Turbo Demodulation of Zero-Padded OFDM Transmissions

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Abstract

We extend in this paper the turbo-demodulation procedure proposed in [5, 6] to OFDM systems with cyclic prefix (CP-OFDM) or Zero-Padding (ZP-OFDM). The extensions account for the noise color introduced by linear equalization. Resorting to realistic simulations, we also show that the turbo-demodulation procedure used with set partitioned labeling can significantly outperform non iterative decoding with Gray labeling. We also show that the proposed turbo demodulation scheme increases the performance gap between ZP and CP-OFDM relative to non-iterative decoding because it amplifies the performance gain brought by the guaranteed symbol recovery of ZP-OFDM.

1. Introduction

In modern wireless systems, bit interleaving is preferred over symbol interleaving because it offers improved performance when communicating over Rayleigh channels [3]. The resulting Bit Interleaved Coded Modulation (BICM) scheme [3] includes channel coding, bit-interleaving, and bit to complex symbol mapping stages that are performed separately. With large constellation sizes (e.g. 16- or 64-QAM), it has been shown that optimal BICM decoding is difficult [6, 7] . The reason is that several encoded bits are assigned to the in-phase and quadrature components of the constellation which makes Maximum Likelihood (ML) decoding computationally not feasible. This results in sub-optimal performance and an iterative procedure has been proposed in [5, 6] to reduce the degradation in the case of frequency-flat channels with Additive White Noise (AWN).

It is well known that CP-OFDM offers a simple way to convert frequency-selective Finite Impulse Response (FIR) channels into flat faded subchannels with AWN. Thus, the aforementioned iterative procedure can be applied to most existing OFDM systems implementing BICM at the transmitter. As reported in [6, 7], this can result in significant performance improvement.

Recently, it was proposed to replace the CP by Zero-Padding (ZP) [9]. Unlike CP-OFDM, the resulting ZP-OFDM transmitter guarantees symbol recovery and assures FIR equalization of FIR channels regardless of the channel zero locations. The price paid is increased receiver complexity since the FFT-based equalization of CP-OFDM is replaced by FIR filtering if MMSE equalization is performed at the receiver. However, it is possible to trade-off complexity with performance by using sub-optimal equalizers. Sufficient sacrifice in complexity results in improved performance as reported in [8] in which only conventional (non-iterative) decoding has been considered.

We propose in this paper to extend the turbo-demodulation algorithm to ZP-OFDM. The problem is that the equalizers for ZP-OFDM color the noise [8], so the BICM decoding which relies on the AWN assumption has to be modified. We here detail the required approximations and modifications. Simulations capitalizing on the constellation design rules of [5] illustrate the gain that can be obtained by using iterative demodulation for ZP-OFDM. Moreover, comparisons show that using ZP-OFDM rather than CP-OFDM for the transmitter allows one to reduce the number of demodulation iterations and hence complexity. In spite of the slight increase in equalization complexity, the overall complexity of ZP-OFDM receivers with turbo demodulation can be smaller than that of CP-OFDM ones for equivalent or higher levels of performance.

The rest of this paper is organized as follows: Section 2 reviews the iterative demodulation procedure; Section 3 briefly presents the ZP-OFDM transmitter and extends the iterative demodulation procedure to it. Section 4 presents simulations and comparisons while conclusions are drawn in Section 5.

2. Turbo-demodulation for BICM

In this paper, lower case letters are used to represent bits or complex symbols. Lower case boldface letters denote sequences or vectors of either bits or symbols. Upper case boldface letters are reserved for matrices.

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Figure 5 depicts the discrete time block diagram of a bit-interleaved transmission system in the simple case of a rate $R = 1/2$ convolutional code and a 16QAM modulation. The extension to other coding rates, coherent modulations (e.g., PSK, APSK and QAM modulations of any order) and other kinds of codes (e.g., block codes) for which soft decoding algorithms exist, is straightforward. The modulator is composed of three sub-blocks which operates independently of the others: a convolutional encoder, an interleaver and a bit to symbol mapper.

Each information bit b_i is first encoded into 2 bits c_i^0 and c_i^1 which are interleaved in order to gain resilience against error bursts. The interleaved bits are then gathered in subsequences of 4 bits: $d_k^1, \ldots, d_k^4$ which are mapped onto the complex symbols s_k according to a given labeling map $\mathcal{S}$ (Figure 3 provides two labeling examples). Note that different subscripts i and k, are used in order to highlight that several encoded bits c_i^l correspond to one complex symbol s_k. The resulting complex symbols s_k are then sent through the channel where they are distorted and received in AWN. For frequency flat Rayleigh fading channels with AWN, the received signal is given by: $y_k = h_k s_k(d_k^1, \ldots, d_k^4) + n_k$ where the channel coefficients h_k are independent and identically distributed (i.i.d.) random scalar complex gains drawn from a complex Gaussian distribution and the noise coefficients n_k are scalar i.i.d. complex Gaussian with zero-mean and variance σ_n^2.

Based on this transmission model, the receiver has to estimate the sequence of transmitted bits $\mathbf{b}$ from the sequence of received symbols $\mathbf{y}$ so as to minimize a given criterion (e.g., Bit Error Rate (BER)). Towards this goal, one has to maximize $P(\mathbf{y}|\mathbf{b})$. In practice, one works with the encoded bits since there is an one to one correspondence between information sequences and codewords. Thus, one looks for the ML interleaved codeword $\hat{\mathbf{d}}$ given by:

$$\begin{aligned}\hat{\mathbf{d}} &= \underset{\mathbf{d}\in\mathcal{D}}{\operatorname{argmax}}\, P(\mathbf{y}|\mathbf{d}) \\ &= \underset{\mathbf{d}\in\mathcal{D}}{\operatorname{argmax}} \prod_k P\left(y_k | s_k(d_k^1, \ldots, d_k^4)\right) \\ &\approx \underset{\mathbf{d}\in\mathcal{D}}{\operatorname{argmax}} \prod_k \prod_{l=1}^{4} P(d_k^l | y_k)\end{aligned}$$

where $\mathcal{D}$ stands for the set of interleaved codewords, and the last approximation allows the bit de-interleaving to be performed. The bit metrics $P(d_k^l|y_k)$ can be computed using Baye's rule as:

$$P(d_k^1|y_k) = \sum_{d_k^2, d_k^3, d_k^4 \in \{0,1\}} P(d_k^1, \ldots, d_k^4 | y_k)$$

$$= \sum_{d_k^2, d_k^3, d_k^4 \in \{0,1\}} \frac{P(y_k|s(d_k^1, \ldots, d_k^4))P(d_k^1, \ldots, d_k^4)}{P(y_k)}$$

With large interleaver sizes, $P(d_k^1, \ldots, d_k^4) \approx P(d_k^1) \ldots P(d_k^4)$ which enables computation of the bit metrics. Conventional non-iterative procedures do not take into account the fact that bits d_k^l are encoded bits and that some combinations of bits for various k and l are prohibited by the encoder. In this case, one makes no assumption on the a priori probabilities of the encoded bits which are assumed to take values 0 and 1 with the same probability. Thus, the bit metrics are given by:

$$P(d_k^1|y_k) \propto \sum_{d_k^2, \ldots, d_k^4 \in \{0,1\}} e^{-\frac{1}{2\sigma^2}|y_k - h_k s_k(d_k^1, \ldots, d_k^4)|^2}.$$

At this point, an estimate of the information bits can be obtained from the bit metrics by applying either the Viterbi or the BCJR algorithm [1].

The turbo-demodulation algorithm improves the estimate of the sequence of bits by iteratively taking into account the code structure in the inverse mapping process. For this, the a posteriori information obtained at the decoder output is used as an a priori information on the encoded bits in the inverse mapping sub-block. The corresponding demodulation scheme is depicted in Figure 6 and is summarized below:

1. initialize the *a priori* information on the bits to be uniform.

2. perform the symbol to bit inverse mapping by taking into account the actual *a priori* information on the encoded bits (sub-block 1).

3. de-interleave the resulting bit probabilities to get the probabilities on the encoded bits $P(c_i^l|y_{\pi_i^l})$ where $y_{\pi_i^l}$ is the complex symbol carrying c_i^l. Then feed them to the decoding algorithm (sub-block 2).

4. use the SISO decoding algorithm [2] to obtain: **i)** the a posteriori probability (APP) of the information bits $P^{APP}(b_i|\mathbf{y})$ (output 3a of sub-block 3); and **ii)** the extrinsic probabilities of the encoded bits which are defined as: $\gamma(d_k^l) \propto P^{\text{APP}}(d_k^l|\mathbf{y})/P(d_l^k|y_k)$ (output 3b).

5. re-interleave the extrinsic probabilities in order to use them as *a priori* information $\gamma(c_i^l)$ in sub-block 1 (sub-block 4) to improve the inverse mapping.

6. iterate between steps 2 through 5 as previously described.

7. after a given number of iterations, compute an estimate of the information bits $\hat{b}_i$ from the a posteriori probabilities $P^{APP}(b_i|\mathbf{y})$ (sub-block 5).

Next section extends this procedure to ZP-OFDM.

3. Application to ZP-OFDM systems

With CP-OFDM, it is well known that the effect of channel propagation simplifies to a simple scalar multiplication of the symbol sent on each subcarrier by the corresponding channel attenuation and the addition of AWN. Thus, the previous method applies straightforwardly to CP-OFDM as reported in [6, 7] and results in significant performance improvement if the interleaver and constellation design rules of [5] are applied. Since symbol recovery is not guaranteed when some subcarriers are hit by channel nulls, it has been proposed [9] to replace the CP by ZP which ensures symbol recovery and leads to improved performance [8]. Up to now, only non-iterative decoding has been considered and we extend in what follows the turbo-demodulation procedure to ZP-OFDM.

Figure 7 depicts the baseband discrete-time model of ZP-OFDM. The $M \times 1$ digital input $\bar{\mathbf{s}}_M(i)$ is first modulated by the IFFT matrix $\mathbf{F}_M^{\mathcal{H}}$ with entries $M^{-1/2}\exp\{j2\pi mk/M\}$. Then L trailing zeroes are padded at the end of the resulting vector $\mathbf{s}_M(i)$. The corresponding $P \times 1$ transmitted vector $\mathbf{s}_{\text{ZP}}(i) = \mathbf{F}_{\text{ZP}}^{\mathcal{H}}\bar{\mathbf{s}}_M(i)$, where $\mathbf{F}_{\text{ZP}}^{\mathcal{H}} = [\mathbf{F}_M \; \mathbf{0}]^{\mathcal{H}}$, is then serialized and transmitted through the Lth-order FIR channel with impulse response $h_l = 0, \forall l \notin [0, L]$. The all-zero $L \times M$ matrix $\mathbf{0}$ eliminates inter-block interference. Let $\mathbf{H} = [\mathbf{H}_0 \, , \mathbf{H}_{\text{ZP}}]$ denote a partition of the $P \times P$ convolution matrix $(\mathbf{H})_{ij} = h_{i-j}$ with $\mathbf{H}_0$ and $\mathbf{H}_{\text{ZP}}$ constituting its first M and last L columns, respectively. The received $P \times 1$ vector is then:

$$\mathbf{x}_P(i) = \mathbf{H}\mathbf{F}_{\text{ZP}}^{\mathcal{H}}\bar{\mathbf{s}}_M(i) + \mathbf{n}_P(i) = \mathbf{H}_0\mathbf{F}_M^{\mathcal{H}}\bar{\mathbf{s}}_M(i) + \mathbf{n}_P(i)$$

where $\mathbf{n}_P(i)$ represents the noise samples. The $P \times M$ matrix $\mathbf{H}_0$ is Toeplitz and is always guaranteed to be invertible, which enables symbol recovery regardless of the channel zero locations. In what follows, only MMSE equalization is considered but it is possible to extend the procedure to other equalizers. Usually the noise is assumed to be white with variance σ_n^2 and the MMSE equalizer is given by [9]: $\mathbf{G} = \mathbf{R}_{ss}\mathbf{F}_M\mathbf{H}_0^{\mathcal{H}}(\sigma_n^2\mathbf{I}_P + \mathbf{H}_0\mathbf{F}_M^{\mathcal{H}}\mathbf{R}_{ss}\mathbf{F}_M\mathbf{H}_0^{\mathcal{H}})^{-1}$, where $\mathbf{R}_{ss}$ stands for the signal autocorrelation matrix. From $\mathbf{x}_P(i)$, an estimate of $\bar{\mathbf{s}}_M(i)$ is given by: $\hat{\bar{\mathbf{s}}}_M(i) = \mathbf{G}\mathbf{x}_P(i) \approx \bar{\mathbf{s}}_M(i) + \mathbf{G}\mathbf{n}_P(i)$. Let $\bar{\mathbf{n}}_M(i) = \mathbf{G}\mathbf{n}_P(i)$ be the noise affecting $\hat{\bar{\mathbf{s}}}_M(i)$. Noise $\bar{\mathbf{n}}_M(i)$ is not white even if $\mathbf{n}_P(i)$ is white and hence the AWN assumption used in the demodulation procedure does not hold. In order to solve this problem, we propose to approximate the noise autocorrelation: $\mathbf{R}_{nn} = \sigma_n^2\mathbf{G}\mathbf{G}^{\mathcal{H}}$ by its main diagonal: $\mathbf{R}_{nn} \approx \sigma_n^2\text{Diag}(\mathbf{G}\mathbf{G}^{\mathcal{H}})$. This allows to account for the noise power disparity (different diagonal entries) by assuming that the noise samples are independent but not identically distributed (different variances). In this case, the bit metrics for ZP-MMSE can be expressed as:

$$P(d_k^1|y_k) \propto \sum_{d_k^2,\ldots,d_k^4 \in \{0,1\}} e^{-\frac{1}{2\sigma_k^2}\left|\hat{s}_k - s_k(d_k^1,\ldots,d_k^4)\right|^2}$$

where $\sigma_k^2 = [\text{Diag}(\mathbf{R}_{nn})]_{k,k}$. Approximating the noise as white may lead to problems, especially with small interleaver sizes. We thus resort to realizations to verify that our approximations are reasonable and that turbo-demodulation effectively works with ZP-OFDM.

4. Simulations

Simulations have been conducted for a multicarrier system with $M = 64$ subcarriers and $L = 16$ CP or ZP samples, based on Monte Carlo simulations with each trial corresponding to a different realization of the channel models provided in [4]. During a frame, the channel is time-invariant and is exactly known at the receiver. Most results are given in Packet Error Rate (PER) because standards classically specify targeted PER rather than BER.

Figure 1 depicts PER for channel model C. Results have been obtained for a 16QAM constellation with Set Partitioning (SP) labeling for a random interleaver of size 1024 and a memory 2 convolutional encoder with rate $R = 3/4$ defined in octal form by (5,7). It can be observed that the SNR to obtain a PER of 10^{-2} is reduced by about 2dB if ZP-OFDM is used instead of CP-OFDM. Note that the performance of ZP-OFDM after 2 decoding steps is close to that of CP-OFDM after 4 and 8 iterations. Hence, equalization complexity is not the only point to consider if iterative decoding is used because ZP-OFDM may exhibit a smaller overall complexity. For reference purpose the results obtained using Gray labeling and conventional non-iterative decoding are also provided. It can be inferred that the performance gap between CP and TZ is enlarged with iterative decoding. Figure 2 illustrates the influence of the interleaver size by displaying the results obtained with smaller interleavers operating respectively on 256 and 512 bits. It can be observed that the gap between TZ and CP-OFDM is slightly reduced (0.5dB less) with small size interleavers.

Figure 4 illustrates the performance gain that could be obtained in the Hiperlan/2 (HL2) context if iterative decoding and ZP-OFDM would be used. HL2 specifies a CP-OFDM transmitter with Gray labeling for the modulation. In this case, the performance gain brought by iterative decoding is small as reported in [6]. The figure depicts the results of ZP-OFDM with SP labeling. Note that the encoder and the interleaver used for the simulation are the ones specified in HL2: the interleaver operates on 288 bits and the encoder is the memory-6 encoder with rate 3/4 defined in octal form by (133,171). The only differences is that a different puncturing scheme and a random interleaver are used with set partitioning because those currently designed in HL2 are not appropriate for iterative decoding. Note however, that none of the proposed changes (ZP, labeling, puncturing, interleaving) has an impact on the transmitter complexity, on the decoding delay or on the data rate. This shows that ZP-OFDM and iterative decoding should be considered in the future for robust transmissions.

5. Conclusion

This paper has extended the turbo-demodulation procedure to ZP-OFDM by accounting for the noise color introduced by the linear equalization. Simulations have shown that iterative demodulation increases the performance gap between ZP and CP-OFDM. Indeed, the gain in performance brought by the robustness of ZP-OFDM to channel zero locations is amplified by iterative decoding. Finally, it has been shown that these ideas could be successfully applied in real contexts.

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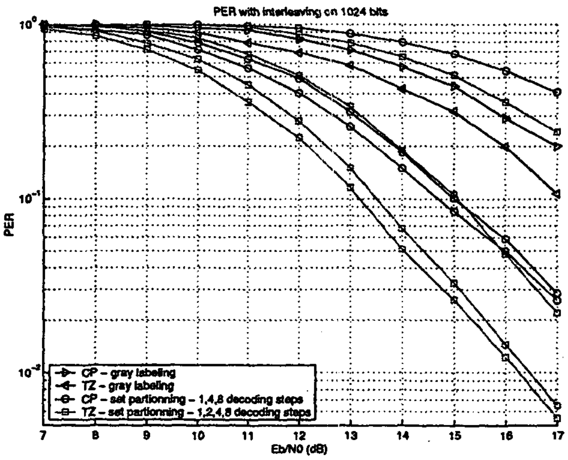

Figure 1. CP and ZP-OFDM with iterative decoding

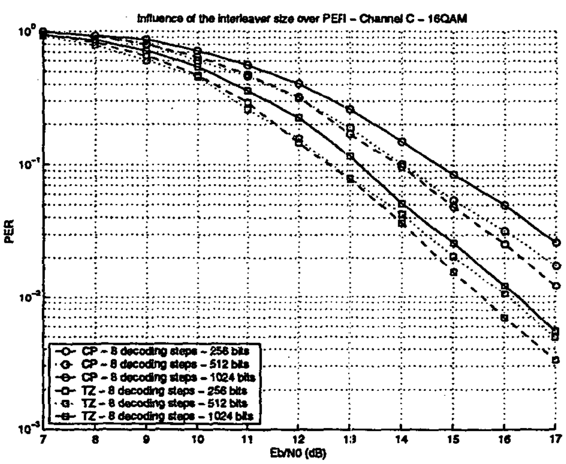

Figure 2. Influence of the interleaver size

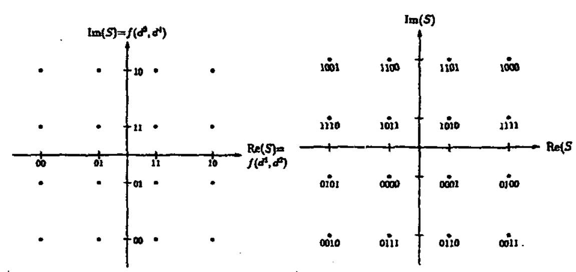

Figure 3. Gray and set partitionning labelings for 16QAM

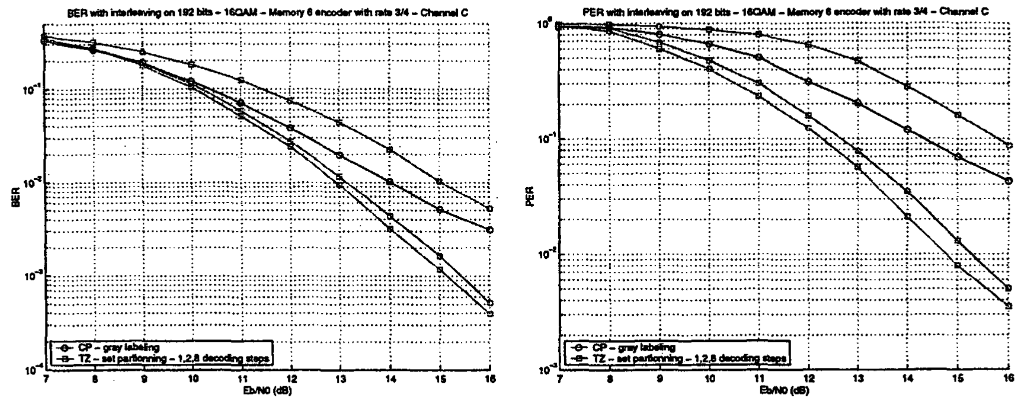

Figure 4. Potential gain in the Hiperlan/2 context

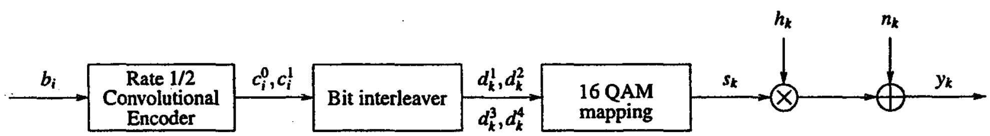

Figure 5. Transmission scheme for a bit-interleaved modulation with rate 1/2 encoding and 16QAM

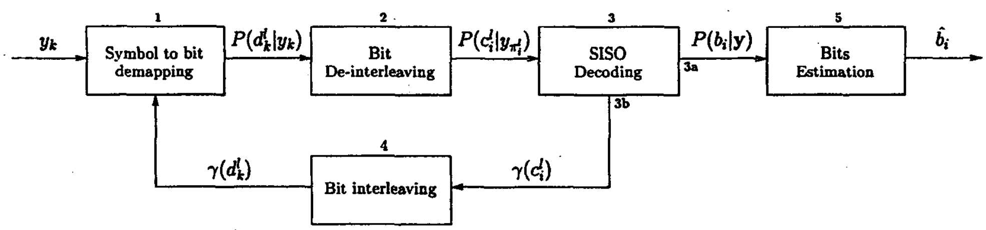

Figure 6. Iterative demodulation scheme

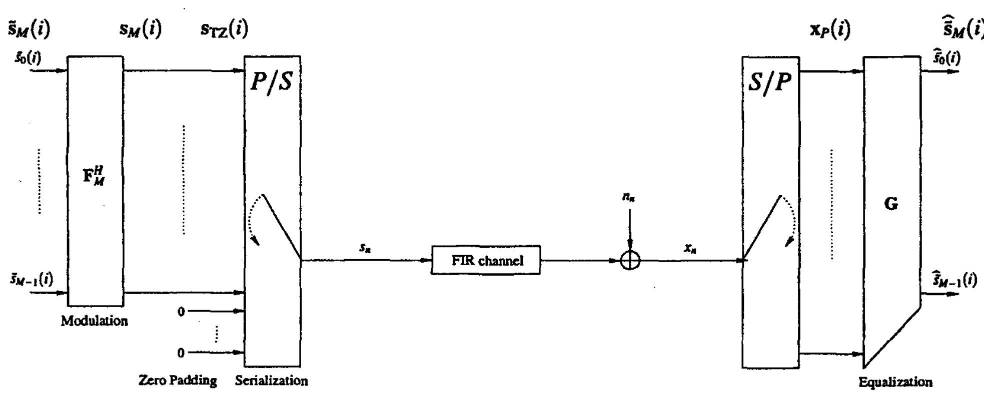

Figure 7. Discrete model of a TZ transceiver