N_p)\}=0;\\ &\text{When } m\in[m_0,N_p-1],\\ &\quad R_{2x}(m)=E\{s(n)s(n)\}E\{\bar{g}_0(m)\bar{g}_0(N_p)\}>0.\end{aligned} \tag{21}$$

In practice, $R_{2x}(m)$ can be estimated using noisy pulse-rate samples averaged over N symbols as follows:

$$\hat{R}_{2x}(m)=\frac{1}{N}\sum_{n=0}^{N-1}x_1(nN_f;m)x_1(nN_f;N_p). \tag{22}$$

To exploit the symbol transition property, we construct these decision statistics: $\lambda(m) := \sum_{i=0}^{m}R_{2x}(i)$, $m=0,1,\ldots N_p-1$. It follows from (21) that

$$\lambda(m)=\begin{cases}0, & m\leq m_0-1\\ \sum_{i=m_0}^{m}E\{\bar{g}_0(m)\bar{g}_0(N_p)\}>0, & m\geq m_0\end{cases} \tag{23}$$

As a result, we can find the critical point m_0 by energy detection on $\lambda(m)$, or, by counting the number of positive values in $\{\lambda(m)\}_{m=0}^{N_p=1}$:

$$m_0 = \arg\min_m\{m:\ \lambda(m)>0\} \tag{24}$$
$$\quad = N_p-\sum_{m=0}^{N_p-1}(\lambda(m)>0). \tag{25}$$

Proposition 2 (Coarse tracking by cross correlation) *Timing can be coarsely tracked by detecting the critical point of symbol transition from the cross-correlation of $x_1(nN_f;m)$ in the starting frames of a symbol period. An estimate can be obtained by $\epsilon=m_0T_p$, where m_0 can be estimated from either (24), or, (25). Such an estimator entails ambiguity up to T_p due to the quantization with pulse period levels.*

The performance of this estimator is not only heavily dependent on the SNR, but also sensitive to the noise correlation between pulses that are $(N_p-m_0)T_f$ apart. In (21), $\hat{R}_{2x}(m)$ is no longer zero when the noise components in $x(nN_f;N_p)$ and $\{x(nN_f;m)\}_{m=0}^{m_0-1}$ are correlated, being in close vicinity. In the ensuing subsection, we will develop a CS-based tracker that is robust to colored noise.

4.2. Tracking by Cyclic Correlation

Here we exploit the CS that is present in our pulse rate samples across different symbols: $\{\cdots x_1(nN_f;0)\cdots x_1(nN_f;N_p-1);x_1((n+1)N_f;0)\cdots x_1((n+1)N_f;N_p-1)\cdots\}$. Let $y(k) := x_1(\lfloor k/N_p\rfloor N_f;[k]_{N_p})$ denote this sequence. From (17), we can express $y(k)$ as

$$y(k)=\begin{cases}\bar{g}_{-1}([k]_{N_p})s(\lfloor k/N_p\rfloor-1), & [k]_{N_p}\in[0,m_0-1];\\ \bar{g}_0([k]_{N_p})s(\lfloor k/N_p\rfloor), & [k]_{N_p}\in[m_0,N_p-1].\end{cases} \tag{26}$$

The time-varying correlation $R_y(k;\nu) := E\{y(k)y(k+\nu)\}$ is given by (assume $\nu \geq 0$ w.l.o.g)

$$R_y(k;\nu) = \begin{cases} R_{g,11}(\nu) := \sigma_s^2 E\{\bar{g}_{-1}([k]_{N_p})\bar{g}_{-1}([k+\nu]_{N_p}), \\ \quad \text{for } k, k+\nu \in [nN_p, nN_p + m_0 - 1]; \\ R_{g,00}(\nu) := \sigma_s^2 E\{\bar{g}_0([k]_{N_p})\bar{g}_0([k+\nu]_{N_p}), \\ \quad \text{for } k, k+\nu \in [nN_p + m_0, (n+1)N_p - 1]; \\ R_{g,01}(\nu) := \sigma_s^2 E\{\bar{g}_0([k]_{N_p})\bar{g}_{-1}([k+\nu]_{N_p}), \\ \quad \text{for } k \in [nN_p + m_0, (n+1)N_p - 1], \\ \quad \text{and } k+\nu \in [(n+1)N_p, (n+1)N_p + m_0 - 1]; \\ 0, \qquad \text{otherwise}, \end{cases} \tag{27}$$

where n is any integer. Adding lN_p to k in (27) does not alter $R_y(k;\nu)$, simply because $[k + lN_p]_{N_p} = [k]_{N_p}$. Therefore, $R_y(k;\nu)$ is indeed periodic in k with period N_p.

The cyclic correlation of $y(k)$ is given by

$$\mathcal{R}_y(l;\nu) = \frac{1}{N_p}\sum_{k=0}^{N_p-1} R_y(k;\nu)e^{-j\frac{2\pi}{N_p}kl} \tag{28}$$

$$= \frac{1}{N_p}[\sum_{k=0}^{m_0-1} R_y(k;\nu)e^{-j\frac{2\pi}{N_p}kl} + \sum_{k=m_0}^{N_p-1} R_y(k;\nu)e^{-j\frac{2\pi}{N_p}kl}].$$

When $\nu = N_p - 1$, the only value of k that contributes non-zero summands in (29) is $k = m_0$, resulting in

$$\mathcal{R}_y(l;N_p-1) = \begin{cases} \frac{1}{N_p}R_{g,00}e^{-j\frac{2\pi}{N_p}lm_0}, & m_0 = 0; \\ \frac{1}{N_p}R_{g,01}e^{-j\frac{2\pi}{N_p}lm_0}, & m_0 \in [1, N_p-1]. \end{cases} \tag{29}$$

Any $m_0 \in [0, N_p - 1]$ can be retrieved from the phase $\angle\mathcal{R}_y(l;\nu)$, for $\nu = N_p - 1$. Hence, we can recover ϵ as

$$\epsilon \approx m_0 T_p = \frac{N_p}{2\pi l} \ \angle\mathcal{R}_y(l; N_p - 1). \tag{30}$$

Proposition 3 (Coarse tracking by cyclic correlation) *Timing can be coarsely tracked by the phase of the cyclic correlation of the pulse-rate samples collected at the first frame out of every N_f frames. An estimate can be obtained using (30), for $\nu = N_p - 1$. To avoid phase wrapping, l should be set to ± 1.*

In practice, $\mathcal{R}_y(l;\nu)$ can be estimated from noisy pulse-rate samples similar to (11). When $\nu = N_p - 1$, we have

$$\hat{\mathcal{R}}_y(l;\nu) = \frac{1}{NN_p}\sum_{n=0}^{N-1} x_1(nN_f;0)x_1(nN_f;N_p-1) + \frac{1}{NN_p}$$

$$\cdot \sum_{n=0}^{N-1}\sum_{m=1}^{N_p-1} x_1(nN_f;m)x_1((n+1)N_f;m-1)e^{-j\frac{2\pi}{N_p}l(nN_f+m)}$$

Our CS-based coarse tracker (30) uses pulse-rate samples. However, its complexity is relatively low compared to a conventional correlator, since it only samples at a fractional time ($1/N_f$) of the whole tracking phase. After coarse tracking, estimating ϵ at a fine scale with sub-pulse resolution could be possible via tracking methods that only require frame-rate sampling; see also [7] for a CS-based fine tracking algorithm.

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