Decoding and Equalization of Unknown Multipath Channels Based on Block Precoding and Transmit-Antenna Diversity*

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Abstract— **Transmit antenna diversity has been exploited recently to develop high-performance space-time coders and simple maximum-likelihood decoders for transmissions over known flat fading channels. Based on blocking and multiple transmit-antennas, this paper derives novel precoders and decoders that eliminate intersymbol interference and achieve transmit diversity gain in the unknown multipath channels. When unknown, channel status information is acquired blindly based on a deterministic variant of the constant-modulus algorithm. System performance is evaluated both analytically and with simulations.**

I. Introduction

Recently, space-time coding has gained much attention as an effective transmit diversity technique to combat fading in wireless communications. Space-time trellis codes were first proposed in [9] to achieve maximum diversity gain. However, for a fixed number of transmit antennas, their decoding complexity increases exponentially with the transmission rate. To reduce decoding complexity, space-time block codes with two transmit antennas were first introduced in [3] and later generalized to an arbitrary number of transmit antennas in [10]. An attractive property of space-time block codes is that maximum-likelihood decoding can be performed using only linear processing. For complex constellations, space-time coding with two transmit antennas is the only block coding that provides full diversity without loss of transmission rate [10].

Space-time codes were originally designed for known flat fading channels. Applications of space-time codes to dispersive channels were dealt with in [1] and [5] for OFDM systems assuming perfect channel knowledge at the receiver. In [4], channel estimation for space-time coded OFDM was implemented using pilot tones. However, it is important to remark that space-time decoding requires multi-channel status information at the receiver. Thus, the achievable diversity gain comes at the price of proportional increase in the amount of training, which incurs efficiency loss especially in a rapidly varying environment. This motivates looking for receivers with blind channel estimation capabilities.

Toward this objective, we propose in this paper a novel generalized space-time OFDM transceiver. We consider a system with two transmit antennas and one receive antenna. Relying on symbol blocking, the space-time block encoder of [3] is incorporated into a generalized OFDM transmitter to achieve transmit diversity in frequency-selective propagation. In addition to performance improvement, the decoding simplicity of the space-time block encoder of [3] is retained in our system, even when communicating over multipath channels. By exploiting the special structure of space-time block codes, we also develop a blind channel estimator based on a deterministic variant of the constant-modulus algorithm (CMA) [11]. Unlike [11], our space-time coded CMA is computationally simple.

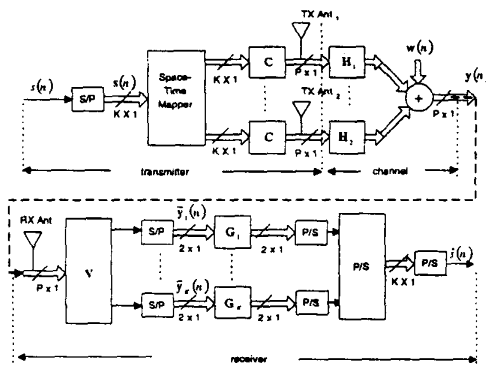

Fig. 1. Discrete-time equivalent baseband system model

The paper is organized as follows. In Section II we describe the system model, while in Section III we present our transceiver design. The blind channel estimation algorithm is developed in Section IV. Bit-error-rate (BER) performance is analyzed and extensive simulations are presented in Section V. Section VI concludes this paper.

II. System Modeling

Consider a wireless system with two transmit antennas at the base station and only one receive antenna at the mobile units. We will concentrate on transmissions from the base station to the mobile users, i.e., on the forward link. Fig. 1 depicts the chip-rate sampled discrete-time equivalent baseband system model. Similar to a conventional OFDM transmission scheme, the information sequence is parsed in blocks $\mathbf{s}(n) := (s(nK), \ldots, s(nK + K - 1))^T$ of size $K \times 1$. Our space-time encoder maps every two consecutive symbol blocks $\mathbf{s}(2n)$ and $\mathbf{s}(2n+1)$ to the following $2K \times 2$ matrix:

$$\begin{pmatrix} \mathbf{s}(2n) & -\mathbf{s}^*(2n+1) \\ \mathbf{s}(2n+1) & \mathbf{s}^*(2n) \end{pmatrix} \quad \begin{matrix} \rightarrow \text{ time} \\ \downarrow \text{ space} \end{matrix} \qquad (1)$$

whose columns are transmitted in successive time intervals with the upper and lower blocks sent through the first and second antenna element respectively. Note that with $K = 1$, no blocking takes place, and (1) then reduces to the standard space-time block codes with two transmit antennas [3]. Prior to transmission, the K-long symbol block in each antenna branch is mapped onto a P-long block

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with $P > K$ through a redundant precoder described by the tall $P \times K$ matrix $\mathbf{C}$. The precoder $\mathbf{C}$ will convert frequency-selective to flat fading channels and will enable us to exploit the transmit diversity built by the space-time mapper in (1). After parallel-to-serial (P/S) conversion and modulation (not shown in Fig. 1), the transmitted symbols propagate through channels with impulse response vectors $\mathbf{h}_i := (h_i(0), \ldots, h_i(L))^T, i = 1, 2$. Each channel's impulse response includes transmit-receive pulse shaping filters, multipath, and relative delay between the two antennas. We will assume that:

(a1) The frequency-selective channels are FIR and an upper bound L on their orders is assumed available; i.e., $h_i(l) = 0, \forall\, l \notin [0, L], \; i = 1, 2$.

Under (a1), each channel in Fig. 1 can be described by a $P \times P$ Toeplitz convolution matrix $\mathbf{H}_i$, with (k, l)th entry $h_i(k - l), \; i = 1, 2$. To avoid channel-induced interblock interference (IBI), we zero pad our transmitted blocks with L null symbols, which corresponds to setting:

(a2) The last $L = P - K$ rows of $\mathbf{C}$ equal to zero; i.e., with $c_k(p)$ denoting the (p, k)th entry of $\mathbf{C}$, we assume $c_k(p) = 0$ for $p = K + 1, \ldots, P$.

Thanks to (a2), we can focus at each received block separately. Alternately, we could have achieved IBI-free reception by inserting an L-long cyclic prefix to each transmitted block (similar to OFDM [4], [5]), and discarding it at the receiver. Either way, after S/P conversion (not shown in Fig. 1), each pair of two consecutive P-long received blocks $\mathbf{y}(2n)$ and $\mathbf{y}(2n+1)$, with $\mathbf{y}(n) := (y(nP), \ldots, y(nP + P - 1))^T$, is given by:

$$\begin{aligned} \mathbf{y}(2n) &= \mathbf{H}_1 \mathbf{C} \mathbf{s}(2n) + \mathbf{H}_2 \mathbf{C} \mathbf{s}(2n + 1) + \mathbf{w}(2n) \\ \mathbf{y}(2n + 1) &= -\mathbf{H}_1 \mathbf{C} \mathbf{s}^*(2n + 1) + \mathbf{H}_2 \mathbf{C} \mathbf{s}^*(2n) + \mathbf{w}(2n + 1), \end{aligned} \quad (2)$$

where $\mathbf{w}(n) := (w(nP), \ldots, w(nP + P - 1))^T$ denotes AWGN. Based on the data model (2), we will design next our transceivers to achieve transmit diversity gain in frequency selective channels.

III. Transceiver Design

In this section we design the precoding matrix $\mathbf{C}$ and the receiver matrices $\mathbf{V}$ and $\{\mathbf{G}_k\}_{k=1}^K$ (see Fig. 1) in order to recover $s(n)$ with transmit diversity gain. We consider that the FIR channels are unknown to the transmitter. Only for this section we will suppose that perfect channel status information (CSI) is available at the receiver.

Orthogonal space-time block codes were possible to design for flat fading channels [3], [10]. However, multipath manifests itself in convolving coded transmissions with FIR channels, and thereby destroys code orthogonality. By judicious design of $\mathbf{C}$, we will see though that code orthogonality can be retained by space-time coding and decoding in the frequency ($\mathcal{Z}$-) domain. Define the $\mathcal{Z}$-transforms: $Y(n; z) = \sum_{p=0}^{P-1} y(nP + p) z^{-p}$; $C_k(z) := \sum_{p=0}^{P-1} c_k(p + 1) z^{-p}$; $H_i(z) = \sum_{l=0}^{L} h_i(l) z^{-l}$; $W(n; z) = \sum_{p=0}^{P-1} w(nP + p) z^{-p}$. Let us focus on the $\mathbf{y}(2n)$ in (2) first, and $\mathcal{Z}$-transform its entries to obtain:

$$\begin{aligned} Y(2n; z) = \sum_{\mu=1}^{K} & [H_1(z) C_\mu(z) s(2nK + \mu - 1) \\ & + H_2(z) C_\mu(z) s((2n + 1)K + \mu - 1)] + W(2n; z). \end{aligned} \quad (3)$$

We seek code polynomials $C_\mu(z)$, $\mu = 1, \ldots, K$, which guarantee that there exist at least K distinct points $\{\rho_k\}_{k=1}^K$ such that for each k, $Y(2n; \rho_k)$ contains the contribution from the kth summand in (3), namely from $s(2nK + k - 1)$ and $s((2n + 1)K + k - 1)$ only, irrespective of $H_1(z)$ and $H_2(z)$. To achieve this goal, we design $C_\mu(\rho_k)$ to satisfy

$$C_\mu(\rho_k) = A \delta(k - \mu), \; \forall \mu, k \in [1, K], \quad (4)$$

where A is a constant chosen to impose the transmission power constraint. For fixed μ, (4) prescribes $C_\mu(z)$ at K points ρ_k. Thus, polynomials $C_\mu(z)$ that satisfy (4) should have degree $deg[C_\mu(z)] \geq K - 1$. When $deg[C_\mu(z)] = K - 1$, the polynomial $C_\mu(z)$ can be uniquely determined by Lagrange interpolation through the points $\{\rho_k\}_{k=1}^K$ as:

$$C_\mu(z) = A \, \frac{\prod_{k=1, k\neq\mu}^{K} (1 - \rho_k z^{-1})}{\prod_{k=1, k\neq\mu}^{K} (1 - \rho_k \rho_\mu^{-1})}, \;\; \mu \in [1, K]. \quad (5)$$

Based on (5) and taking into account the L guard zeros of (a2), the code length is $P = K + L$. Since we deal with transmissions of K-long blocks, the bandwidth efficiency is

$$\eta := \frac{K}{K + L}. \quad (6)$$

Note that for sufficiently large K, we have $\eta \simeq 1$; hence, bandwidth is not over expanded. Using the codes in (5) and evaluating (3) at $z = \rho_k$, we obtain $\forall k \in [1, K]$,

$$\begin{aligned} Y(2n; \rho_k) &= A H_1(\rho_k) s(2nK + k - 1) \\ &+ A H_2(\rho_k) s((2n + 1)K + k - 1) + W(2n; \rho_k). \end{aligned} \quad (7)$$

The same code polynomial design allows us to isolate from $Y(2n+1; \rho_k)$ the symbols $s^*((2n+1)K+k-1)$ and $s^*(2nK + k - 1)$; hence, similar to (7) we can write $\forall k \in [1, K]$,

$$\begin{aligned} Y(2n + 1; \rho_k) &= -A H_1(\rho_k) s^*((2n + 1)K + k - 1) \\ &+ A H_2(\rho_k) s^*(2nK + k - 1) + W(2n + 1; \rho_k). \end{aligned} \quad (8)$$

To express the $\mathcal{Z}$-transforms evaluated at $z = \rho_k$ in (7) and (8), let the $P \times 1$ Vandermonde vector $\boldsymbol{v}(\rho, P)$ built from the complex constant ρ as: $\boldsymbol{v}(\rho, P) := (1, \rho^{-1}, \ldots, \rho^{-(P-1)})^T$. With z replacing ρ, the $\mathcal{Z}$-transform of any $P \times 1$ vector $\mathbf{x}(n)$ can be represented by $X(n; z) = \boldsymbol{v}^T(z, P) \mathbf{x}(n)$. We can thus express $\{Y(n; \rho_k)\}_{k=1}^K$ as outputs of the receive filterbank described by the $K \times P$ Vandermonde matrix:

$$\mathbf{V} := (\boldsymbol{v}(\rho_1, P), \ldots, \boldsymbol{v}(\rho_K, P))^T. \quad (9)$$

Defining $\bar{\mathbf{y}}_k(n) := (Y(2n; \rho_k), \; Y^*(2n + 1; \rho_k))^T$, it follows from (7) and (8) that

$$\bar{\mathbf{y}}_k(n) = A \, \tilde{\mathbf{H}}_k \, \mathbf{s}_k(n) + \bar{\mathbf{w}}_k(n), \; \forall k \in [1, K], \quad (10)$$

where $\mathbf{s}_k(n) := (s(2nK+k-1),\ s((2n+1)K+k-1))^T$, $\bar{\mathbf{w}}_k(n) := (W(2n;\rho_k),\ W^*(2n+1;\rho_k))^T$ and

$$\bar{\mathbf{H}}_k := \begin{pmatrix} H_1(\rho_k) & H_2(\rho_k) \\ H_2^*(\rho_k) & -H_1^*(\rho_k) \end{pmatrix}. \quad (11)$$

Note that (10) reveals an important feature of our design: $2K$-long block precoded transmissions over frequency selective fading channels have been converted to K parallel 2-long block transmissions over flat fading channels, so that complexity of channel estimation and signal detection can be reduced considerably. Observe also that $\bar{\mathbf{H}}_k$ in (11) is a unitary matrix. Hence, the Zero-Forcing (ZF) equalizer for $\mathbf{s}_k(n)$ can be described by the 2×2 matrix:

$$\mathbf{G}_k := \bar{\mathbf{H}}_k^{\mathcal{H}}, \qquad \forall k \in [1,\ K], \quad (12)$$

and the decision vector $\mathbf{z}_k(n) = \mathbf{G}_k\bar{\mathbf{y}}_k(n)$ is given by:

$$\mathbf{z}_k(n) = A(|H_1(\rho_k)|^2 + |H_2(\rho_k)|^2)\mathbf{s}_k(n) + \boldsymbol{\eta}_k(n), \quad (13)$$

where $\boldsymbol{\eta}_k(n) := \bar{\mathbf{H}}_k^{\mathcal{H}}\bar{\mathbf{w}}_k(n)$. Eq. (13) implies that diversity gain of order two has been achieved for every $\mathbf{s}_k(n)$. After detecting $\hat{\mathbf{s}}_k(n)$ from $\mathbf{z}_k(n)$, the symbols $s(n)$ can be retrieved by the P/S conversion of $\hat{\mathbf{s}}_k(n)$ as shown in Fig. 1.

Let us now consider a particular choice of the points $\{\rho_k\}_{k=1}^K$. Specifically, let ρ_k's be chosen regularly spaced around the unit circle on the complex plane,

$$\rho_k = e^{j\frac{2\pi}{K}(k-1)}, \quad \forall k \in [1, K]. \quad (14)$$

It can be shown that the corresponding matrix $\mathbf{C}$ coincides with the IFFT matrix with $A = \sqrt{K}$. Accordingly, matrix $\mathbf{V}$ in (9) becomes the FFT matrix. Thus, matrix multiplication by $\mathbf{C}$ or $\mathbf{V}$ can be replaced by FFTs which have low computational complexity. Eq. (14) corresponds to OFDM-like transmissions but with the cyclic prefix replaced by trailing zeros. In contrast to the conventional OFDM systems of [4] and [5], where $\{\rho_k\}_{k=1}^K$ are fixed and equispaced around the unit circle, the flexibility to choose different ρ_k's in our system may offer some advantages. For example, as suggested in [8], we can periodically rotate (hop) the ρ_k's to ameliorate consistent fading effects caused by common channel zeros [6]. Henceforth, instead of assuming channel knowledge at the receiver, we will equip our receiver with blind channel estimation capabilities.

IV. Blind Channel Estimation

We pursue *blind* estimation of the two channels $\mathbf{h}_i, i = 1,2$, based on (a1), (a2) and the following assumptions:

(a3) Constant modulus (CM) modulation is applied;

(a4) Channels $\mathbf{h}_i$, $i = 1,2$, do not share common zeros, i.e., $H_1(z)$, $H_2(z)$ are coprime polynomials;

(a5) Block size is chosen to satisfy: $K \geq 6L+3$.

Given $\bar{\mathbf{y}}_k(n)$, channel estimation will be sought in two steps: First, we will exploit the CM property of transmitted symbols to develop a deterministic Constant Modulus Algorithm (CMA) that yields two estimates for two channel ratios $H_1^*(\rho_k)/H_2(\rho_k)$ and $-H_2^*(\rho_k)/H_1(\rho_k)$ for every k, with an ambiguity of knowing which estimates correspond to what ratio. Second, we will exploit the channel's finite support and develop an exhaustive search to resolve this ambiguity and estimate the two channels $\mathbf{h}_1$ and $\mathbf{h}_2$ jointly.

A. Estimating Channel Ratios Using CMA

Consider (10) and simplify the notation by absorbing A into $\bar{\mathbf{H}}_k$ and ρ_k in the subscript k, to rewrite it as:

$$\bar{\mathbf{y}}_k(n) := (Y_{1k}(n),\ Y_{2k}^*(n))^T = \bar{\mathbf{H}}_k\mathbf{s}_k(n) + \bar{\mathbf{w}}_k(n), \quad (15)$$

where $\mathbf{s}_k(n) := (s_{1k}(n),\ s_{2k}(n))^T$. Without loss of generality [1], we will fix the modulus $|s_{1k}(n)|^2 = 1$, $|s_{2k}(n)|^2 = 1$. Given $\bar{\mathbf{y}}_k(n)$, we seek an equalizer $\mathbf{G}_k$ such that $\hat{\mathbf{s}}_k(n) := \mathbf{G}_k\bar{\mathbf{y}}_k(n)$ has entries with unit constant modulus. Our *deterministic* CM equalization is equivalent to a generalized eigenvalue problem which can be solved by the analytical CMA (ACMA) of [11]. However, we develop here a simpler algorithm taking advantage of the specific structure of $\bar{\mathbf{H}}_k$.

We will first consider blind channel estimation at sufficiently high SNR, where the noise $\bar{\mathbf{w}}_k(n)$ can be neglected (noise effects will be tested in the simulations). Because $\bar{\mathbf{H}}_k$ is unitary and $\mathbf{G}_k\bar{\mathbf{H}}_k = \mathbf{I}$, we look for an equalizer:

$$\mathbf{G}_k := \begin{pmatrix} g_{1k} & g_{2k} \\ g_{2k}^* & -g_{1k}^* \end{pmatrix}. \quad (16)$$

Writing $\hat{\mathbf{s}}_k(n) := [\hat{s}_{1k}(n),\ \hat{s}_{2k}(n)]^T = \mathbf{G}_k\ (Y_{1k}(n), Y_{2k}^*(n))^T$ component-wise, and imposing: $|\hat{s}_{1k}(n)|^2 = 1$, $|\hat{s}_{2k}(n)|^2 = 1$, we arrive at $\bar{\mathbf{Y}}_k(n)\mathbf{g}_k = (1,\ 1)^T$, where

$$\mathbf{g}_k := \begin{pmatrix} g_{1k}g_{1k}^* \\ g_{2k}g_{2k}^* \\ g_{1k}g_{2k}^* \\ g_{1k}^*g_{2k} \end{pmatrix}, \quad (17)$$

and $\bar{\mathbf{Y}}_k(n)$ is a 2×4 matrix consisting of $Y_{1k}(n)$ and $Y_{2k}(n)$. Next, we stack N blocks of data $\{\bar{\mathbf{y}}_k(n)\}_{n=0}^{N-1}$ and concatenate $\bar{\mathbf{Y}}_k(n)\mathbf{g}_k$ to arrive at:

$$\bar{\mathbf{Y}}_k\mathbf{g}_k = (1,\ldots,1)_{2N\times 1}^T, \quad (18)$$

where $\bar{\mathbf{Y}}_k := (\bar{\mathbf{Y}}_k^T(0),\ldots,\bar{\mathbf{Y}}_k^T(N-1))^T$. If the $2N\times 4$ matrix $\bar{\mathbf{Y}}_k$ had full column rank, $\bar{\mathbf{Y}}_k^{\dagger}$ would have yielded a unique solution of $\mathbf{g}_k$ from (18). Unfortunately, the maximum column rank of $\bar{\mathbf{Y}}_k$ is only three [11]. Nevertheless, the solution of (18) can still be sought in the form $\mathbf{g}_k = \mathbf{g}_{0k} + \lambda\mathbf{g}_{1k}$, where $\mathbf{g}_{0k}$ is a particular solution of (18) and $\mathbf{g}_{1k}$ spans the one-dimensional kernel of $\bar{\mathbf{Y}}_k$. With $g_k(i)$ denoting the ith entry of $\mathbf{g}_k$, λ can be determined by noting that in (17) we must have $g_k(1)g_k(2) = g_k(3)g_k(4)$. Because solving the latter leads to a second-order equation in λ, we end up with two possible solutions, $\tilde{\mathbf{g}}_k$ and $\tilde{\mathbf{g}}_k'$, which correspond to:

$$\tilde{\mathbf{G}}_k := \begin{pmatrix} \tilde{g}_{1k} & \tilde{g}_{2k} \\ \tilde{g}_{2k}^* & -\tilde{g}_{1k}^* \end{pmatrix} = \frac{1}{\sum_{i=1}^2 |H_{ik}|^2}\begin{pmatrix} H_1^*(\rho_k) & H_2(\rho_k) \\ H_2^*(\rho_k) & -H_1(\rho_k) \end{pmatrix}$$
$$\tilde{\mathbf{G}}_k' := \begin{pmatrix} \tilde{g}_{1k}' & \tilde{g}_{2k}' \\ \tilde{g}_{2k}'^* & -\tilde{g}_{1k}'^* \end{pmatrix} = \frac{1}{\sum_{i=1}^2 |H_{ik}|^2}\begin{pmatrix} H_2^*(\rho_k) & -H_1(\rho_k) \\ H_1^*(\rho_k) & H_k(\rho_k) \end{pmatrix} \quad (19)$$

[1] In this paper, we deal with complex modulations only. Nonetheless, the extension to real modulations, e.g., BPSK, is straightforward and the constraints to be imposed are $s_{1k}^2(n) = 1$ and $s_{2k}^2(n) = 1$.

Since we can not distinguish between $\tilde{\mathbf{g}}_k$ and $\tilde{\mathbf{g}}'_k$, it follows that solving (18) leads to an ambiguity in choosing between the two possible equalizers $\tilde{\mathbf{G}}_k$ and $\tilde{\mathbf{G}}'_k$ (and thus the two channels) for every k. According to (a5), we can have at least $6L+3$ solution pairs $\{\tilde{\mathbf{g}}_k, \tilde{\mathbf{g}}'_k\}_{k=1}^{6L+3}$. But (a1) implies that $H_1(\rho_k) = 0$ on at most L values of k, and likewise for $H_2(\rho_k)$. Hence, we can find at least $6L+3-2L = 4L+3$ solution pairs $\{\tilde{\mathbf{g}}_k, \tilde{\mathbf{g}}'_k\}_{k=1}^{4L+3}$ for which both $H_1(\rho_k)$ and $H_2(\rho_k)$ are non-zero. For these k's we can define the ratio: $r(k) = g_k(3)/g_k(2)$, and use (19) to infer that:

$$r(k) = \begin{cases} \bar{r}(k) := H_1^*(\rho_k)/H_2(\rho_k) & \text{if } \mathbf{g}_k = \tilde{\mathbf{g}}_k, \\ \bar{r}'(k) := -H_2^*(\rho_k)/H_1(\rho_k) & \text{if } \mathbf{g}_k = \tilde{\mathbf{g}}'_k \end{cases}. \quad (20)$$

Now the ambiguity between $\tilde{\mathbf{g}}_k$ and $\tilde{\mathbf{g}}'_k$ in solution pairs $\{\tilde{\mathbf{g}}_k, \tilde{\mathbf{g}}'_k\}_{k=1}^{4L+3}$ translates to the ambiguity between $\bar{r}(k)$ and $\bar{r}'(k)$ for every $r(k)$. To resolve this ambiguity, one approach is to estimate one channel by training, and retrieve the other channel by inspection from (20). Because only training for one channel is needed, such a "partially trained" CMA saves 50% overhead. Note that so far, we do not require (a4) to obtain channel ratios and further estimate the two channels if partial training is applied. Looking for a full blind channel estimator, we resort next to an exhaustive search to resolve the ambiguity between $r(k)$ and $r'(k)$ and estimate the two channels blindly.

B. Resolving Channel Ambiguity

Recalling that for each $k \in [1, 4L+3]$, $r(k)$ can be either $\bar{r}(k)$ or $\bar{r}'(k)$, we infer that there are 2^{4L+3} possible collections of $r(k)$'s. With i_c denoting collection-index, we represent each of them with the $(4L+3) \times 1$ vector $\mathbf{r}_{i_c} := (r_{i_c}(1), \ldots, r_{i_c}(4L+3))^T$, and the entire collection with the set $\mathcal{R} = \{\mathbf{r}_{i_c}, i_c = 1, \ldots, 2^{4L+3}\}$. Because each entry of $\mathbf{r}_{i_c}$ comes either from $\bar{r}(k)$ or from $\bar{r}'(k)$, the $4L+3$ entries of every $\mathbf{r}_{i_c}$ can be divided into two groups, namely the $\bar{r}$ group that contains $\bar{r}(k)$'s and the $\bar{r}'$ group that consists of $\bar{r}'(k)$'s. Thinking in terms of a coin-flipping experiment, we term the entries in each group as "same-side" entries. To resolve channel ambiguity, we will use the following lemma:

Lemma 1: *Under (a1)-(a5), $2L+2$ same-side entries of any $\mathbf{r}_{i_c}$, denoted by $\{r_{i_c}(k_j), 1 \le k_j \le 4L+3\}_{j=1}^{2L+2}$, enable identifiability of the two channels (within a scalar ambiguity) either as $\mathbf{h}_{12} := (\mathbf{h}_1^{\mathcal{H}}, \mathbf{h}_2^T)^T$, or, as $\mathbf{h}_{21} := (-\mathbf{h}_2^{\mathcal{H}}, \mathbf{h}_1^T)^T$. Using a common notation $(\mathbf{b}_{i_c}^{\mathcal{H}}, \mathbf{a}_{i_c}^T)^T$, either $\mathbf{h}_{12}$ or $\mathbf{h}_{21}$ can be found by solving for the eigenvector corresponding to the minimum eigenvalue of the $(2L+2) \times (2L+2)$ matrix Θ_{i_c}:*

$$\Theta_{i_c} := \begin{pmatrix} \boldsymbol{v}^T(\rho_{k_1}^*, L+1) & -r_{i_c}(k_1)\boldsymbol{v}^T(\rho_{k_1}, L+1) \\ \vdots & \vdots \\ \boldsymbol{v}^T(\rho_{k_{2L+2}}^*, L+1) & -r_{i_c}(k_{2L+2})\boldsymbol{v}^T(\rho_{k_{2L+2}}, L+1) \end{pmatrix}$$

Because each $\mathbf{r}_{i_c}$ has $4L+3$ entries, we infer that $2L+2$ same-side entries can always be found. However, because an ambiguity appears for every k, we do not know where the $2L+2$ same-side entries are. To locate these $2L+2$ same-side entries, we resort to an exhaustive search following these steps:

s1) For each $i_c \in [1, 2^{4L+3}]$, use $\{r_{i_c}(k)\}_{k=1}^{2L+2}$ and obtain $(\mathbf{b}_{i_c}^{\mathcal{H}}, \mathbf{a}_{i_c}^T)^T$ (with k_j replaced by j for $j = 1, \ldots, 2L+2$ in Θ_{i_c}), as in Lemma 1.

s2) Use $(\mathbf{b}_{i_c}, \mathbf{a}_{i_c})$ from s1) to form $\forall k \in [1, 4L+3]$ the difference:

$$\Delta_{i_c}(k) := \boldsymbol{v}^T(\rho_k^*, L+1)\mathbf{b}_{i_c}^* - r_{i_c}(k)\boldsymbol{v}^T(\rho_k, L+1)\mathbf{a}_{i_c}. \quad (21)$$

s3) Select the index: $\bar{i}_c := \arg\min_{i_c} \sum_{k=1}^{4L+3} |\Delta_{i_c}(k)|^2$.

The two possible solutions for our channels are either

$$\begin{pmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \end{pmatrix} = \alpha \begin{pmatrix} \mathbf{b}_{\bar{i}_c} \\ \mathbf{a}_{\bar{i}_c} \end{pmatrix}, \quad \text{or,} \quad \begin{pmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \end{pmatrix} = \alpha \begin{pmatrix} \mathbf{a}_{\bar{i}_c}^* \\ -\mathbf{b}_{\bar{i}_c}^* \end{pmatrix} \quad (22)$$

where α denotes a scalar complex constant.

We show in [6] that the channel estimates in (22) are unique, and their mean-square error are close to Cramér-Rao Bound at high SNR. Having reduced the ambiguity to choosing between these two solutions and resolving the scalar ambiguity, enables (almost blind) channel estimation using the received data and two pilot tones only. Specifically, with two pilot tones we can estimate $H_1(\rho_1)$ and $H_2(\rho_1)$ from which the two channels can be identified uniquely.

The computational complexity of our fully blind channel estimation algorithm is relatively high. The CMA requires an SVD of size $2N \times 4$ for every k, which is much simpler than the ACMA in [11], while the exhaustive search involves 2^{2L+2} EVDs of size $(2L+2) \times (2L+2)$. Fortunately, L is small in typical applications and the smallest null vector can be computed online [2]. Note also that unlike existing statistical CMA variants that require long data sets, our space-time coded deterministic CMA is data-efficient and does not impose any input whiteness assumption.

V. Analytical and Simulated Performance

When the noise $w(n)$ is AGN, theoretical BER evaluation is possible for a given constellation if we adopt a ZF equalizer. Starting from (13), we can compute the covariance matrix of $\boldsymbol{\eta}_k(n)$ as:

$$\mathbf{R}_{\eta_k} = (|H_1(\rho_k)|^2 + |H_2(\rho_k)|^2)\boldsymbol{v}^{\mathcal{H}}(\rho_k, P)\boldsymbol{v}(\rho_k, P)\sigma_w^2\mathbf{I}, \quad (23)$$

and derive the BER assuming, e.g., a QPSK modulation. Our figure of merit is the average BER, defined as $\bar{P}_e = (2K)^{-1}\sum_{k=1}^{K}\sum_{i=1}^{2}\bar{P}_{i,k}$, where $\bar{P}_{1,k}$ and $\bar{P}_{2,k}$ denote the BERs for the sequences $s(2nK+k-1)$ and $s((2n+1)K+k-1)$, respectively. It follows from (13) and (23) that

$$\bar{P}_e = \frac{1}{K}\sum_{k=1}^{K}\mathcal{Q}\left(\sqrt{\frac{A^2E_b(|H_1(\rho_k)|^2 + |H_2(\rho_k)|^2)}{N_0\boldsymbol{v}^{\mathcal{H}}(\rho_k, P)\boldsymbol{v}(\rho_k, P)}}\right), \quad (24)$$

where $\mathcal{Q}(\cdot)$ denotes the $\mathcal{Q}$-function and $2E_b/N_0$ denotes bit SNR. Note that because each symbol is transmitted twice, we divide the transmit power by two for each transmit antenna to obtain (24). We next resort to simulations to test performance and reveal salient features of our design.

In all simulations, QPSK modulation was employed. The channels were assumed to be Raleigh faded and $\{\rho_k\}_{k=1}^{K}$

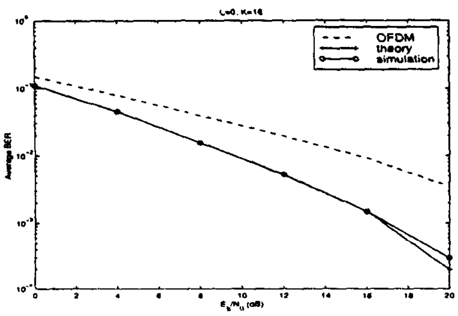

Fig. 2. Flat fading channels

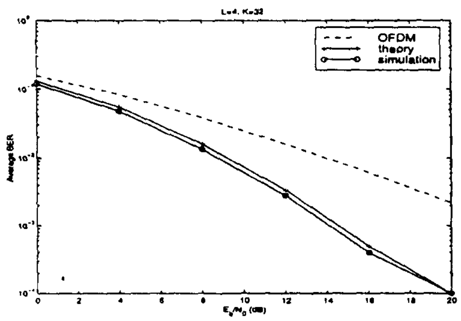

Fig. 3. Frequency selective channels

were chosen as in (14). All curves were averaged over 200 independent channel realizations.

Example 1 (performance comparison with OFDM): Fig. 2 compares the BER of our proposed transceivers with conventional (single transmit antenna) OFDM, assuming that two channels are flat and known to the receiver. In OFDM, 16 subcarriers are used, and correspondingly in our system, we choose the block length $K = 16$. Fig. 2 shows that our design outperforms OFDM significantly. For frequency selective channels of order $L = 4$, we choose 32 subcarriers for OFDM and block parameter $K = 32$ for our system. The result is shown in Fig. 3, where the performance of our system is seen to be much better than OFDM. The conventional OFDM transmits each symbol through a single fading channel. In contrast, our proposed system transmits each symbol twice through two different fading channels. Eq. (13) shows that the equivalent channel gain for each symbol is the sum of the squares of two different channels which is more reliable than a single channel. This explains why the proposed system outperforms the OFDM.

Example 2 (estimation of unknown channels): To simulate the performance of our blind channel estimation algorithm in Section IV, we choose $L = 2$ and $K = 16$. For estimating channel ratios, $N = 64$ received blocks are used for each k. The performance of our system with estimated channels is shown against that with perfect channel status information. The results are shown in Fig. 4 where we observe that the blind method entails a small penalty ($<$ 2dB).

VI. Conclusions

In this paper we designed novel space-time transceivers suitable for multipath channels. Relying on symbol blocking, space-time block codes designed for flat fading channels were extended to frequency-selective channels. By exploiting the specific structure of space-time block coding, a blind channel estimation algorithm was also developed. In addition to simplicity, numerical simulations demonstrated superior performance of the proposed receiver over competing alternatives.

Because symbol recovery and blind channel estimation in this paper depend on channel zero locations, we currently investigate space-time coded transceiver designs irrespective of the channel nulls. Preliminary results relying on symbol blocking and long codes will be reported in [7].

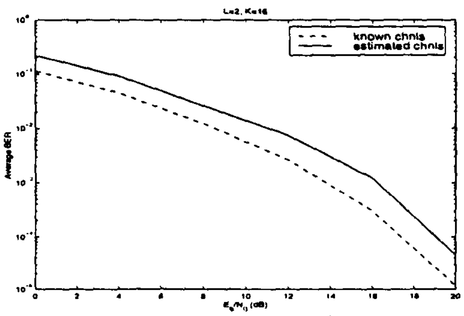

Fig. 4. Known vs. estimated channels

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