Subspace-based Estimation of Frequency-Selective Channels for Space-Time Block Precoded Transmissions

Bertrand Muquet
Motorola Labs Paris, Espace Technologique
Saint-Aubin, 91193 Gif-sur-Yvette, France
email: muquet@crm.mot.com

Shengli Zhou and *Georgios B. Giannakis*
Dept. of ECE, Univ. of Minnesota
200 Union Str. SE, MLPS, MN 55455, US
email: [shengli,georgios]@ece.umn.edu

Abstract

Space Time Coding has been by now well documented as an attractive means of achieving high data rate transmissions with high quality of service, provided that the underlying channels can be accounted for. We propose in this paper a (semi-)blind channel estimation algorithm that is suitable for space time (ST) block precoded transmissions over frequency-selective channels. We show that multi-channel identifiability is guaranteed up to only one scalar ambiguity regardless of channel zero locations. Simulation results illustrate that the proposed algorithm is capable of tracking slow channel variations.

1. Introduction

New applications such as high speed Internet access or wireless digital television call for very high data rate transmissions. Using multiple antennas both at the transmitter and at the receiver has recently been shown to increase the channel capacity by an order of magnitude or more [6] and thus appears as an appealing solution for future wireless communications. In order to provide diversity and coding gains over single antenna transmissions, many Space-Time Coding (STC) schemes relying on appropriate signal processing have recently been proposed (see [7] and references cited therein).

For most STC approaches, channel knowledge is indispensable at the receiver to decode the transmitted signal and specific multi-channel estimation algorithms are needed. Traditionally, known training symbols are transmitted periodically and thus the receiver can deduce the multiple channel responses. However, training sequences consume bandwidth and thus incur spectral efficiency losses especially in rapidly varying environments. For this reason, a plethora of blind channel estimation methods have been proposed in various ST uncoded contexts. However, only few works have been reported so far on channel estimation by capitalizing on the specific properties of STC. Relying on nonredundant precoding, a blind channel identification and equalization algorithm was proposed in [2] for OFDM-based multiple-antenna systems using cyclostationary statistics. For ST-OFDM (i.e., OFDM with Alamouti's ST encoding [1] over each subcarrier), a deterministic constant modulus blind channel estimator was proposed in [5], which can identify the channels deterministically if the channels are coprime (no common zeros) and the transmitted signals have constant-modulus.

Here, we build on the ST-OFDM approach of [5] and derive a blind channel identification algorithm for frequency-selective Finite Impulse Response (FIR) channels. Based on a subspace approach, this algorithm possesses three attractive features: i) it can be applied to arbitrary signal constellations; ii) by proper system design, it guarantees channel identifiability regardless of channel zero locations; and iii) it can identify multiple channels up to a scalar ambiguity only.

The rest of this paper is organized as follows: Section 2 presents the system model and defines notations; Section 3 details the proposed algorithm; Section 4 addresses identifiability issues. Finally, Section 5 presents simulation results, while Section 6 gathers conclusions.

2 System Description

Figure 1 depicts the wireless system considered in this paper, where the ST transceiver is equipped with two transmit antennas and one receive antenna. It is very close to the counterpart of [1] for FIR channels that was originally derived in [4]. The information symbols are grouped into blocks $\mathbf{s}(n)$ of size $K \times 1$. Two different linear block precoders, one for odd block indices $2n$ and one for even indices $2n+1$, described by the tall $J \times K$ matrices $\mathbf{\Theta}_1$ and $\mathbf{\Theta}_2$ are used to add redundancy. The corresponding $J \times 1$ precoded blocks $\bar{\mathbf{s}}(2n) := \mathbf{\Theta}_1\mathbf{s}(2n)$ and $\bar{\mathbf{s}}(2n+1) := \mathbf{\Theta}_2\mathbf{s}(2n+1)$ are fed to the ST encoder $\mathcal{M}(\cdot)$. As reported in [4], the redundancy introduced by these block precoders facilitates ISI elimination and symbol recovery regardless of the underlying frequency-selective FIR channels. The purpose of this paper is to show that redundant precoding also enables blind identification of the multiple channels. The ST encoder takes as input two consecutive precoded blocks, $\bar{\mathbf{s}}(2n)$ and $\bar{\mathbf{s}}(2n+1)$, to output

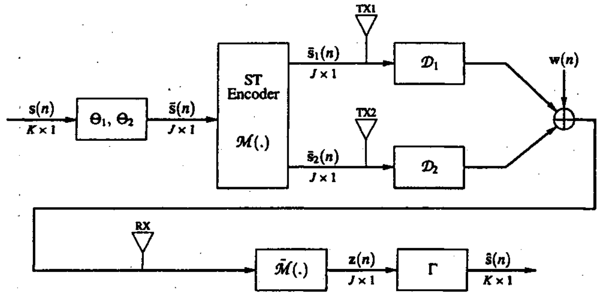

Figure 1. ST transceiver model

the following $2J \times 2$ code matrix (* stands for conjugation):

$$\begin{bmatrix} \bar{s}_1(2n) & \bar{s}_1(2n+1) \\ \bar{s}_2(2n) & \bar{s}_2(2n+1) \end{bmatrix} = \begin{bmatrix} \bar{s}(2n) & -\bar{s}^*(2n+1) \\ \bar{s}(2n+1) & \bar{s}^*(2n) \end{bmatrix},$$

where each block column is transmitted over successive time intervals with the blocks $\bar{s}_1(n)$ and $\bar{s}_2(n)$ sent through transmit-antennas 1 and 2, respectively. Note that without blocking ($J = 1$) this code matrix is the same as the well known Alamouti's block STC [1].

We assume in what follows that the channels are frequency-selective and that their equivalent effect in discrete time is an FIR tap-delay line filter with channel impulse response $\mathbf{h}_i := [h_i(0), \ldots, h_i(L)]$, $i = 1, 2$, where L is the channel order. Moreover, we assume that an OFDM modulator at the transmitter together with the corresponding demodulator at the receiver has been used to convert the FIR channels to a set of parallel flat faded subchannels (see e.g., [9] for details). Let $\mathcal{D}_1$ and $\mathcal{D}_2$ be the diagonal matrices corresponding to the subchannel attenuations: $\mathcal{D}_i := \text{diag}[H_i(0) \ldots H_i(J-1)]$, where $H_i(k) := \sum_{l=0}^{L} h_i(l) e^{-j2\pi l i/J}$. Considering two successive received blocks: $\bar{y}(2n)$ and $\bar{y}(2n+1)$, let us define $\check{y}(n)$ and $\check{s}(n)$ as: $\check{y}(n) := [\bar{y}(2n)^T, \bar{y}^{\mathcal{H}}(2n+1)]^T$ and $\check{s}(n) := [s(2n)^T, s^T(2n+1)]^T$, where ${}^{\mathcal{H}}$ denotes Hermitian transpose. Let also $\check{w}(n)$ be the noise added to the the noise-free version of $\check{y}(n)$, that is denoted by $\check{x}(n)$. The received noisy block $\check{y}(n)$ can then be expressed as:

$$\begin{aligned} \check{y}(n) &= \mathcal{D}\Phi_{12}\check{s}(n) + \check{w}(n) \\ &= \mathcal{H}\check{s}(n) + \check{w}(n) := \check{x}(n) + \check{w}(n), \end{aligned} \tag{1}$$

where $\mathcal{D}, \Phi_{12}, \mathcal{H}$ are defined as:

$$\mathcal{D} = \begin{bmatrix} \mathcal{D}_1 & \mathcal{D}_2 \\ \mathcal{D}_2^* & -\mathcal{D}_1^* \end{bmatrix}, \quad \Phi_{12} = \begin{bmatrix} \Theta_1 & 0 \\ 0 & \Theta_2 \end{bmatrix}, \quad \mathcal{H} = \mathcal{D}\Phi_{12}.$$

Assuming that channel matrices $\mathcal{D}_1$ and $\mathcal{D}_2$ are available at the receiver, it is possible to demodulate with diversity gains by a simple matrix multiplication:

$$\check{z}(n) = \mathcal{D}^{\mathcal{H}}\check{y}(n) = \begin{bmatrix} \mathcal{D}_0\Theta_1 & 0 \\ 0 & \mathcal{D}_0\Theta_2 \end{bmatrix} \check{s}(n) + \mathcal{D}^{\mathcal{H}}\check{w}(n) \tag{2}$$

where $\mathcal{D}_0 := \mathcal{D}_1^*\mathcal{D}_1 + \mathcal{D}_2^*\mathcal{D}_2$. Observing that $\mathcal{D}_0 = \text{diag}[\sum_{i=1}^{2} |H_i(e^{j0})|^2, \ldots, \sum_{i=1}^{2} |H_i(e^{j2\pi(J-1)/J})|^2]$, we infer that diversity advantage of order two has been achieved. We also deduce from (2) that the zero-forcing recovery of $\check{s}(n)$ from $\check{z}(n)$ in the noiseless case requires the matrices $\mathcal{D}_0\Theta_i$, $i = 1, 2$, to be full column rank. Since $\mathcal{D}_0$ has at most L zero diagonal entries, this full rank condition can be assured if we adopt the following design conditions on the block lengths and the linear precoders [9]:

a1) $J > K + L$;

a2) $\Theta_i, i \in \{1, 2\}$, is designed so that any K rows of Θ_i are linearly independent.

We show next that these conditions enable (even blind) multichannel identification regardless of their zero locations.

3 Blind multichannel estimation

We will start from the noiseless vectors $\check{x}(n)$, since we are concerned with channel identifiability questions first.

To estimate the channels $\{\mathbf{h}_i\}_{i=1}^{2}$ (or equivalently $\mathcal{H}$ in (1)), the receiver collects N blocks of $\check{x}(n)$ to a $2J \times N$ matrix $\mathbf{X}_N := [\check{x}(0), \ldots, \check{x}(N-1)]$ and forms $\mathbf{X}_N\mathbf{X}_N^{\mathcal{H}} = \mathcal{H}\mathbf{S}_N\mathbf{S}_N^{\mathcal{H}}\mathcal{H}^{\mathcal{H}}$, where $\mathbf{S}_N := [\check{s}(0), \ldots, \check{s}(N)]$. At the receiver-end, we also select the number of blocks:

a3) N ($\geq 2K$) to be large enough so that $\mathbf{S}_N\mathbf{S}_N^{\mathcal{H}}$ has full rank $2K$.

Condition a3) is known as the "persistence of excitation" assumption and is usually satisfied for values of N comparable to $2K$.

Under a1) and a2), matrix $\mathcal{H}$ has always full column rank. Indeed, $\mathcal{D}^{\mathcal{H}}\mathcal{H} = [\mathbf{I}_2 \otimes \mathcal{D}_0]\Phi_{12}$ where $\otimes$ stands for the Kronecker product and $\mathbf{I}_2$ denotes a 2×2 identity matrix. Since $\mathcal{D}_0$ has at most L zero diagonal entries, each matrix $\mathcal{D}_0\Theta_i$ has full column rank because any K rows of Θ_i are linearly independent. Thus $\mathcal{D}^{\mathcal{H}}\mathcal{H}$ has full column rank and hence $\mathcal{H}$ is also full column rank since any matrix pre-multiplication can only reduce the rank. Therefore, together with a3), we have $\text{rank}(\mathbf{X}_N\mathbf{X}_N^{\mathcal{H}}) = 2K$ and the range space $\mathcal{R}(\mathbf{X}_N\mathbf{X}_N^{\mathcal{H}}) = \mathcal{R}(\mathcal{H})$. Thus, the nullity of $\mathbf{X}_N\mathbf{X}_N^{\mathcal{H}}$ is $\nu(\mathbf{X}_N\mathbf{X}_N^{\mathcal{H}}) = 2J - 2K$. Further, the eigen decomposition

$$\mathbf{X}_N\mathbf{X}_N^{\mathcal{H}} = [\mathbf{U}\ \tilde{\mathbf{U}}] \begin{bmatrix} \Sigma & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U}^{\mathcal{H}} \\ \tilde{\mathbf{U}}^{\mathcal{H}} \end{bmatrix}, \tag{3}$$

where Σ is a diagonal matrix of size $2K \times 2K$ with non-zeros diagonal entries, yields the $2J \times (2J - 2K)$ matrix $\tilde{\mathbf{U}}$ whose columns span the null space $\mathcal{N}(\mathbf{X}_N\mathbf{X}_N^{\mathcal{H}})$. Because the latter is orthogonal to $\mathcal{R}(\mathbf{X}_N\mathbf{X}_N^{\mathcal{H}}) = \mathcal{R}(\mathcal{H})$, it follows that $\tilde{\mathbf{u}}_k^{\mathcal{H}}\mathcal{H} = \mathbf{0}_{1\times 2K}^T$, $k \in [1, 2J - 2K]$, where $\tilde{\mathbf{u}}_k$ denotes the kth column of $\tilde{\mathbf{U}}$.

Let us now split the vector $\tilde{\mathbf{u}}_k$ to its upper and lower parts as: $\tilde{\mathbf{u}}_k = [\hat{\mathbf{u}}_k^T, \check{\mathbf{u}}_k^T]^T$, where $\hat{\mathbf{u}}_k$ and $\check{\mathbf{u}}_k$ are $J \times 1$ vectors. Define $\tilde{\mathbf{h}}_i := [H_i(e^{j0}), \ldots, H_i(e^{j2\pi(J-1)/J})]^T$ and let $\mathbf{D}(\mathbf{v})$ stand for the diagonal matrix with diagonal entries from the elements of the vector $\mathbf{v}$. Therefore, $\tilde{\mathbf{u}}_k^{\mathcal{H}}\mathcal{H}$ can be factored as

$$[\hat{\mathbf{u}}_k^{\mathcal{H}}, \check{\mathbf{u}}_k^{\mathcal{H}}] \begin{bmatrix} \mathcal{D}_1 & \mathcal{D}_2 \\ \mathcal{D}_2^* & -\mathcal{D}_1^* \end{bmatrix} \begin{bmatrix} \Theta_1 & \mathbf{0} \\ \mathbf{0} & \Theta_2 \end{bmatrix} = \mathbf{0}, \quad (4)$$

and rewritten as

$$[\tilde{\mathbf{h}}_1^T, \tilde{\mathbf{h}}_2^{\mathcal{H}}] \begin{bmatrix} \mathbf{D}(\hat{\mathbf{u}}_k^*) & -\mathbf{D}(\check{\mathbf{u}}_k) \\ \mathbf{D}(\check{\mathbf{u}}_k^*) & \mathbf{D}(\hat{\mathbf{u}}_k) \end{bmatrix} \underbrace{\begin{bmatrix} \Theta_1 & \mathbf{0} \\ \mathbf{0} & \Theta_2^* \end{bmatrix}}_{:=\Psi} = \mathbf{0}. \quad (5)$$

With $\mathbf{V}$ being the $J \times (L+1)$ Vandermonde matrix with $(p+1, l+1)$st entry $\exp(-j2\pi pl/J)$, one can write $\tilde{\mathbf{h}}_i = \mathbf{V}\mathbf{h}_i$ and express (5) as:

$$\underbrace{[\mathbf{h}_1^T, \mathbf{h}_2^{\mathcal{H}}] \begin{bmatrix} \mathbf{V}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{V}^{\mathcal{H}} \end{bmatrix}}_{:=\mathcal{F}} \underbrace{\begin{bmatrix} \mathbf{D}(\hat{\mathbf{u}}_k^*) & -\mathbf{D}(\check{\mathbf{u}}_k) \\ \mathbf{D}(\check{\mathbf{u}}_k^*) & \mathbf{D}(\hat{\mathbf{u}}_k) \end{bmatrix}}_{:=\mathcal{D}(\tilde{\mathbf{u}}_k)} \Psi = \mathbf{0}. \quad (6)$$

Stacking (6) for each $\tilde{\mathbf{u}}_k$, $k \in [1, 2J-2K]$, we obtain:

$$[\mathbf{h}_1^T, \mathbf{h}_2^{\mathcal{H}}]\mathcal{F}[\mathcal{D}(\tilde{\mathbf{u}}_1)\Psi, \ldots, \mathcal{D}(\tilde{\mathbf{u}}_{2J-2K})\Psi] = \mathbf{0}, \quad (7)$$

from which one can solve for $[\mathbf{h}_1^T, \mathbf{h}_2^{\mathcal{H}}]^T$.

In the presence of white noise, we replace $\mathbf{X}_N\mathbf{X}_N^{\mathcal{H}}$ in (3) by the received data covariance matrix $\mathbf{R}_{\check{\mathbf{y}}\check{\mathbf{y}}} := E\{\check{\mathbf{y}}(n)\check{\mathbf{y}}^{\mathcal{H}}(n)\}$ and we sort the eigenvalues in (3) in decreasing order. In practice, the ensemble correlation matrix is replaced by the sample average: $\mathbf{R}_{\check{\mathbf{y}}\check{\mathbf{y}}}^{(N)} = (1/N)\sum_{n=0}^{N-1}\check{\mathbf{y}}(n)\check{\mathbf{y}}^{\mathcal{H}}(n)$, which converges in the mean square sense to the true correlation matrix since $\check{\mathbf{x}}(n)$ has finite moments.

We summarize the algorithm in the following steps:

s1. Collect the received data blocks $\check{\mathbf{y}}(n)$ and compute $\mathbf{R}_{\check{\mathbf{y}}\check{\mathbf{y}}}^{(N)} = (1/N)\sum_{n=0}^{N-1}\check{\mathbf{y}}(n)\check{\mathbf{y}}^{\mathcal{H}}(n)$;

s2. Determine the eigenvectors $\tilde{\mathbf{u}}_k$, $k = 1, \ldots, 2J-2K$ corresponding to the smallest $2J-2K$ eigenvalues of matrix $\mathbf{R}_{\check{\mathbf{y}}\check{\mathbf{y}}}^{(N)}$;

s3. From these eigenvectors, estimate $[\mathbf{h}_1^T, \mathbf{h}_2^{\mathcal{H}}]^T$ as the non-trivial solution of (7) by determining the left eigenvector corresponding to the smallest eigenvalue of the matrix $\mathcal{F} \cdot [\mathcal{D}(\tilde{\mathbf{u}}_1)\Psi \cdots \mathcal{D}(\tilde{\mathbf{u}}_{2J-2K})\Psi]$.

4 Channel identifiability

The natural question here is whether the solution of (7) is unique. We explore channel identifiability in two directions; namely with identical, or, distinct antenna precoders.

4.1. Identical Precoders

We first study the case where $\Theta_1 = \Theta_2 = \Theta$. It turns out that channel identifiability is not guaranteed as stated in the following theorem:

Theorem 1: *If the same precoding matrix is used for odd and even indexed blocks of symbols, the solution provided by* (7) *belongs to a two-dimensional vector space that is spanned by* $\mathbf{h}_{12} = [\mathbf{h}_1^T, \mathbf{h}_2^{\mathcal{H}}]^T$ *and* $\mathbf{h}_{21} = [\mathbf{h}_2^T, -\mathbf{h}_1^{\mathcal{H}}]$.

Proof: Let $(\mathbf{h}_3, \mathbf{h}_4)$ be a pair of channels satisfying (7) or equivalently having the same signal subspace as $(\mathbf{h}_1, \mathbf{h}_2)$. There exists a full rank $2K \times 2K$ matrix $\mathbf{A}$ such that: $\mathcal{H}(\mathbf{h}_3, \mathbf{h}_4) = \mathcal{H}(\mathbf{h}_1, \mathbf{h}_2)\mathbf{A}$. Let us split $\mathbf{A}$ into four sub-matrices of size $K \times K$ implicitly defined by the following equation:

$$\begin{bmatrix} \mathcal{D}_3 & \mathcal{D}_4 \\ \mathcal{D}_4^* & -\mathcal{D}_3^* \end{bmatrix} \Phi_{12} = \begin{bmatrix} \mathcal{D}_1 & \mathcal{D}_2 \\ \mathcal{D}_2^* & -\mathcal{D}_1^* \end{bmatrix} \Phi_{12} \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_3 & \mathbf{A}_4 \end{bmatrix}, \quad (8)$$

from which one can arrive at:

$$(\mathcal{D}_1^*\mathcal{D}_3 + \mathcal{D}_2\mathcal{D}_4^*)\Theta = (|\mathcal{D}_1|^2 + |\mathcal{D}_2|^2)\Theta\mathbf{A}_1, \quad (9)$$

$$(\mathcal{D}_2^*\mathcal{D}_3 - \mathcal{D}_1\mathcal{D}_4^*)\Theta = (|\mathcal{D}_1|^2 + |\mathcal{D}_2|^2)\Theta\mathbf{A}_3, \quad (10)$$

$$(\mathcal{D}_1\mathcal{D}_3^* + \mathcal{D}_2^*\mathcal{D}_4)\Theta = (|\mathcal{D}_1|^2 + |\mathcal{D}_2|^2)\Theta\mathbf{A}_4, \quad (11)$$

$$(-\mathcal{D}_2\mathcal{D}_3^* + \mathcal{D}_1^*\mathcal{D}_4)\Theta = (|\mathcal{D}_1|^2 + |\mathcal{D}_2|^2)\Theta\mathbf{A}_2. \quad (12)$$

Let $D_i(u)$, $i \in [1,2,3,4]$, denote the uth diagonal entry of $\mathcal{D}_i$ and θ_u^T as the uth row of Θ. Since $|D_1(u)|^2 + |D_2(u)|^2$ has at most L zeros for $u = 1, \ldots, J$, we can find from (9) at least $J - L \geq K+1$ equations satisfying:

$$\frac{D_1^*(u)D_3(u) + D_2(u)D_4^*(u)}{|D_1(u)|^2 + |D_2(u)|^2}\theta_u^T = \theta_u^T\mathbf{A}_1. \quad (13)$$

At this point, we need to introduce the following lemma:

Lemma 1: *If for any* $K \times K$ *matrix* $\mathbf{A}'$, *there exist at least* $K+1$ *rows of* Θ *satisfying:* $\lambda_k\theta_k^T = \theta_k^T\mathbf{A}'$, *then* $\mathbf{A}' = \lambda\mathbf{I}_K$ *for some* λ.

The proof of Lemma 1 can be directly extracted from [3, Appendix 2], where $\mathbf{A}'$ was unnecessarily assumed to have full rank. Applying Lemma 1 to (13), we obtain $\mathbf{A}_1 = \lambda_1\mathbf{I}$. Similarly, we obtain from (10), (11), and (12) that : $\mathbf{A}_3 = \lambda_3\mathbf{I}$, $\mathbf{A}_4 = \lambda_4\mathbf{I}$, $\mathbf{A}_2 = \lambda_2\mathbf{I}$. Substituting $\mathbf{A}_1$ and $\mathbf{A}_4$ into (9) and (11), we obtain

$$(\mathcal{D}_1^*\mathcal{D}_3 + \mathcal{D}_2\mathcal{D}_4^*) = \lambda_1(|\mathcal{D}_1|^2 + |\mathcal{D}_2|^2), \quad (14)$$

$$(\mathcal{D}_1\mathcal{D}_3^* + \mathcal{D}_2^*\mathcal{D}_4) = \lambda_4(|\mathcal{D}_1|^2 + |\mathcal{D}_2|^2), \quad (15)$$

which implies that $\lambda_4 = \lambda_1^*$. Similarly, from (10) and (12) we obtain $\lambda_3 = -\lambda_2^*$. Therefore, (8) can be re-expressed as:

$$\begin{bmatrix} \mathcal{D}_3 & \mathcal{D}_4 \\ \mathcal{D}_4^* & -\mathcal{D}_3^* \end{bmatrix} \Phi_{12} = \begin{bmatrix} \mathcal{D}_1\lambda_1 - \mathcal{D}_2\lambda_2^* & \mathcal{D}_1\lambda_2 + \mathcal{D}_2\lambda_1^* \\ \mathcal{D}_2^*\lambda_1 + \mathcal{D}_1^*\lambda_2^* & \mathcal{D}_2^*\lambda_2 - \mathcal{D}_1^*\lambda_1^* \end{bmatrix} \Phi_{12},$$

from which we obtain:

$$\begin{bmatrix} \mathcal{D}_3 \\ \mathcal{D}_4^* \end{bmatrix} = \lambda_1 \begin{bmatrix} \mathcal{D}_1 \\ \mathcal{D}_2^* \end{bmatrix} - \lambda_2^* \begin{bmatrix} \mathcal{D}_2 \\ -\mathcal{D}_1^* \end{bmatrix}. \quad (16)$$

Therefore, both $\mathbf{h}_{12} := [\mathbf{h}_1^T, \mathbf{h}_2^{\mathcal{H}}]^T$ and $\mathbf{h}_{21} := [\mathbf{h}_2^T, -\mathbf{h}_1^{\mathcal{H}}]$, as well as their linear combinations, satisfy (7). Thus, the channel estimator of (7) does not yield the desired output. This fact is due to the inherent symmetry between antenna pairs and can be easily understood by noting that:

$$\begin{bmatrix} \mathcal{D}_1 & \mathcal{D}_2 \\ \mathcal{D}_2^* & -\mathcal{D}_1^* \end{bmatrix} \begin{bmatrix} \bar{s}(2n) \\ \bar{s}(2n+1) \end{bmatrix} = \begin{bmatrix} \mathcal{D}_2 & -\mathcal{D}_1 \\ -\mathcal{D}_1^* & -\mathcal{D}_2^* \end{bmatrix} \begin{bmatrix} \bar{s}(2n+1) \\ -\bar{s}(2n) \end{bmatrix}$$

For symmetric constellations, it is impossible to determine whether the transmitter has sent $\bar{s}(2n)$ and $\bar{s}(2n+1)$ on channels $\mathbf{h}_1$ and $\mathbf{h}_2$ or $\bar{s}(2n+1)$ and $-\bar{s}(2n)$ on channels $\mathbf{h}_2$ and $-\mathbf{h}_1$, respectively (see also [5] for related remarks). ■

4.2. Different Precoders

To break the symmetry and identify the channel uniquely, a pre-weighting approach that loads different power over different antennas was proposed in [5]. However, the price paid is degraded bit error rate (BER) performance. Here, we break this symmetry by introducing distinct linear precoders Θ_1 and Θ_2, that can be designed to have balanced power. The channel identifiability can then be guaranteed as summarized in the following theorem:

Theorem 2: *Suppose a1), a2) and a3) hold true; let $\bar{\mathbf{D}}$ denote any diagonal matrix with unit amplitude diagonal entries, and $\bar{\Theta}_1$, $\bar{\Theta}_2$ be formed from any $J-L$ rows of Θ_1, Θ_2, respectively. If $\bar{\Theta}_1$ and $\bar{\Theta}_2$ satisfy: $\bar{\mathbf{D}}\bar{\Theta}_1 \notin \mathcal{R}(\bar{\Theta}_2)$, the solution of (7) is unique up to a constant and thus channel identifiability (within a scalar) is guaranteed.*

Proof: With Θ_1 and Θ_2, equations (9)-(12) turn into:

$$(\mathcal{D}_1^*\mathcal{D}_3 + \mathcal{D}_2\mathcal{D}_4^*)\Theta_1 = (|\mathcal{D}_1|^2 + |\mathcal{D}_2|^2)\Theta_1\mathbf{A}_1, \quad (17)$$

$$(\mathcal{D}_2^*\mathcal{D}_3 - \mathcal{D}_1\mathcal{D}_4^*)\Theta_1 = (|\mathcal{D}_1|^2 + |\mathcal{D}_2|^2)\Theta_2\mathbf{A}_3, \quad (18)$$

$$(\mathcal{D}_1\mathcal{D}_3^* + \mathcal{D}_2^*\mathcal{D}_4)\Theta_2 = (|\mathcal{D}_1|^2 + |\mathcal{D}_2|^2)\Theta_2\mathbf{A}_4, \quad (19)$$

$$(-\mathcal{D}_2\mathcal{D}_3^* + \mathcal{D}_1^*\mathcal{D}_4)\Theta_2 = (|\mathcal{D}_1|^2 + |\mathcal{D}_2|^2)\Theta_1\mathbf{A}_2. \quad (20)$$

From (17) and (19), we obtain $\mathbf{A}_1 = \lambda_1\mathbf{I}$, $\mathbf{A}_4 = \lambda_4\mathbf{I}$ and $\lambda_4 = \lambda_1^*$ following the proof of Theorem 1.

The right-hand sides of (18) and (20) have at least $J-L$ nonzero rows each. Let us collect only $J-L$ nonzero rows with the row numbers denoted as $u_1, u_2, \ldots, u_{J-L}$. Define the $(J-L) \times (J-L)$ matrix $\bar{\mathbf{D}}'$ as the diagonal matrix with (i,i) entry $[D_2^*(u_i)D_3(u_i) - D_1(u_i)D_4^*(u_i)]/[|D_1(u_i)|^2 + |D_2(u_i)|^2]$ and let $\bar{\Theta}_1'$ and $\bar{\Theta}_2'$ be formed by the $u_1, u_2, \ldots, u_{J-L}$ rows of Θ_1, Θ_2. We then obtain from (18) and (20):

$$\bar{\mathbf{D}}'\bar{\Theta}_1' = \bar{\Theta}_2'\mathbf{A}_3, \qquad -(\bar{\mathbf{D}}')^*\bar{\Theta}_2' = \bar{\Theta}_1'\mathbf{A}_2. \quad (21)$$

From (21), we deduce that $-|\bar{\mathbf{D}}|^2\Theta_2 = \Theta_2\mathbf{A}_3\mathbf{A}_2$. Applying Lemma 1, we find that $\mathbf{A}_3\mathbf{A}_2 = -\lambda\mathbf{I}$ and $|\bar{\mathbf{D}}'|^2 = \lambda\mathbf{I}$. If $\lambda \neq 0$, then there exists a $(\bar{\mathbf{D}}'/\sqrt{\lambda}, \bar{\Theta}_1', \bar{\Theta}_2')$ triplet within the class of $(\bar{\mathbf{D}}, \bar{\Theta}_1, \bar{\Theta}_2)$ triplets specified by Theorem 3, which satisfies $\bar{\mathbf{D}}'\bar{\Theta}_1' \in \mathcal{R}(\bar{\Theta}_2')$. Since this is impossible under the design constraints of Theorem 2, we arrive at $\lambda = 0$, which allows only: $[\mathbf{D}_3\ \mathbf{D}_4^*] = \lambda_1 [\mathbf{D}_1\ \mathbf{D}_2^*]$. ■

If $\Theta_1 = \Theta_2$, the scalar λ can be any nonnegative number and a simple class of $\mathbf{A}_3, \mathbf{A}_2$ can be: $\mathbf{A}_3 = \sqrt{\lambda}e^{j\phi}\mathbf{I}$, $\mathbf{A}_2 = -\sqrt{\lambda}e^{-j\phi}\mathbf{I}$, where ϕ is arbitrary. Therefore, channel identifiability is not guaranteed as per Theorem 1.

Note that Theorem 2 considers $J-K \geq L+1$. Now let us focus on minimally redundant precoders with **a1')** $J = K + \tilde{L}$, $\tilde{L} \in [1, L]$.

We assume that channels $\mathbf{h}_1$ and $\mathbf{h}_2$ have Z common zeros that are located on the FFT grid. Note that $|D_1(u)|^2 + |D_2(u)|^2$ for $u = 1, 2, \ldots, J$ have Z zeros. If $Z < \tilde{L}$, we can find $J - Z \geq K + 1$ equations like (13). Following the same arguments as in the proof of Theorem 2, we obtain the following identifiability result:

Theorem 3: *Suppose a1'), a2) and a3) hold true; let $\bar{\mathbf{D}}$ denote any diagonal matrix with unit amplitude diagonal entries, and $\bar{\Theta}_1$, $\bar{\Theta}_2$ be formed from any $K+1$ rows of Θ_1, Θ_2 respectively. If $\bar{\Theta}_1$ and $\bar{\Theta}_2$ satisfy: $\bar{\mathbf{D}}\bar{\Theta}_1 \notin \mathcal{R}(\bar{\Theta}_2)$, channel identifiability (within a scalar) is guaranteed for those channel pairs with $Z < \tilde{L}$ common zeros located on the FFT grid.*

Under a1'), Theorem 3 shows that channel identifiability is now related to channel zero locations. Because a channel of order L has only L zeros, the case $J = K + L$ turns out to be a special case as follow: From (8), we infer that $\mathcal{D}_3\Theta_1 = \mathcal{D}_1\Theta_1 + \mathcal{D}_2\Theta_2\mathbf{A}_3$. From this equation, if $\mathbf{h}_1$, $\mathbf{h}_2$ have the same $Z = L$ roots located on the FFT grid indexed by $u_1, u_2, \ldots, u_L$, we have $D_3(u_l) = 0$ (similarly $D_4(u_l) = 0$), since $D_1(u_l) = D_2(u_l) = 0$ for $l = 1, \ldots, L$. Thus, $\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3$ and $\mathbf{h}_4$ have the same L roots and are all proportional to each other, which indicates that channel identifiability is also guaranteed. However, in this case it is not difficult to show that we need to resolve two scalar ambiguities $\mathbf{h}_3 = \alpha_1\mathbf{h}_1$ and $\mathbf{h}_4 = \alpha_2\mathbf{h}_2$ compared to one that we had with Theorems 2 and 3.

To select the appropriate Θ_1 and Θ_2 precoders, we can, for instance construct them as Vandermonde matrices with distinct generators $[\rho_{i,1}, \cdots, \rho_{i,J}]_{i=1,2}$:

$$\Theta_i = \begin{bmatrix} 1 & \rho_{i,1}^{-1} & \cdots & \rho_{i,1}^{-(K-1)} \\ \vdots & & & \vdots \\ 1 & \rho_{i,J}^{-1} & \cdots & \rho_{i,J}^{-(K-1)} \end{bmatrix},$$

and then check whether $\bar{\mathbf{D}}\bar{\Theta}_1 \notin \mathcal{R}(\bar{\Theta}_2)$ by simulation. Thus, from the received data and one pilot tone, which is used to resolve the scalar ambiguity inherent to all blind methods, multiple channels can be estimated simultaneously with linearly block precoded ST-OFDM transmissions.

5 Simulations

To test the proposed channel estimation algorithm we resort to simulations. The figures of merit here are the averaged Normalized Mean Square Error (NMSE) of the channels defined as: $(1/2)\sum_{i=1}^{2} \|\hat{\tilde{\mathbf{h}}}_i - \tilde{\mathbf{h}}_i\|^2/\|\tilde{\mathbf{h}}_i\|^2$ and the uncoded Bit Error Rate (BER). We set $L = 8$, $K = 3L$, $J = K + L = 32$; and generate the channels according to the Channel Model A specified by ETSI. Fig. 2 shows the NMSE for the semi-blind version in [4] of the proposed algorithm and the training based approach proposed in [8] (with one training block from each antenna) at a typical SNR of $E_b/N_0 = 10$dB and a terminal speed $v = 3$m/s. The implementation of the semi-blind channel estimator is detailed in [4] except that a sliding window over 100 blocks of symbols has been used here instead of an exponential window. Figs. 3 and 4 depict the average NMSE and BER averaged over independent channel realizations for frames of 400 blocks of symbols. An 1dB gain for a BER of 10^{-3} can be observed. We can infer from these simulations that the semi-blind channel estimator outperforms the training based approach in this setup and is also able to track slow channel variations. Fig. 4 also shows the BER for the ideal case where the two channels are known. The large gap (especially at high SNR) between the BERs with known channels and estimated channels highlights the importance of accurate channel estimation.

6 Conclusions

We have developed a blind channel estimation algorithm for space-time block precoded transmissions over frequency selective channels. Based on a subspace approach, this algorithm can be applied to any signal constellation and guarantees channel identifiability regardless of channel zero locations. Simulations have illustrated that the method is effective and capable of tracking slow channel variations in its semi-blind implementation.

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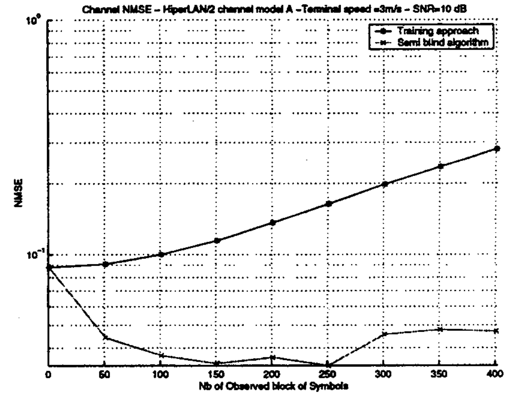

Figure 2. Channel MSE along the frame

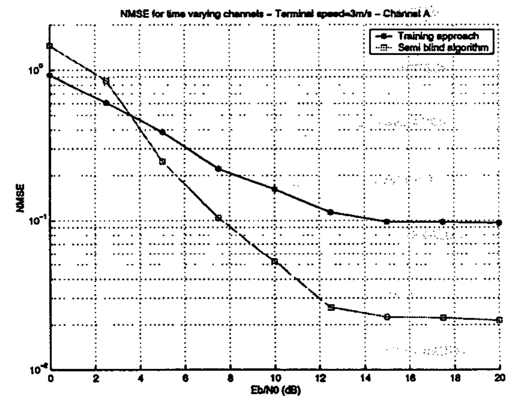

Figure 3. MSE as a function of the SNR

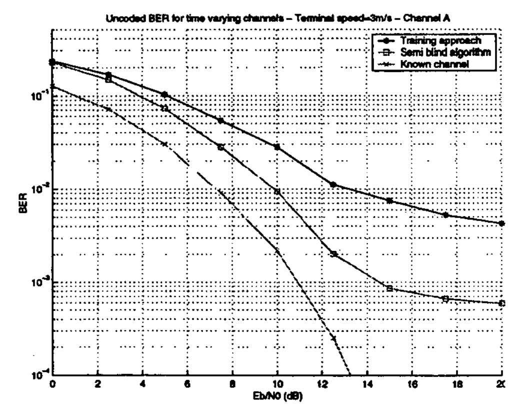

Figure 4. BER as a function of the SNR