Space-Time-Frequency Coded OFDM Over Frequency-Selective Fading Channels

Zhiqiang Liu, Yan Xin, *Student Member, IEEE*, and Georgios B. Giannakis, *Fellow, IEEE*

***Abstract*—This paper proposes novel space-time-frequency (STF) coding for multiantenna orthogonal frequency-division multiplexing (OFDM) transmissions over frequency-selective Rayleigh fading channels. Incorporating subchannel grouping and choosing appropriate system parameters, we first convert our system into a set of group STF (GSTF) systems. This enables simplification of STF coding within each GSTF system. We derive design criteria for STF coding and exploit existing ST coding techniques to construct both STF block and trellis codes. The resulting codes are shown to be capable of achieving maximum diversity and coding gains, while affording low-complexity decoding. The performance merits of our design is confirmed by corroborating simulations and compared with existing alternatives.**

***Index Terms*—Diversity methods, fading channels, space-time-frequency codes, space-time codes, space-frequency codes, transmit antenna diversity.**

I. Introduction

THE major driver for broadband wireless communications has been reliable high-data-rate services (e.g., real-time multimedia services). This, together with the scarcity of bandwidth resources, motivate research toward developing efficient coding and modulation schemes that improve the quality and bandwidth efficiency of wireless systems. In wireless links, multipath fading causes performance degradation and constitutes the bottleneck for increasing data rates. Traditionally, the most popular technique to combat fading has been the exploitation of diversity.

Space-time (ST) coding has been proved effective in combating fading, and enhancing data rates; see e.g., [16], [19], and references therein. Exploiting the presence of spatial diversity offered by multiple transmit and/or receive antennas, ST coding relies on simultaneous coding across space and time to achieve diversity gain without necessarily sacrificing precious bandwidth. Two typical examples of ST codes are ST trellis codes [22] and ST block codes [2], [21]. In ST coding, the maximum achievable diversity advantage is equal to the product of the number of transmit and receive antennas; therefore, it is constrained by the size and cost a system can afford. The latter motives exploitation of extra diversity dimensions, such as multipath diversity.

Multipath diversity becomes available when frequency selectivity is present, which is the typical situation for broadband wireless channels [16]. As proved in [4], [18], and [27], multiantenna transmissions over frequency-selective fading channels can potentially provide a maximum diversity gain that is multiplicative in the number of transmit antennas, receive antennas, and the channel length. Inspired by this result, a number of coding schemes have been proposed recently to exploit multipath diversity. Because they offer low-complexity equalization decoding and facilitate the support of multirate services, multicarrier transmissions[1] are typically adopted by those schemes [1], [4], [14], [18], [27]. Among them, [14] and [27] rely on combining ST codes with redundant or nonredundant linear precoders. Maximum diversity gain is achieved in [14] and [27] at the expense of bandwidth efficiency [14] or increased decoding complexity [14], [27]. On the other hand, [1], [4], [5], [12], and [18] are based on space-frequency (SF) coding, which amounts to simultaneously coding over space and frequency. However, due to the prohibitive complexity in constructing the codes, no SF codes have been designed in [1], [4], [12], or [18]. Instead, [1], [4], [12], and [18] simply adopt existing codes [ST block codes in [4] and trellis-coded modulation (TCM) codes in [1], [4], [12], [18]], without maximum diversity gain guarantees. In [5], an SF code is proposed to achieve maximum diversity gain at the expense of bandwidth efficiency. Moreover, issues pertaining to maximizing the coding gain of ST-coded transmissions over frequency-selective channels have yet to be addressed.

Focusing on multiantenna orthogonal frequency-division multiplexing (OFDM) transmissions through frequency-selective Rayleigh fading channels, this paper pursues a novel path: joint space-time-frequency (STF) coding over space, time, and frequency. Resorting to subchannel grouping [10], [17], [24] and by choosing proper system parameters, we first divide the set of generally correlated OFDM subchannels into groups of subchannels. We thus convert our system into a set of what we term group STF (GSTF) subsystems, within which STF coding is considered. By deriving design criteria for STF codes, we provide a link between STF codes and existing ST codes. We prove that subchannel grouping does preserve maximum diversity gains while simplifying not only the code construction but the decoding algorithm significantly as well. Aiming at

Manuscript received May 14, 2001; revised May 13, 2002. This work was supported by the National Science Foundation under Wireless Initiative grant 9979442, ARO grant DAAG55-98-1-0336, and a grant from the ARL/CTA. Part of this work appears in the *Proceedings of the International Conference on Acoustics, Speech, and Signal Processing*, Orlando, FL, May 13–17, 2002 and in the *Proceedings of Vehicular Technology Conference*, Birmingham, AL, May 4–9, 2002. The associate editor coordinating the review of this paper and approving it for publication was Dr. Thomas L. Marzetta.

Z. Liu is with the Department of Electrical and Computer Engineering, University of Iowa, Iowa City, IA 52244 USA (e-mail: zhiqiang-liu@uiowa.edu).

Y. Xin and G. B. Giannakis are with the Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN 55455 USA (e-mail: yxin@ece.umn.edu; georgios@ece.umn.edu).

Publisher Item Identifier 10.1109/TSP.2002.803332.

[1]This paper will focus on multicarrier transmissions and will not cover the single-carrier schemes in, e.g., [13] and [27].

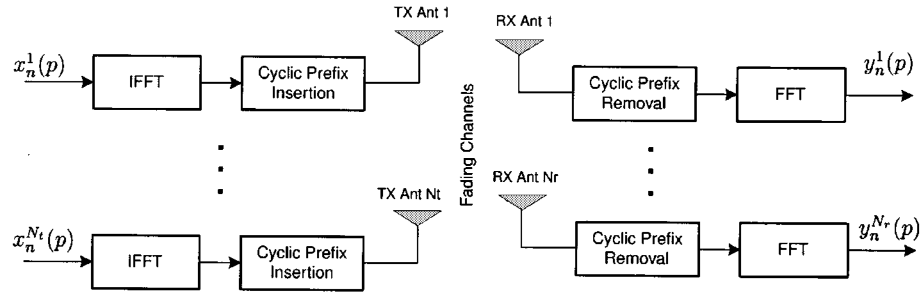

Fig. 1. Multi-antenna space-time OFDM system.

maximizing both diversity and coding gains, we construct two types of STF codes [STF block (STFB) codes and STF trellis (STFT) codes], whose performance is investigated both by theoretical analyzes and by corroborating simulations.

The major contributions of this paper are the following.

- For multiantenna OFDM systems, we introduce the concept of STF coding to enable maximum diversity, high coding gains, and low decoding complexity.
- For multiantenna OFDM systems, we incorporate subchannel grouping to create GSTF systems and justify that subchannel grouping does not impair the potential for achieving maximum diversity gain, while resulting in GSTF coded subsystems that are "friendly" to design.
- For GSTF systems, we derive design criteria of STF codes and explicitly link them to those of ST codes. The latter facilitates exploitation of existing ST coding techniques in designing STF codes.
- Under the established design criteria, we construct STFB and STFT codes, which perform well in various channel environments.

The paper is organized as follows. In Section II, we present the system model and introduce the concept of STF coding. In Section III, we analyze the performance, describe the GSTF system, and derive design criteria for STF coding. Sections IV and V, respectively, deal with the construction of STF block codes and STF trellis codes. Several practical issues are considered in Section VI. Simulations are carried out in Section VII, whereas Section VIII concludes this paper.

Notation: Column vectors (matrices) are denoted by boldface lower (upper) case letters. Superscripts ^{T}, *, and $^{\mathcal{H}}$ stand for transpose, conjugate, and conjugate transpose, respectively; $\mathrm{diag}(d_1, \ldots, d_P)$ denotes a $P \times P$ diagonal matrix with diagonal entries $d_1, \ldots, d_P$; and $\boldsymbol{I}_P$ stands for the $P \times P$ identity matrix.

II. Preliminaries

A. System Model

Fig. 1 depicts a multiantenna wireless communication system with N_t transmit antennas and N_r receive antennas, where OFDM utilizing N_c subcarriers is employed per antenna transmission. The fading channel between the μth transmit antenna and the νth receive antenna is assumed to be frequency-selective but time-flat and is described by the discrete-time baseband equivalent impulse response vector $\boldsymbol{h}_{\mu\nu} := [h_{\mu\nu}(0), \ldots, h_{\mu\nu}(L)]^T \in \mathbb{C}^{(L+1)\times 1}$, with L standing for the channel order. The channel impulse response includes the effects of transmit receive filters, physical multipath, and relative delays among antennas.

Let $x_n^\mu(p)$ be the data symbol transmitted on the pth subcarrier (frequency bin) from the μth transmit antenna during the nth OFDM symbol interval. As defined, the symbols $\{x_n^\mu(p),\ \mu = 1, \ldots, N_t,\ p = 0, 1, \ldots, N_c - 1\}$ are transmitted in parallel on N_c subcarriers by N_t transmit antennas. Notice that three variables μ, n and p have been introduced to, respectively, index the antenna- (space-), time-, and frequency- dimensions associated with the transmission of $x_n^\mu(p)$. Thus, $x_n^\mu(p)$ can be viewed as a point in a three-dimensional (3-D) space-time-frequency (STF) parallelepiped.

At the receiver, each antenna receives a noisy superposition of the multiantenna transmissions through the fading channels. We assume ideal carrier synchronization, timing, and perfect symbol-rate sampling. We also suppose that a cyclic prefix (CP) of length L has been inserted per OFDM symbol and is removed at the receiver end. After FFT processing, the received data sample $y_n^\nu(p)$ at the νth receive antenna can be expressed as

$$y_n^\nu(p) = \sum_{\mu=1}^{N_t} H_{\mu\nu}(p) x_n^\mu(p) + w_n^\nu(p), \quad \nu = 1, \ldots, N_r \qquad p = 0, \ldots, N_c - 1 \tag{1}$$

where $H_{\mu\nu}(p)$ is the subchannel gain from the μth transmit antenna to the νth receive antenna evaluated on the pth subcarrier

$$H_{\mu\nu}(p) := \sum_{l=0}^{L} h_{\mu\nu}(l) e^{-j(2\pi/N_c)lp} \tag{2}$$

and the additive noise $w_n^\nu(p)$ is circularly symmetric, zero-mean, complex Gaussian with variance N_0 that is also assumed to be statistically independent with respect to n, ν, and p.

Equation (1) represents a general model for multiantenna OFDM systems, including those considered in [1], [4], [5], [12], [15], [18], and [27]. The difference among those systems lies in how $x_n^\mu(p)$s are generated from the information symbols s_n, which eventually leads to corresponding tradeoffs among performance, decoding complexity, and transmission rate. In our system, the generation of $x_n^\mu(p)$ is performed via what we term STF coding, which is we describe next.

B. STF Coding

Recalling that each $x_n^{\mu}(p)$ is a point in 3-D, we define each STF codeword as the collection of transmitted symbols within the parallelepiped, spanned by N_t transmit antennas, N_x OFDM symbol intervals, and N_c subcarriers. Thus, one STF codeword contains $N_t N_x N_c$ transmitted symbols $\{x_n^{\mu}(p),\ \mu = 1,\ldots,N_t,\ n=0,\ldots,N_x-1,\ p=0,\ldots,N_c-1\}$, which for mathematical convenience can be organized in a block matrix

$$\boldsymbol{X} := [\boldsymbol{X}(0) \quad \boldsymbol{X}(1) \quad \ldots \quad \boldsymbol{X}(N_c-1)] \in \mathbb{C}^{N_t \times N_c N_x} \tag{3}$$

where

$$\boldsymbol{X}(p) := \begin{bmatrix} x_0^1(p) & \ldots & x_{N_x-1}^1(p) \\ \vdots & & \vdots \\ x_0^{N_t}(p) & \ldots & x_{N_x-1}^{N_t}(p) \end{bmatrix} \in \mathbb{C}^{N_t \times N_x}. \tag{4}$$

Let us define the multi-input multi-output (MIMO) channel matrix $\boldsymbol{H}(p) \in \mathbb{C}^{N_r \times N_t}$ with (ν,μ)th entry $[\boldsymbol{H}(p)]_{\nu\mu} = H_{\nu\mu}(p)$, the received sample matrix $\boldsymbol{Y}(p) \in \mathbb{C}^{N_r \times N_x}$ with $[\boldsymbol{Y}(p)]_{\nu n} = y_n^{\nu}(p)$, and the noise matrix $\boldsymbol{W}(p) \in \mathbb{C}^{N_r \times N_x}$. It follows from (1) that

$$\boldsymbol{Y}(p) = \boldsymbol{H}(p)\boldsymbol{X}(p) + \boldsymbol{W}(p), \quad p = 0,\ldots,N_c-1 \tag{5}$$

which confirms that OFDM yields parallel $\boldsymbol{X}(p)$ transmissions over different frequencies. Because each $\boldsymbol{X}(p)$ can be thought of as being transmitted using an ST system, the N_c in (5) provides a model for our 3-D STF (transmission) system (see also Fig. 2). It is important to point out that the transmissions of $x_n^{\mu}(p)$s are separable in both time and frequency but not in space. This separability will prove useful when we develop a space-virtual-time system in Section V.

Suppose that $\boldsymbol{X}$ has been generated by $\bar{N}_I$ information symbols collected in the block $\boldsymbol{s} := [s_0,\ldots,s_{\bar{N}_I-1}]^T \in \mathbb{C}^{\bar{N}_I \times 1}$. STF coding is then defined as an one-to-one mapping Ψ

$$\Psi : \boldsymbol{s} \rightarrow \boldsymbol{X}. \tag{6}$$

Because $\boldsymbol{X}$ in (3) is described by three dimensions, STF coding simultaneously encodes information over space, time, and frequency, as its name reveals. The design of Ψ will be discussed in Sections IV and V.

Let $\mathcal{A}_s \ni s_n$ be the alphabet set to which the information symbol s_n belongs, and let $|\mathcal{A}_s|$ be the cardinality of $\mathcal{A}_s$. Since $\boldsymbol{X}$ is uniquely mapped from $\boldsymbol{s}$, the number of possible STF codewords $\boldsymbol{X}$ is $|\mathcal{A}_s|^{\bar{N}_I}$, which we collect into a finite set $\mathcal{A}_x$ with $|\mathcal{A}_x| = |\mathcal{A}_s|^{\bar{N}_I}$. From a conceptual point of view, STF coding is equivalent to constructing the finite set $\mathcal{A}_x$, as well as specifying the mapping Ψ. By the definition of $\boldsymbol{X}$, it is clear that transmitting $\bar{N}_I$ information symbols uses N_c subcarriers and occupies N_x OFDM symbols. Therefore, the STF *code rate* is defined as

$$R = \frac{\bar{N}_I}{N_c N_x}. \tag{7}$$

Accounting for the CP and the constellation size $|\mathcal{A}_s|$, the transmission rate is therefore $R_T = \bar{N}_I/((N_c + L)N_x) \log_2 |\mathcal{A}_s|$ bps/Hz. Based on (1) or its matrix counterpart in (5), our goal is to strike the best tradeoffs among performance, rate, and complexity by carefully designing Ψ and properly choosing system parameters. To achieve this goal, we first have to link the design of STF coding with the performance of the STF transmission in (5).

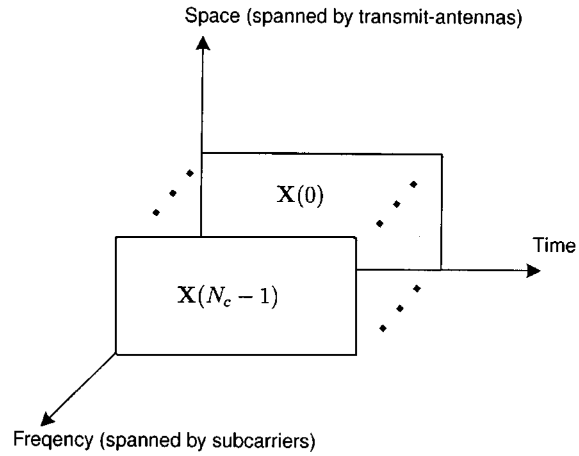

Fig. 2. Illustration of STF-coded transmissions.

III. Design Criteria

In this section, we will derive design criteria for our STF coding while keeping in mind the importance of simplifying the code design as much as possible without sacrificing system performance. Our derivations are based on the following assumptions.

as1) Maximum likelihood (ML) detection is performed with channel state information (CSI) that is known at the receiver. CSI can be acquired either via preamble training or via inserted pilots, as in [12] and [20].

as2) High SNR is observed at the receiver.

as3) The $N_t(L+1) \times 1$ channel vector $\boldsymbol{h}_{\nu} := \left[\boldsymbol{h}_{1\nu}^T,\ldots,\boldsymbol{h}_{N_t\nu}^T\right]^T$ is zero-mean, complex Gaussian with full-rank correlation matrix $E\left(\boldsymbol{h}_{\nu}\boldsymbol{h}_{\nu}^{\mathcal{H}}\right) = (1/L+1)\boldsymbol{R}_h$. However, $\boldsymbol{h}_{\nu}$s for different ν are statistically independent, which can be satisfied by well separating the multiple receive antennas.

Notice that $\boldsymbol{R}_h$ in as3) allows for correlated wireless channel taps with, e.g., an exponential power profile. However, as we prove later, our design of STF coding will turn out to be independent of $\boldsymbol{R}_h$ as long as $\boldsymbol{R}_h$ has full rank. Thus, we will be able to simplify our design as we will show in Section III-C. Note that channel taps are assumed independent in [5], [13], and [18], which is quite idealistic for frequency-selective channels.

A. Pairwise Error Probability Analysis

We consider here the optimal performance possible when STF transmissions obey as1)–as3). Even when exact performance analysis is possible, it may not lead to meaningful design criteria. Alternately, and similar to [11], [18], [22], and [27], we will rely on pairwise error probability (PEP) analysis. Recalling (5), the PEP $P(\boldsymbol{X} \rightarrow \boldsymbol{X}')$ is defined as the probability that ML decoding of $\boldsymbol{X}$ erroneously decides $\boldsymbol{X}'$ in favor of the actually transmitted $\boldsymbol{X}$.

According to (5), ML decoding of $\boldsymbol{X}$ from $\{\boldsymbol{Y}(p)\}_{p=0}^{N_c-1}$ yields

$$\hat{\boldsymbol{X}} = \arg\min_{\boldsymbol{X}\in\mathcal{A}_x} \sum_{p=0}^{N_c-1} \|\boldsymbol{Y}(p) - \boldsymbol{H}(p)\boldsymbol{X}(p)\|^2 \tag{8}$$

where $\|\cdot\|$ denotes the Frobenius norm. Conditioned on $\boldsymbol{H}(p)$s, it then follows that

$$P\left(\boldsymbol{X}\to\boldsymbol{X}'|\boldsymbol{H}(0),\ldots,\boldsymbol{H}(N_c-1)\right) \le \exp\left[-\frac{d^2(\boldsymbol{X},\boldsymbol{X}')}{4N_0}\right] \tag{9}$$

where

$$d^2(\boldsymbol{X},\boldsymbol{X}') = \sum_{p=0}^{N_c-1} \|\boldsymbol{H}(p)\boldsymbol{\Delta}(p)\|^2 \tag{10}$$

and $\boldsymbol{\Delta}(p) := \boldsymbol{X}(p) - \boldsymbol{X}'(p)$. Defining $\boldsymbol{\omega}(p) := [1, \exp(-j2\pi p/N_c), \ldots, \exp(-j2\pi Lp/N_c)]^T \in \mathbb{C}^{(L+1)\times 1}$, we can express (2) as

$$H_{\mu\nu}(p) = \boldsymbol{h}_{\mu\nu}^T \boldsymbol{\omega}(p). \tag{11}$$

Because $\boldsymbol{R}_h$ is positive definite Hermitian symmetric [cf. as3)], we can decompose $\boldsymbol{R}_h$ as $\boldsymbol{R}_h = \boldsymbol{B}_h\boldsymbol{B}_h^{\mathcal{H}}$, where $\boldsymbol{B}_h \in \mathbb{C}^{N_t(L+1)\times N_t(L+1)}$ is the square root of $\boldsymbol{R}_h$ with full rank. We further define the $N_t(L+1)\times 1$ prewhitened channel vector $\bar{\boldsymbol{h}}_\nu := [\bar{h}_{1\nu}(0), \ldots, \bar{h}_{1\nu}(L), \ldots, \bar{h}_{N_t\nu}(0), \ldots, \bar{h}_{N_t\nu}(L)]^T = \boldsymbol{B}_h^{-1}\boldsymbol{h}$. It follows from as3) that $\bar{h}_{\mu\nu}(l)$s are independent and identically distributed (i.i.d), zero mean, complex Gaussian with variance $1/(2L+2)$ per dimension. Plugging (11) into (10) and based on eigenanalysis and standard derivations, $d^2(\boldsymbol{X},\boldsymbol{X}')$ can be rewritten as

$$d^2(\boldsymbol{X},\boldsymbol{X}') = \sum_{\nu=1}^{N_r} \boldsymbol{h}_\nu^T \boldsymbol{\Lambda}_e \boldsymbol{h}_\nu^* = \sum_{\nu=1}^{N_r} \bar{\boldsymbol{h}}_\nu^T \bar{\boldsymbol{\Lambda}}_e \bar{\boldsymbol{h}}_\nu^* \tag{12}$$

where

$$\begin{aligned}\bar{\boldsymbol{\Lambda}}_e &:= \boldsymbol{B}_h^T \boldsymbol{\Lambda}_e \boldsymbol{B}_h^* \in \mathbb{C}^{N_t(L+1)\times N_t(L+1)}\\ \boldsymbol{\Lambda}_e &:= \sum_{p=0}^{N_c-1} \boldsymbol{\Omega}(p)\boldsymbol{\Delta}(p)\boldsymbol{\Delta}^{\mathcal{H}}(p)\boldsymbol{\Omega}^{\mathcal{H}}(p)\\ &\quad\in \mathbb{C}^{N_t(L+1)\times N_t(L+1)}\\ \boldsymbol{\Omega}(p) &:= \boldsymbol{I}_{N_t} \otimes \boldsymbol{\omega}(p) \in \mathbb{C}^{N_t(L+1)\times N_t}\end{aligned} \tag{13}$$

with $\otimes$ denoting the Kronecker product.

Because the design of STF coding should not depend on particular channel realizations, it is appropriate to consider the expected PEP averaged over all channel realizations. Let us suppose that $\bar{\boldsymbol{\Lambda}}_e$ has rank rank$(\bar{\boldsymbol{\Lambda}}_e)$ and denote its nonzero eigenvalues as $\lambda_{e,i}$, $i = 1, \ldots, \text{rank}(\bar{\boldsymbol{\Lambda}}_e)$. Using as2) and as3), the expected PEP is given by

$$\bar{P}(\boldsymbol{X}\to\boldsymbol{X}' \le \left(G_{e,c}\frac{1}{4N_0}\right)^{-G_{e,d}} \tag{14}$$

where

$$\begin{aligned}G_{e,d} &= N_r \cdot \text{rank}(\bar{\boldsymbol{\Lambda}}_e), \text{ and}\\ G_{e,c} &= \left[\prod_{i=1}^{\text{rank}(\bar{\boldsymbol{\Lambda}}_e)} \frac{1}{L+1}\lambda_{e,i}\right]^{1/\text{rank}(\bar{\boldsymbol{\Lambda}}_e)}\end{aligned} \tag{15}$$

are, respectively, the pairwise diversity and coding advantages. Noting that both $G_{e,d}$ and $G_{e,c}$ depend on choice of the pair $\{\boldsymbol{X},\boldsymbol{X}'\}$, we further define the (overall) diversity and coding advantages, respectively, as

$$G_d = \min_{\forall\, \boldsymbol{X}\neq\boldsymbol{X}'\in\mathcal{A}_x} G_{e,d}, \quad \text{and} \quad G_c = \min_{\forall\, \boldsymbol{X}\neq\boldsymbol{X}'\in\mathcal{A}_x} G_{e,c} \tag{16}$$

where the minimization is taken over all pairs of distinct STF codewords in 3-D.

It is well known that at reasonably high SNR, the diversity advantage plays a more important role than the coding advantage when it comes to improving the performance in wireless fading channels [22]. Thus, our STF coding will focus on achieving maximum diversity advantage while improving coding advantage as much as possible.

The expression of $G_{e,d}$ and $G_{e,c}$ in (15) has important implications on system design. First, checking the dimensionality of $\bar{\boldsymbol{\Lambda}}_e$ reveals that the maximum diversity advantage in our system is $G_d^{\max} = N_r N_t(L+1)$, which is the same as that proved in [27] (as well as in [4] and [18] but without providing a full-rate ST system that achieves it). Second, because $\boldsymbol{B}_h$ has full rank, the maximum diversity advantage can be achieved if and only if $\boldsymbol{\Lambda}_e$ has full rank. Since the maximum rank of each summand $\boldsymbol{\Omega}(p)\boldsymbol{\Delta}(p)\boldsymbol{\Delta}^{\mathcal{H}}(p)\boldsymbol{\Omega}^{\mathcal{H}}(p)$ in $\boldsymbol{\Lambda}_e$ is N_t, the number of linearly independent rows or columns contributed by each summand is no more than N_t. Therefore, in order to maximize the diversity advantage, we need to have $N_c \ge L+1$. The latter suggests that we should consider joint coding across multiple (at least $L+1$) frequency slots (subcarriers). On the other hand, since $\boldsymbol{\Lambda}_e$ could have full rank even if $N_x = 1$, coding across multiple time slots is not "a must." In other words, it is possible to achieve $G_d^{\max}$ with space-frequency coding (as suggested in [1], [4], and [18]), which is certainly subsumed by our STF coding. However, as we discuss later, taking into account the time dimension will considerably simplify the code design and will offer design flexibility. Third, when the maximum diversity advantage is achieved, i.e., $\boldsymbol{\Lambda}_e$ has full rank, $G_{e,c}$ can be factored into

$$\begin{aligned}G_{e,c} &= \frac{1}{L+1}\left[\det(\bar{\boldsymbol{\Lambda}}_e)\right]^{1/N_t(L+1)}\\ &= \frac{1}{L+1}\left[\det(\boldsymbol{R}_h)\det(\boldsymbol{\Lambda}_e)\right]^{1/N_t(L+1)}.\end{aligned} \tag{17}$$

Therefore, maximizing the coding advantage translates into maximizing the determinant $\det(\boldsymbol{\Lambda}_e)$, which is independent of the channel correlation as long as $\boldsymbol{R}_h$ has full rank.

Thus far, we have seen how the codeword set $\mathcal{A}_x$ is linked to the diversity advantage G_d, and the coding advantage G_c [cf. (15) and (16)]. As in [22], one could certainly propose rank and determinant design criteria for STF coding, which would involve designing the set $\mathcal{A}_x$ with codewords $\boldsymbol{X}$ of size $N_t \times N_x N_c$ so that both G_d and G_c are maximized. It is noted that these two design criteria are reduced to those proposed for SF coding when $N_x = 1$. However, N_c is typically a large number in practical OFDM systems (e.g., $N_c = 48$ in HIPERLAN 2 [7]). Therefore, design of STF codes or SF codes for such systems would entail large size codewords. Thinking of the difficulties already encountered in designing ST or SF codes of a much smaller size, it is expected that this design will be far more challenging, without any effort to alleviate the "curse of dimensionality." The tool we will use to reduce the dimensionality, and thus facilitate design and decoding, is subchannel grouping, which we describe next.

B. Subchannel Grouping

Subchannel grouping was suggested in [17] to reduce design and decoding complexity while preserving both diversity and coding advantages for *single-antenna* linear constellation precoded OFDM systems. Subcarrier grouping was originally proposed in [24] for multiuser interference elimination and later in [10] for peak-to-average ratio (PAR) reduction. For STF coding, the first step toward subchannel grouping is to choose the number of subcarriers equal to an integer multiple of the channel length

$$N_c = N_g(L+1) \tag{18}$$

for a certain positive integer N_g denoting the number of groups. The second step is to split the $N_t \times N_c N_x$ STF codeword $\boldsymbol{X}$ into N_g group STF (GSTF) codewords $\boldsymbol{X}_g$, $g = 0, 1, \ldots, N_g - 1$:

$$\boldsymbol{X}_g = [\boldsymbol{X}_g(0), \boldsymbol{X}_g(1), \ldots, \boldsymbol{X}_g(L)] \in \mathbb{C}^{N_t \times N_x(L+1)} \tag{19}$$

where $\boldsymbol{X}_g(l) := \boldsymbol{X}(N_g l + g)$. Accordingly, we divide the STF system of (5) into N_g GSTF (sub)systems, which we describe through the input-output relationships

$$\begin{aligned}\boldsymbol{Y}_g(l) =& \boldsymbol{H}_g(l)\boldsymbol{X}_g(l) + \boldsymbol{W}_g(l),\\ l =& 0, \ldots, L, \quad g = 0, 1, \ldots, N_g - 1\end{aligned} \tag{20}$$

where,

$$\begin{aligned}\boldsymbol{Y}_g(l) :=& \boldsymbol{Y}(N_g l + g),\\ \boldsymbol{H}_g(l) :=& \boldsymbol{H}(N_g l + g), \quad \boldsymbol{W}_g(l) := \boldsymbol{W}(N_g l + g).\end{aligned}$$

Each GSTF subsystem is nothing but a simplified STF system with a much smaller size in the frequency dimension, as compared with the original STF system. To take advantage of subchannel grouping, we will consider STF coding within each GSTF subsystem, i.e., we will perform STF coding to generate $\boldsymbol{X}_g$s individually, rather than generating $\boldsymbol{X}$ as a whole. As we will show in the next subsection, doing so will not incur any reduction in the diversity advantage, whereas it will reduce the design complexity considerably. To distinguish GSTF from the STF coding in (6), we hereafter name the STF coding for each GSTF subsystem as GSTF coding and denote it by the unique mapping

$$\Psi_g \colon \boldsymbol{s}_g \rightarrow \boldsymbol{X}_g \tag{21}$$

where $\boldsymbol{s}_g \in \mathbb{C}^{N_I \times 1}$ is the information symbol block used to generate $\boldsymbol{X}_g$. It is clear that we have $\bar{N}_I = N_g N_I$, and thus, the code rate in (7) can be re-expressed as

$$R = \frac{N_I}{(L+1)N_x}. \tag{22}$$

So far, we have converted the design of Ψ into the design of the set $\{\Psi_g\}_{g=0}^{N_g-1}$. Since all Ψ_gs are basically uniform, we will only focus on one of them in the ensuing discussion.

C. Design Criteria

We next derive design criteria for the GSTF codes $\boldsymbol{X}_g$ by analyzing the expected PEP $\bar{P}(\boldsymbol{X}_g \rightarrow \boldsymbol{X}'_g)$ for two distinct GSTF codewords $\boldsymbol{X}_g := [\boldsymbol{X}_g(0), \ldots, \boldsymbol{X}_g(L)]^T$ and $\boldsymbol{X}'_g := [\boldsymbol{X}'_g(0), \ldots, \boldsymbol{X}'_g(L)]^T$. Before doing so, let us introduce the following notation:

$$\begin{aligned}\boldsymbol{\Delta}_g(l) :=& \boldsymbol{X}_g(l) - \boldsymbol{X}'_g(l) \in \mathbb{C}^{N_t \times N_x}\\ \boldsymbol{\Omega}_g(l) :=& \boldsymbol{\Omega}(N_g l + g) \in \mathbb{C}^{N_t(L+1) \times N_t}\\ \boldsymbol{\Lambda}_{e,g} :=& \sum_{l=0}^{L} \boldsymbol{\Omega}_g(l)\boldsymbol{\Delta}_g(l)\boldsymbol{\Delta}_g^{\mathcal{H}}(l)\boldsymbol{\Omega}_g^{\mathcal{H}}(l)\\ &\in \mathbb{C}^{N_t(L+1) \times N_t(L+1)}\\ \bar{\boldsymbol{\Lambda}}_{e,g} :=& \boldsymbol{B}_h^T \boldsymbol{\Lambda}_{e,g} \boldsymbol{B}_h^* \in \mathbb{C}^{N_t(L+1) \times N_t(L+1)}.\end{aligned} \tag{23}$$

Following steps similar to those used to derive (14), we obtain the pairwise diversity advantage

$$G_{g,e,d} = N_r \text{rank}(\bar{\boldsymbol{\Lambda}}_{e,g}) \tag{24}$$

where rank$(\bar{\boldsymbol{\Lambda}}_{e,g})$ denotes the rank of $\bar{\boldsymbol{\Lambda}}_{e,g}$. When $\bar{\boldsymbol{\Lambda}}_{e,g}$ has full rank $N_t(L+1)$, the coding advantages for GSTF subsystems is given by

$$G_{g,e,c} = \frac{1}{L+1}\left[\det(\boldsymbol{R}_h)\det(\boldsymbol{\Lambda}_{e,g})\right]^{1/N_t(L+1)} \tag{25}$$

which is the per group counterpart of (17). As mentioned before, our design goal here is to preserve maximum diversity advantage and achieve the largest possible coding advantage. Let $\mathcal{A}_{x_g}$ be the set of all possible $\boldsymbol{X}_g$s. Following our arguments in Section III-A, this goal can be translated into designing $\mathcal{A}_{x_g}$ such that i) $\bar{\boldsymbol{\Lambda}}_{e,g}$ has full rank, and ii) $\det(\boldsymbol{\Lambda}_{e,g})$ is maximized. Although the design of $\boldsymbol{X}_g$ is clearly simpler than that of $\boldsymbol{X}$, it is still challenging. By exploiting the fact that our design is irrespective of the channel correlation, we next simplify our design further by letting $\boldsymbol{R}_h = \boldsymbol{I}$ in our STF system. For convenience, we term the resulting system as "dummy" STF system.

Since $\boldsymbol{R}_h = \boldsymbol{I}$ in our dummy system, the $N_t N_r(L+1)$ channel taps $h_{\mu\nu}(l)$ for $l = 0, \ldots, L$, $\mu = 1, \ldots, N_t$, $\nu = 1, \ldots, N_r$ become independent and identically distributed (i.i.d.), zero-mean, complex Gaussian with variance $1/(2L+2)$ per dimension per as3). It then follows that. $\forall\ \mu, \mu', \nu, \nu'$, and $p_1 \neq p_2$

$$E\left[H_{\mu\nu}(p_1)H^*_{\mu'\nu'}(p_2)\right] = 0, \text{ if } \mod(p_1 - p_2, N_g) = 0 \tag{26}$$

from which it is deduced that $H_{\mu\nu}(p_1)$ and $H_{\mu'\nu'}(p_2)$ are statistically independent. Consequently, our "dummy" GSTF subsystem possesses the following property.

Property 1: When $\boldsymbol{R}_h = \boldsymbol{I}$ and parameters in (18) are chosen, all subchannels within $\{\boldsymbol{H}_g(l)\}_{l=0}^{L}$ of each GSTF subsystem are statistically independent.

By utilizing Property 1, we reperform the PEP analysis to obtain the counterparts of (24) and (25), respectively, as

$$\begin{aligned} G_{g,e,d} &= N_r \sum_{l=0}^{L} \text{rank}(\boldsymbol{\Lambda}_e(l)) \\ G_{g,e,c} &= \left[\prod_{l=0}^{L} \det(\boldsymbol{\Lambda}_e(l))\right]^{1/N_t(L+1)} \end{aligned} \quad (27)$$

where $\boldsymbol{\Lambda}_e(l) := \boldsymbol{\Delta}_g(l)\boldsymbol{\Delta}_g^{\mathcal{H}}(l)$. Since the "dummy" STF system is a special case of our STF system, the diversity and coding advantages in (27) are identical to those in (24) and (25) with $\boldsymbol{R}_h = \boldsymbol{I}$. Targeting maximum diversity and coding advantages, we are therefore able to deduce the following two design criteria.

C1) (Sum-of-ranks criterion) Design $\mathcal{A}_{x_g}$ such that $\forall\, \boldsymbol{X}_g \neq \boldsymbol{X}'_g \in \mathcal{A}_{x_g}$, the matrices

$$\boldsymbol{\Lambda}_e(l) = [\boldsymbol{X}_g(l) - \boldsymbol{X}'_g(l)][\boldsymbol{X}_g(l) - \boldsymbol{X}'_g(l)]^{\mathcal{H}}, \quad \forall\, l \in [0, L] \quad (28)$$

have full rank.

C2) (Product-of-determinants criterion) For the set of matrices satisfying C1), design $\mathcal{A}_{x_g}$ such that $\forall\, \boldsymbol{X}_g \neq \boldsymbol{X}'_g \in \mathcal{A}_{x_g}$, which is the minimum of

$$\prod_{l=0}^{L} \det[\boldsymbol{\Lambda}_e(l)] \quad (29)$$

is maximized.

Regarding the design criteria C1) and C2), three remarks are due at this point.

Remark 1: In order to achieve maximum diversity gain, it follows from C1) that we must have $N_x \geq N_t$. Therefore, coding across different time slots is indispensable in GSTF coding. As compared with SF coding where no time-dimension processing is performed, allowing for coding across time in our design enjoys several advantages. First, checking the dimensionality of $\boldsymbol{X}_g$ [cf. (19)] reveals that the minimum size of the GSTF codewords is $N_t \times N_t(L+1)$, whereas the SF codeword size in [1], [4], [5], [12], and [18] is $N_t \times N_c$. Because codeword size affects directly the design complexity, GSTF coding enjoys lower design complexity relative to [4] and [18] since $N_c > N_t(L+1)$ in typical applications. Second, and more importantly, $\boldsymbol{\Lambda}_e(l)$ in both C1) and C2) is related to $\boldsymbol{X}_g(l)$ in a much simpler way, as compared with those in SF coding [4], [5], and [18]. This enables us to construct GSTF codes that achieve better performance than SF coding [4], [18],[2] as we will verify by simulations.

Remark 2: Checking the dimensionality of $\boldsymbol{\Lambda}_e(l)$ reveals that the maximum diversity advantage of each GSTF subsystem is $N_t N_r(L+1)$, which coincides with that of STF systems without subchannel grouping [27]. Thus, our subchannel grouping does not sacrifice the diversity order. It is not difficult to show that this result holds true even with arbitrary subchannel grouping [instead of (19)], as long as each GSTF subsystem contains $L+1$ subcarriers. However, arbitrary subchannel grouping generally involves correlated subchannels per "dummy" GSTF system [cf. Property 1], which will decrease the coding advantage. Thinking of the fact that the STF codes design does not depend on channel correlation, it is thus inferred that our subchannel grouping scheme in (19) is optimal in the sense of maximizing coding advantage for a GSTF system of a given size. Although we believe that the achieved coding advantage in each GSTF system comes close to that in the STF system without subchannel grouping, we have not been able to provide a general proof that certainly constitutes an interesting future research topic. Nevertheless, it is important to point out that even if subchannel grouping preserves both diversity and coding advantage, it does not necessarily preserve the BER performance. The reason is twofold. First, diversity and coding advantage is only a good approximation of the BER performance at high SNR. Second, the BER performance is also affected by other parameters such as the kissing number [6] that may increase due to subchannel grouping.

Remark 3: When $L = 0$, our STF design criteria C1) and C2) are, respectively, reduced to the rank and determinant criteria proposed for flat Rayleigh fading channels in [22]. When $N_x = 1$, C1) and C2) become the distance and product criteria used to construct ST codes for fast fading channels [22].

Having obtained the design criteria, we are in a position to design GSTF codes. Two approaches will be pursued: GSTF block (GSTFB) coding and GSTF trellis (GSTFT) coding.

IV. GSTF Block Codes

Based on the observation that the design of GSTF codewords $\boldsymbol{X}_g$ can be accomplished by the joint design of the ST codewords $\{\boldsymbol{X}_g(l)\}_{l=0}^{L}$ [cf. C1) and C2)], the encoding of GSTF codes is carried out in two successive stages: constellation precoding and ST component coding. Constellation precoding will enable multipath diversity, whereas the ST component coding will collect spatial diversity. With proper design, it will be shown that the resulting GSTF codes are capable of achieving maximum diversity and coding advantages while allowing one to perform low-complexity two-stage optimal decoding. We next describe our design in detail, starting from the encoding process.

A. Encoding

We first choose the parameter $N_I = N_s(L+1)$, where N_s depends on N_t, and will be specified soon. Then, we demultiplex the information symbol block $\boldsymbol{s}_g$ into $\{\boldsymbol{s}_{g,i} \in \mathbb{C}^{(L+1)\times 1}, i = 0, \ldots, N_s - 1\}$ sub-blocks so that $\boldsymbol{s}_g := \left[\boldsymbol{s}_{g,0}^T, \ldots, \boldsymbol{s}_{g,N_s-1}^T\right]^T$. The stage of constellation precoding is first invoked to distribute information symbols over multiple subcarriers by precoding $\boldsymbol{s}_{g,i}$ to obtain $\tilde{\boldsymbol{s}}_{g,i} := \boldsymbol{\Theta}\boldsymbol{s}_{g,i}$, where $\boldsymbol{\Theta} \in \mathbb{C}^{(L+1)\times(L+1)}$ denotes our square constellation precoder. The precoded blocks $\tilde{\boldsymbol{s}}_{g,i} \in \mathbb{C}^{(L+1)\times(L+1)}$ are subsequently processed to form the GSTF codeword $\boldsymbol{X}_g$ via the stage of ST component coding. Combining these two stages, we will be able to achieve $G_d^{\max}$, if constellation precoding and ST component coding are designed properly, as we describe next.

[2] In fact, [4] and [18] offer no code construction due to their difficulty.

1) ST Component Coding: The design of ST component coding is extended from the generalized complex orthogonal design (GCOD) in [21], which we review briefly for convenience.

Definition 1—Generalized Complex Orthogonal Design: Let $\boldsymbol{\mathcal{O}}_{N_t}$ be an $N_d \times N_t$ matrix with its nonzero entries drawn from the set $\{d_i, d_i^*, i = 0, \ldots, N_s - 1\}$. If

$$\boldsymbol{\mathcal{O}}_{N_t}^{\mathcal{H}} \boldsymbol{\mathcal{O}}_{N_t} = \alpha \sum_{i=0}^{N_s-1} |d_i|^2 \boldsymbol{I}_{N_t} \tag{30}$$

for some positive constant α, then $\boldsymbol{\mathcal{O}}_{N_t}$ is called a GCOD in variables $d_0, \ldots, d_{N_s-1}$.

On the existence of GCODs, it has been asserted by [21] that a GCOD can be found with size (N_s, N_d)

$$(N_s, N_d) = \begin{cases} (2,2), & \text{if } N_t = 2 \\ (3,4), & \text{if } N_t = 3,4 \\ (N_t, 2N_t), & \text{if } N_t > 4. \end{cases} \tag{31}$$

Using the nonzero entries $\{d_i, d_i^*, i = 0, \ldots, N_s - 1\}$, $\boldsymbol{\mathcal{O}}_{N_t}$ can be represented as [14]

$$\boldsymbol{\mathcal{O}}_{N_t} = \sum_{i=0}^{N_s-1} (\boldsymbol{A}_i d_i + \boldsymbol{B}_i d_i^*) \tag{32}$$

where, the real matrices $\{\boldsymbol{A}_i \in \mathbb{R}^{N_d \times N_t}, \boldsymbol{B}_i \in \mathbb{R}^{N_d \times N_t}, i = 0, \ldots, N_s - 1\}$ satisfy the following property: $\forall\, i,\ i'$

$$\begin{aligned} \boldsymbol{A}_i^T \boldsymbol{A}_{i'} + \boldsymbol{B}_i^T \boldsymbol{B}_{i'} &= \alpha \boldsymbol{I}_{N_t} \delta(i - i') \\ \boldsymbol{A}_i^T \boldsymbol{B}_{i'} &= \boldsymbol{0}. \end{aligned}$$

This property will come handy at the decoding stage. For example, when $N_t = 2$, the GCOD $\boldsymbol{\mathcal{O}}_2$ reduces to Alamouti's ST block codematrix [2]

$$\boldsymbol{\mathcal{O}}_2 = \begin{bmatrix} d_0 & d_1 \\ -d_1^* & d_0^* \end{bmatrix}$$

which can be represented as in (32) using

$$\begin{aligned} \boldsymbol{A}_0 &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{A}_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \\ \boldsymbol{B}_0 &= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{B}_1 = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}. \end{aligned}$$

Let us define $\tilde{\boldsymbol{s}}_{g,i} := [\tilde{s}_{g,i,0}, \ldots, \tilde{s}_{g,i,L}]^T$. To perform ST component coding, we first use $\boldsymbol{\mathcal{O}}_{N_t}$ to obtain matrix pairs $\{\boldsymbol{A}_i, \boldsymbol{B}_i\}_{i=0}^{N_s-1}$, and then, we construct $\boldsymbol{X}_g(l)$ as

$$\boldsymbol{X}_g^T(l) = \sum_{i=0}^{N_s-1} \left(\boldsymbol{A}_i \tilde{s}_{g,i,l} + \boldsymbol{B}_i \tilde{s}_{g,i,l}^*\right) \tag{33}$$

which is nothing but a GCOD in variables $\tilde{s}_{g,i,l}, i = 0, \ldots, N_s - 1$. We deduce from (33) that $N_x = N_d$. Thus, the code rate for GSTFB coding is

$$R = \frac{N_s}{N_d}. \tag{34}$$

According to (31), we have $R < 1$ if $N_t > 2$. This implies that our design will induce a loss in bandwidth efficiency if more than two transmit antennas are deployed.

Having specified the ST component coding, we move on to design the constellation precoder.

2) Constellation Precoding: With $\boldsymbol{\Theta} := [\boldsymbol{\theta}_0, \ldots, \boldsymbol{\theta}_L]^T$ denoting our precoder, we can write $\tilde{s}_{g,i,l} = \boldsymbol{\theta}_l^T \boldsymbol{s}_{g,i}$, where $\boldsymbol{\theta}_l^T \in \mathbb{C}^{1 \times (L+1)}$ is the lth row of $\boldsymbol{\Theta}$. Recalling that $\boldsymbol{X}_g^T(l)$ is a GCOD by construction, we use (30) to rewrite (28) as

$$\boldsymbol{\Lambda}_e(l) = \alpha \left[\sum_{i=0}^{N_s-1} |\boldsymbol{\theta}_l^T (\boldsymbol{s}_{g,i} - \boldsymbol{s'}_{g,i})|^2\right] \boldsymbol{I}_{N_t} \tag{35}$$

where the meaning of $\boldsymbol{s'}_{g,i} \neq \boldsymbol{s}_{g,i}$ is clear from the PEP analysis.

Because C1) will be automatically satisfied if C2) is satisfied, we consider C2) only. Plugging (35) into C2), we have

$$\begin{aligned} \xi(\boldsymbol{s}_g, \boldsymbol{s}'_g) :=& \prod_{l=0}^{L} \det[\boldsymbol{\Lambda}_e(l)] \\ =& \alpha^{N_t} \prod_{l=0}^{L} \left[\sum_{i=0}^{N_s-1} |\boldsymbol{\theta}_l^T (\boldsymbol{s}_{g,i} - \boldsymbol{s}'_{g,i})|^2\right]^{N_t}. \end{aligned} \tag{36}$$

Using the arithmetic-geometric mean inequality, (36) can be lower bounded by

$$\xi(\boldsymbol{s}_g, \boldsymbol{s}'_g) \geq \alpha^{N_t} N_s^{N_t} \prod_{l=0}^{L} \prod_{i=0}^{N_s-1} |\boldsymbol{\theta}_l^T (\boldsymbol{s}_{g,i} - \boldsymbol{s}'_{g,i})|^{N_t/N_s} \tag{37}$$

where the equality is satisfied when $|\boldsymbol{\theta}_l^T (\boldsymbol{s}_{g,i} - \boldsymbol{s}'_{g,i})|^2 = |\boldsymbol{\theta}_l^T (\boldsymbol{s}_{g,i'} - \boldsymbol{s}'_{g,i'})|^2, \forall\, i \neq i'$. According to C2), we are dealing with a max–min problem for all possible $\boldsymbol{s}_g \neq \boldsymbol{s}'_g$. Thus, it is meaningful to maximize only the lower bound in (37). Because the lower bound is attained when $|\boldsymbol{\theta}_l^T (\boldsymbol{s}_{g,i} - \boldsymbol{s}'_{g,i})|$s are equal for different is, we use $\bar{\boldsymbol{s}}$ to denote a generic $\boldsymbol{s}_{g,i}$. From C2), the design criterion for $\boldsymbol{\Theta}$ can be reduced to the following.

C3) (Product distance criterion) Design $\boldsymbol{\Theta}$ to maximize

$$\min_{\forall\, \bar{\boldsymbol{s}} \neq \bar{\boldsymbol{s}}'} \prod_{l=0}^{L} |\boldsymbol{\theta}_l^T (\bar{\boldsymbol{s}} - \bar{\boldsymbol{s}}')|. \tag{38}$$

Interestingly, C3) is exactly the design criterion used in [6], [9], and [25] to construct the so-called constellation precoder for flat fading channels. Therefore, we will not pursue the detailed construction of $\boldsymbol{\Theta}$ in this paper. Instead, we will simply borrow the precoders found in [25]. For example, when $L = 1, 3$ and QPSK modulation is employed, the precoders $\boldsymbol{\Theta}$ are given, respectively, by [25]

$$\begin{aligned} \boldsymbol{\Theta} =& \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & e^{j(\pi/4)} \\ 1 & e^{j(5\pi/4)} \end{bmatrix} \\ \boldsymbol{\Theta} =& \frac{1}{2} \begin{bmatrix} 1 & e^{j(\pi/8)} & e^{j(2\pi/8)} & e^{j(3\pi/8)} \\ 1 & e^{j(5\pi/8)} & e^{j(10\pi/8)} & e^{j(15\pi/8)} \\ 1 & e^{j(9\pi/8)} & e^{j(18\pi/8)} & e^{j(27\pi/8)} \\ 1 & e^{j(13\pi/8)} & e^{j(26\pi/8)} & e^{j(39\pi/8)} \end{bmatrix}. \end{aligned} \tag{39}$$

Different from the redundant and constellation-irrespective linear precoders used in [24], it is worthwhile to underscore that $\boldsymbol{\Theta}$ is square (thus nonredundant), and its constellation-specific design depends on the finiteness of $\mathcal{A}_s$.

Before we proceed to describe STFB decoding, we first summarize the design steps of GSTFB codes, as follows.

d1) Given N_t, N_r, and L, choose N_g, and $N_c = N_g(L+1)$.
d2) Determine N_s and N_d according to (31).

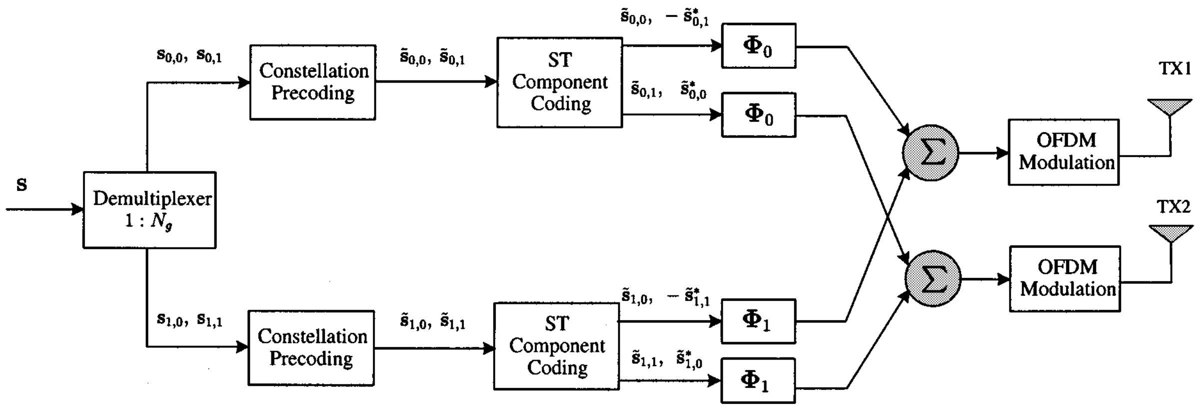

Fig. 3. STF block-coded OFDM with $N_t = 2$ and $N_g = 2$.

d3) Choose $N_I = N_s(L+1)$, $N_x = N_d$, and $\bar{N}_I = N_g N_I$.
d4) Design the $(L+1) \times (L+1)$ constellation precoder $\boldsymbol{\Theta}$ as in [25].
d5) Construct GSTF codewords according to (33).

For example, when $N_t = 2$ and $N_g = 2$, we illustrate our STF block coded OFDM system in Fig. 3, where the $N_c \times (L+1)$ matrices $\{\boldsymbol{\Phi}_g\}_{g=0}^{N_g-1}$ represent our subcarrier selectors that are used to assign $L+1$ subcarriers to each GSTF subsystem according to our subcarrier grouping scheme in Section III-B. Let us denote $\boldsymbol{e}(g,l) := [\boldsymbol{I}_{N_c}]_{(N_g l+g)} \in \mathbb{C}^{N_c \times 1}$ as the $(N_g l + g + 1)$st column of the $N_c \times N_c$ identity matrix $\boldsymbol{I}_{N_c}$. Mathematically, $\boldsymbol{\Phi}_g$ is given by

$$\boldsymbol{\Phi}_g = [\boldsymbol{e}(g,0), \boldsymbol{e}(g,1), \dots, \boldsymbol{e}(g,L)].$$

B. Decoding of GSTF Block Codes

Following the reverse order of the encoding process, we perform the decoding starting with ST component decoding to obtain decision statistics of $\tilde{\boldsymbol{s}}_{g,i}$ from which $\boldsymbol{s}_{g,i}$ is recovered by using ML decoding (or the lower complexity sphere decoding algorithm [8], [23]). Our decoding algorithm exploits the separability of GSTF transmissions in the frequency dimension.

Let us focus on $\boldsymbol{X}_g(l)$ and combine (20) and (33) to obtain

$$\boldsymbol{Y}_g^T(l) = \left[\sum_{i=0}^{N_s-1} (\boldsymbol{A}_i \tilde{s}_{g,i,l} + \boldsymbol{B}_i \tilde{s}^*_{g,i,l})\right] \boldsymbol{H}_g^T(l) + \boldsymbol{W}_g^T(l). \quad (40)$$

In order to decode $\tilde{s}_{g,i,l}$, we further define $\boldsymbol{\phi}_{l,i,\nu} \in \mathbb{C}^{2N_d \times 1}$ as

$$\boldsymbol{\phi}_{l,i,\nu}^T := \left[\sum_{\mu=1}^{N_t} [\boldsymbol{A}_i]_\mu^T H^*_{\nu\mu}(N_g l + g) \right| \times \left. \sum_{\mu=1}^{N_t} [\boldsymbol{B}_i]_\mu^T H_{\nu\mu}(N_g l + g)\right] \quad (41)$$

where, $[\boldsymbol{A}_i]_\mu$ and $[\boldsymbol{B}_i]_\mu$ denote the μth columns of $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$, respectively. Based on (41), the soft decision statistics of $\tilde{s}_{g,i,l}$ can be formed as

$$\hat{\tilde{s}}_{g,i,l} = \sum_{\nu=1}^{N_r} \boldsymbol{\phi}_{l,i,\nu}^T \begin{bmatrix} \boldsymbol{y}_{l,\nu} \\ \boldsymbol{y}^*_{l,\nu} \end{bmatrix} \quad (42)$$

with $\boldsymbol{y}_{l,\nu} \in \mathbb{C}^{N_d \times 1}$ denoting the νth column of $\boldsymbol{Y}_g^T(l)$. Using (30), one can verify by direct substitution that in the absence of noise

$$\hat{\tilde{s}}_{g,i,l} = \alpha \sum_{\nu=1}^{N_r} \sum_{\mu=1}^{N_t} |H_{\nu\mu}(N_g l + g)|^2 \tilde{s}_{g,i,l}. \quad (43)$$

After obtaining $\hat{\tilde{\boldsymbol{s}}}_{g,i} := [\hat{\tilde{s}}_{g,i,0}, \dots, \hat{\tilde{s}}_{g,i,L}]^T$, as discussed in [25] and [26], ML decoding of $\boldsymbol{s}_{g,i}$ can be performed by using the sphere decoding algorithm whose complexity is irrespective of the constellation size and polynomial in $L+1$. Because L is small relative to N_c, the decoding complexity of GSTFB codes is relatively low.

Having designed STF block codes, we turn our attention to STF trellis codes.

V. STF Trellis Coding

In this section, we design STF trellis codes by applying multiple trellis coded modulation (M-TCM) that has been used for developing ST trellis codes [22]. To enable application of M-TCM, we first build a link between our STF system and the conventional ST system.

A. Space Virtual-Time Transmissions

Recalling that (19) represents 3-D GSTF codes $\boldsymbol{X}_g$ as a set of 2-D codes, we have implicitly suggested that the transmission of $\boldsymbol{X}_g$ can be thought of as being carried out in an equivalent 2-D (what we term) space-virtual-time (SVT) system with N_t transmit antennas and N_r receive antennas, where the virtual-time dimension corresponds to the joint dimension of time and frequency in a way we detail next.

First, let us define $\boldsymbol{x}_t := [x_t^1, \dots, x_t^{N_t}]^T \in \mathbb{C}^{N_t \times 1}$ as the symbol block transmitted by N_t transmit antennas during the tth virtual time interval and $\boldsymbol{y}_t := [y_t^1, \dots, y_t^{N_r}]^T \in \mathbb{C}^{N_r \times 1}$ as the corresponding block of received samples. The SVT system is modeled as

$$\boldsymbol{y}_t = \boldsymbol{H}_t \boldsymbol{x}_t + \boldsymbol{w}_t, \quad t = 0, \dots, N_x(L+1) - 1 \quad (44)$$

where $\boldsymbol{H}_t \in \mathbb{C}^{N_r \times N_t}$ is the MIMO channel matrix, and $\boldsymbol{w}_t \in \mathbb{C}^{N_r \times 1}$ is the noise vector. To link the SVT system to the GSTF system, let us define [cf. (20)]

$$\boldsymbol{Y}_g := [\boldsymbol{Y}_g(0), \ldots, \boldsymbol{Y}_g(L)] \in \mathbb{C}^{N_r \times N_x(L+1)}$$
$$\boldsymbol{W}_g := [\boldsymbol{W}_g(0), \ldots, \boldsymbol{W}_g(L)] \in \mathbb{C}^{N_r \times N_x(L+1)}$$

and specify $\boldsymbol{y}_t$, $\boldsymbol{x}_t$, $\boldsymbol{w}_t$, and $\boldsymbol{H}_t$ as

$$\boldsymbol{y}_t = [\boldsymbol{Y}_g]_{\zeta(t)} \qquad \boldsymbol{x}_t = [\boldsymbol{X}_g]_{\zeta(t)}$$
$$\boldsymbol{w}_t = [\boldsymbol{W}_g]_{\zeta(t)} \qquad \boldsymbol{H}_t := \boldsymbol{H}_g(\tau(t)). \tag{45}$$

The index functions $\zeta(t)$ and $\tau(t)$ in (45) are given by (with $\lfloor \cdot \rfloor$ denoting integer floor)

$$\zeta(t) := N_x t - \left\lfloor \frac{t}{L+1} \right\rfloor (N_x L + N_x - 1) + 1$$
$$\tau(t) := t - \left\lfloor \frac{t}{L+1} \right\rfloor (L+1) \tag{46}$$

respectively. It is not difficult to recognize that except for the difference in the ordering of transmissions, the model in (44) is mathematically equivalent to that in (20). It is important to point out that in modeling our STF system as an SVT system, we have exploited the property that STF transmissions are separable in both frequency and time but not in space.

Although the underlying fading channels are time-invariant, we notice that $\boldsymbol{H}_t$ varies with virtualv time, due to the fact each GSTF codeword is transmitted over different subcarriers. Therefore, (44) can be thought of as an ST system transmitting over time-selective (but frequency-flat) fading channels. This provides an explicit link between each GSTF subsystem and the well-developed ST system. Furthermore,(44) implies that i) due to the presence of time diversity (or, precisely, virtual-time diversity), the SVT (or STF) system can potentially achieve diversity advantage higher than $N_t N_r$, which corroborates the results in Sections III and IV, and ii) it is possible to take advantage of existing ST coding techniques in designing GSTFT codes.

The so-called "smart-greedy" ST trellis codes have already been designed in [22] to achieve acceptable performance for both flat-fading and time-varying channels. Unlike [22], we are dealing with a different channel situation. Because the time-varying channel $\boldsymbol{H}_t$ herein is artificially created and its time-variations are well structured, we do not have to design ST codes that are "smart." Instead, we only need them to be "greedy" in order to take advantage of time selectivity. Therefore, it suffices for us to stay with the design criteria C1) and C2).

B. Code Construction

Based on (44), the design of STFT codes is equivalent to building a trellis to generate $\boldsymbol{x}_t$s continuously. Before pursuing their design, we state an important property of STFT codes.

Theorem 1: Suppose that each transmitted symbol x_t^μ belongs to the constellation set $\mathcal{A}_t$ with $|\mathcal{A}_t| = 2^b$ elements. If the maximum diversity advantage $G_d^{\max} = N_t N_r (L+1)$ is achieved, then the transmission rate is at most $R_t^{\max} = \log_2 |\mathcal{A}_t| / (L+1) = b/(L+1)$ bits/s/Hz.

Proof: The proof can be readily extended from [22, Th. 3.3.1, Corol. 3.3.1]. □

Because $R_t^{\max}$ is related to R by $R_t^{\max} = R \log_2 |\mathcal{A}_s|$, Theorem 1 implicitly suggests two possible design strategies.

Strategy 1: Design an STF trellis code with rate $R = 1$, where the trellis outputs a single block $\boldsymbol{x}_t$ corresponding to each information symbol s_t, and has cardinality $|\mathcal{A}_t| = |\mathcal{A}_s|^{(L+1)}$.

Strategy 2: Design an STF trellis code with code rate $R = 1/(L+1)$, where the trellis outputs $L+1$ blocks $\boldsymbol{x}_t$ corresponding to each s_t, and has cardinality $|\mathcal{A}_s| = |\mathcal{A}_t|$.

These two design strategies are equivalent in the sense that the resulting codes achieve the same transmission rate $R_t^{\max}$. Moreover, both strategies expand either the constellation of the transmitted symbols or that of the information symbols. However, their implementations are drastically different. According to the first strategy, one looks for a trellis involving constellation expansion, which is not the case for most existing ST trellis codes. Because we wish to take advantage of existing techniques developed for ST trellis codes, we will construct our STF trellis codes using the second strategy.

Let us denote with $\mathcal{T}_{\text{ST}}(\cdot)$ an ST trellis encoder with $R = 1$, and $|\mathcal{A}_s| = |\mathcal{A}_t| = 2^{R_t^{\max}(L+1)}$. According to Strategy 2, our STF trellis encoder with $R = 1/(L+1)$ is constructed from $\mathcal{T}_{\text{ST}}(\cdot)$ by simply repeating its output $L+1$ times. In other words, going back to each GSTF subsystem, we basically repeat transmissions over $L+1$ subcarriers. Therefore, the design of STFT trellis codes is reduced to designing conventional ST trellis codes with repeated transmissions. A similar approach was also taken in designing the "smart-greedy" ST trellis codes in [22]. For example, when $N_t = 2$, $L = 1$, and $R_t = 2$ bits/s/Hz, we can use the ST trellis depicted in [22, Fig. 19] to generate STFT codes.

C. Decoding of GSTF Trellis Codes

Because GSTF trellis codes are generated by a trellis, their decoding can be efficiently implemented by using Viterbi decoding. According to (44), the branch metric for $\boldsymbol{x}_t$ is given by

$$\sum_{\nu=1}^{N_r} |y_t^\nu - \sum_{\mu=1}^{N_t} [\boldsymbol{H}_t]_{\nu\mu} x_t^\mu|^2 \tag{47}$$

where $[\boldsymbol{H}_t]_{\nu\mu}$ denotes the (ν, μ)th element of $\boldsymbol{H}_t$. Similar to ST trellis codes [22], the decoding complexity of GSTF trellis codes is exponential in the number of trellis states and the transmission rate.

Thus far, we have designed STF block and trellis codes. Before resorting to simulations for testing their performance in Section VII, we first consider several issues pertaining to their performance and implementation.

VI. Design Issues

In this section, we address several design issues by providing answers to the following two questions.

Q1) How much diversity advantage is sufficient?

Q2) Is it better to use STF block or STF trellis coding?

A. Order of Diversity Advantage

As mentioned in Section I, the major motivation behind STF coding is to improve the diversity advantage from order $N_t N_r$

(in an ST coded system) to order $G_d^{\max} = N_t N_r(L+1)$ by capitalizing also on multipath diversity. On the other hand, intuition suggests that performance improvement by increasing $G_d^{\max}$ will eventually saturate as the diversity order grows beyond a certain level [22]. Hence, STF coding makes more sense when we are dealing with systems having a small number of antennas, which is also commercially preferable. In particular, we are interested in a minimal multiantenna system with $N_t = 2$ and $N_r = 1$ for downlink applications. As argued in [22], when $N_r = 1$, little can gained by using more than $N_t = 4$ transmit antennas in an ST coded system. Hence, $L = 1$ should be enough to provide sufficient diversity advantage in our STF system with $N_t = 2$ and $N_r = 1$, which is desirable if we recall that the smaller the L, the lower the decoding complexity for both STFB and STFT codes.

So far, we have restricted L to be the physical channel order. In the sequel, we lift this restriction by letting L_{real} denote the physical channel order and L the channel order assumed in designing STF codes. Obviously, L can be generally different from L_{real}. In typical wireless environments, we have $L_{\text{real}} > 2$. In order to minimize decoding complexity, we can simply choose $L \leq L_{\text{real}}$ and design our STF codes as detailed in Sections III–V. In fact, if $L_{\text{real}} \geq L$, the resulting system will be able to attain diversity advantage of order $N_r N_t(L+1)$ as well as high coding advantage. Clearly, the achieved performance will be sub-optimal in this case, but it will be sufficiently good for most practical applications.

B. STF Block Coding Versus STF Trellis Coding

Although our design applies to arbitrary N_t and N_r, we focus on the aforementioned minimal multiantenna system with $N_t = 2$ and $N_r = 1$. Accordingly, we compare STF block codes with STF trellis codes based on these parameters. Recall that with $N_t = 2$, there is no rate loss when using STF block coding.

The comparison between STF trellis codes and STF block codes is quite similar to that between ST trellis codes and ST block codes. The notable advantage of STF block codes over STF trellis codes is their low decoding complexity. More important, the decoding complexity of STF block codes is independent of the transmission rate [8], [23], which is not the case for STF trellis codes. In addition, the construction of STF block codes is easier than that of STF trellis codes. On the other hand, STF trellis coding operates in a way similar to conventional channel coding; thus, it can be easily incorporated to existing communication systems. For example, an ST trellis decoder can be directly applied to STF trellis codes.

VII. Simulations

In addition to theoretical analysis and discussions, we present simulations to investigate the performance of our designs in a minimum multiantenna OFDM system with $N_t = 2$ and $N_r = 1$. Our figure of merit is OFDM symbol error rate (OFDM-SER), which we average over 100 000 channel realizations. To maintain fairness, we will fix the transmission rate at $R_t = 2$ bps/Hz in all simulations. The random channels are generated according to two different channel models.

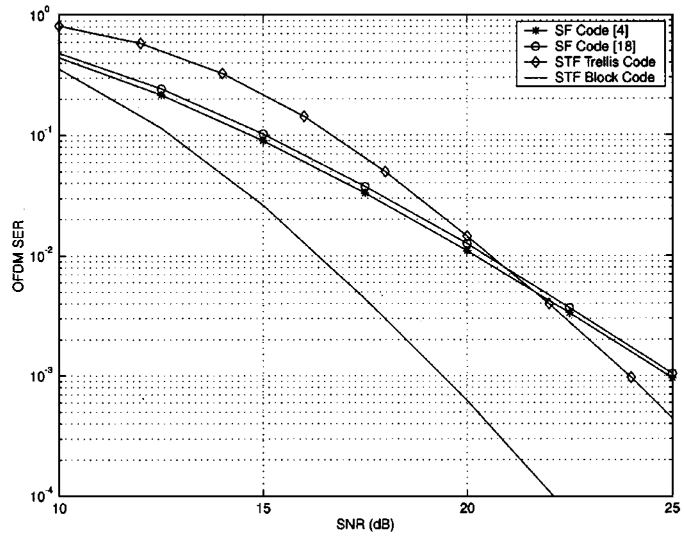

Fig. 4. Comparison with SF codes (multiray channels).

Multiray channel: This corresponds to channel taps that are i.i.d., zero-mean, complex Gaussian with variance $1/(2L_{\text{real}}+2)$ per dimension.

HiperLan 2 channel ($L_{\text{real}} = 8$): These random channels are based on the HiperLan 2 channel model A, which corresponds to a typical office environment [7]. Each channel tap in the profile of channel model A is characterized by the Jakes' Doppler spectrum with a mobile speed at 3 m/s.

In our simulations, we will make distinction between L and L_{real}, as mentioned in Section VI-A.

Example 1—Performance Comparison With Competing Schemes [4], [18]: In this example, we compare our GSTF block and trellis coding schemes to the SF coding schemes reported in [4], [18]. The design of both GSTF block and trellis codes is based on $L = 1$, which could be different from L_{real}. We choose QPSK modulation ($|\mathcal{A}_s| = 4$) for GSTF block coding and for the two SF coding schemes in [4], [18], whereas we select 16-QAM ($|\mathcal{A}_s| = 16$) for GSTF trellis coding. As a result, the transmission rate for all four schemes is $R_t = 2$ bits/s/Hz. We use the ST trellis code in [22, Fig. 19] to construct our STF trellis described in Section V-B. The 16-state TCM code with effective length 3 and the 16-state ST trellis code [22, Fig. 5] are used to generate the SF codes of [18] and [4], respectively. The number of subcarriers $N_c = 64$ is selected for all schemes. We first simulate all schemes in multiray channels with $L_{\text{real}} = 1$. As expected, Fig. 4 confirms that both GSTF trellis and block codes are able to achieve higher diversity gain than SF codes. While our GSTF block code outperforms SF codes consistently for all SNRs, our GSTF trellis code outperforms SF codes only when the SNR is larger than 20 dB. The latter implies that the STF trellis code we designed has poor coding gain, which is due to the fact that the chosen ST trellis code in [22, Fig. 19] is not optimal in terms of coding gain. Performance improvement can be expected if we consider maximizing both diversity and coding gains for GSTF trellis codes (as in GSTF block codes), which will usually involve high-complexity computer search similar to the one performed in e.g., [3].

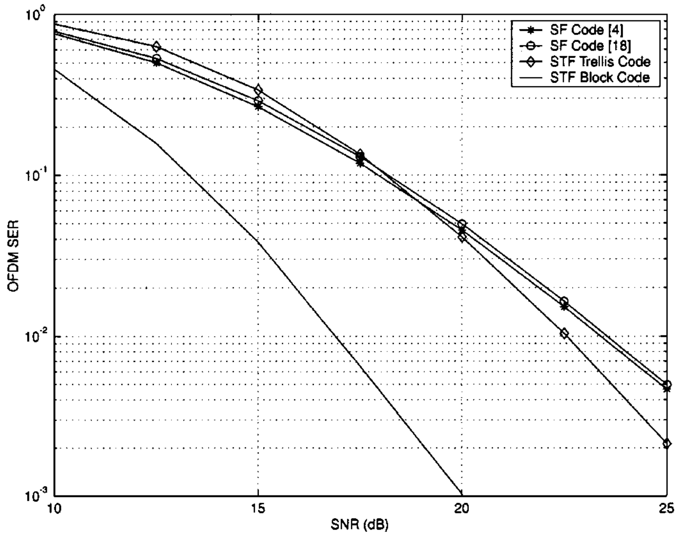

Fig. 5. Comparison with SF codes (HiperLan 2 channels).

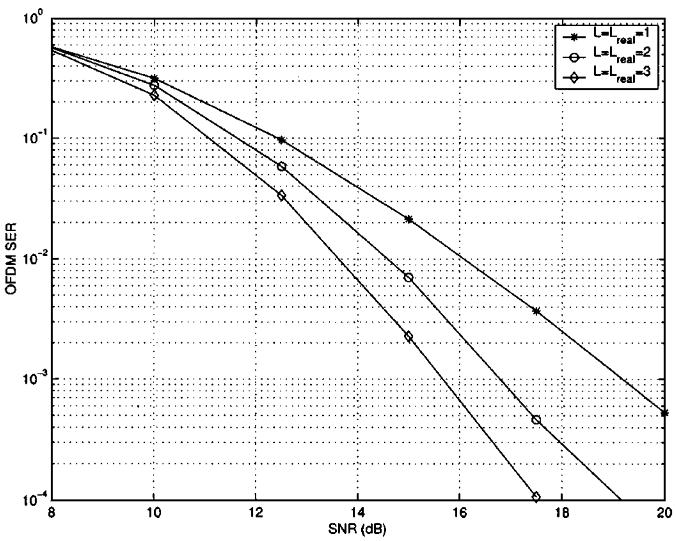

Fig. 6. Importance of multipath diversity (multiray channels).

Using the same setup, we repeat the simulation in more realistic HiperLan 2 channels with $L_{\mathrm{real}} = 8$. As shown in Fig. 5, both GSTF trellis and block codes enjoy higher diversity gain than SF codes. Surprisingly, the performance of our GSTF trellis code is close to that of SF codes, even for low SNR values. It is noted that our GSTF codes are designed regardless of the real channel order $L_{\mathrm{real}} = 8$, channel correlation, and power profile in HiperLan 2 channels, which speaks for the robustness of our GSTF coding design.

Example 2—Performance Improvement With Multipath Diversity: In order to appreciate the importance of multipath diversity, we simulate the performance of GSTF block coding in the presence of multiray channels with different channel orders $L_{\mathrm{real}} = 1, 2, 3$. Assuming perfect knowledge of L_{real}, we design GSTF block codes with $L = L_{\mathrm{real}}$. The number of subcarriers is chosen as $N_c = 48$. Fig. 6 confirms that GSTF codes achieve higher diversity gain as the channel order increases, which justifies the importance of GSTF coding that accounts for multipath diversity.

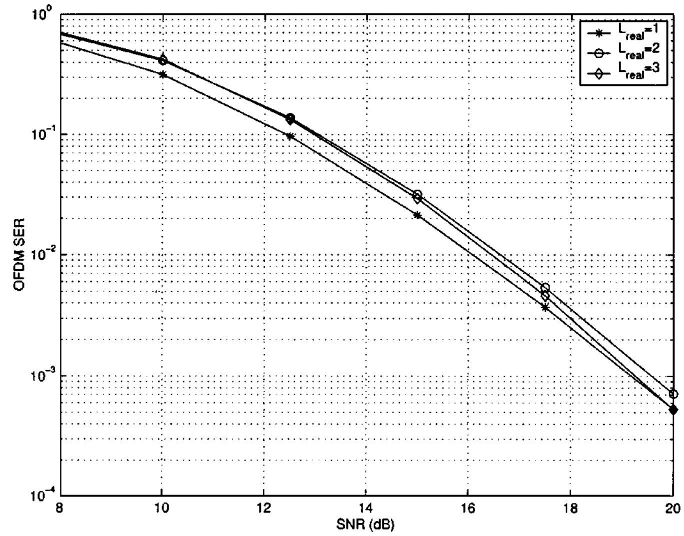

Fig. 7. Effects of channel order under-estimation (multiray channels).

Example 3—Effects of Channel Order Underestimation: In this example, we investigate the performance of GSTF block codes when the channel order is underestimated. In particular, we design our GSTF block code for $L = 1$ and simulate its performance when $L_{\mathrm{real}} = 1$, 2, 3. Multiray channels are used, and $N_c = 48$ is chosen. It is observed from Fig. 7 that the SER performance curves with different L_{real} are quite close. Combining these results with those of Example 2, we deduce that when maximum diversity gain is of interest, we should design GSTF codes with L chosen as the upperbound of all possible L_{real}s.

VIII. Concluding Remarks

We designed STF codes for multiantenna OFDM transmissions over frequency-selective Rayleigh fading channels. In order to simplify the design, we first performed subchannel grouping to convert the complex STF codes design into simpler GSTF designs per group. Based on the derived design criteria, we constructed both GSTF block codes, and GSTF trellis codes. We showed that the resulting GSTF codes are capable of achieving the full diversity gain, which equals the product of the number of transmit and receive antennas times the channel length. In addition to simplified design and low decoding complexity, the performance of our designs has been confirmed by simulations that also illustrate its merits relative to competing schemes.

Acknowledgment

The authors would like to thank X. Ma for her suggestions on the correlated channel case and the anonymous reviewers for their insightful comments.

References

[1] D. Agrawal, V. Tarokh, A. Naguib, and N. Seshadri, "Space-time coded OFDM high data-rate wireless communication over wideband channels," in *Proc. Veh. Technol. Conf.*, Ottawa, ON, Canada, May 18–21, 1998, pp. 2232–2236.

[2] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1451–1458, Oct. 1998.
[3] S. Baro, G. Bauch, and A. Hansmann, "Improved codes for space-time trellis-coded modulation," *IEEE Commun. Lett.*, vol. 4, pp. 20–22, Jan. 2000.
[4] H. Bölcskei and A. Paulraj, "Space-frequency coded broadband OFDM systems," in *Proc. of Wireless Commun. Networking Conf.*, Chicago, IL, Sept. 23–28, 2000, pp. 1–6.
[5] H. Bölcskei and A. J. Paulraj, "Space-frequency codes for broadband fading channels," in *Proc. IEEE Int. Symp. Inform. Theory*, Washington, DC, June 2001.
[6] J. Boutros and E. Viterbo, "Signal space diversity: A power- and bandwidth-efficient diversity technique for the rayleigh fading channel," *IEEE Trans. Inform. Theory*, vol. 44, pp. 1453–1467, July 1998.
[7] "Channel models for Hiperlan/2 in different indoor scenarios," Eur. Telecommun. Stand. Inst., Sophia-Antipolis, Valbonne, France, Norme ETSI, document 3ERI085B, 1998.
[8] M. O. Damen, A. Chkeif, and J. C. Belfiore, "Lattice code decoder for space-time codes," *IEEE Commun. Lett.*, vol. 4, pp. 161–163, May 2000.
[9] X. Giraud, E. Boutillon, and J. C. Belfiore, "Algebraic tools to build modulation schemes for fading channels," *IEEE Trans. Inform. Theory*, vol. 43, pp. 938–952, May 1997.
[10] D. L. Göeckel, "Coded modulation with nonstandard signal sets for wireless ofdm systems," in *Proc. Int. Conf. Commun.*, Vancouver, BC, Canada, June 1999, pp. 791–795.
[11] J. C. Guey, M. P. Fitz, M. R. Bell, and W. Y. Kuo, "Signal design for transmitter diversity wireless communication systems over rayleigh fading channels," *IEEE Trans. Commun.*, vol. 47, pp. 527–537, Apr 1999.
[12] Y. Li, J. C. Chung, and N. R. Sollenberger, "Transmitter diversity for OFDM systems and its impact on high-rate data wireless networks," *IEEE J. Select. Areas Commun.*, vol. 17, pp. 1233–1243, July 1999.
[13] Y. Liu, M. P. Fitz, and O. Y. Takeshita, "Space-time codes performance criteria and design for frequency selective fading channels," in *Proc. . Int. Conf. Commun.*, Helsinki, Finland, June 11–15, 2001, pp. 2800–2804.
[14] Z. Liu and G. B. Giannakis, "Space-time block coded multiple access through frequency-selective fading channels," *IEEE Trans. Commun.*, vol. 49, pp. 1033–1044, June 2001.
[15] Z. Liu, G. B. Giannakis, S. Barbarossa, and A. Scaglione, "transmit antennae space-time block coding for generalized OFDM in the presence of unknown multipath," *IEEE J. Select. Areas Commun.*, vol. 19, pp. 1352–1364, July 2001.
[16] Z. Liu, G. B. Giannakis, B. Muquet, and S. Zhou, "Space-time coding for broadband wireless communications," *Wireless Syst. Mobile Comput.*, vol. 1, no. 1, pp. 33–53, Jan.–Mar. 2001.
[17] Z. Liu, Y. Xin, and G. B. Giannakis, "Linear constellation precoding for OFDM with maximum multipath diversity and coding gains," in *Proc. 35th Asilomar Conf. Signals, Syst., Comput.*, Pacific Grove, CA, Nov 4–7, 2001, pp. 1445–1449.
[18] B. Lu and X. Wang, "Space-time code design in OFDM systems," in *Proc. Global Telecommun. Conf.*, vol. 2, San Francisco, CA, Nov.–Dec. 27–1, 2000, pp. 1000–1004.
[19] A. F. Naguib, N. Seshadri, and R. Calderbank, "Increasing date rate over wireless channels," *IEEE Signal Processing Mag.*, vol. 17, pp. 76–92, May 2000.
[20] S. Ohno and G. B. Giannakis, "Optimal training and redundant precoding for block transmissions with application to wireless OFDM," *IEEE Trans. Commun.*, 2002, to be published.
[21] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1456–1467, July 1999.
[22] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, pp. 744–765, Mar. 1998.
[23] E. Viterbo and J. Boutros, "A universal lattice code decoder for fading channels," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1639–1642, July 1999.
[24] Z. Wang and G. B. Giannakis, "Wireless multicarrier communications: Where fourier meets shannon," *IEEE Signal Processing Mag.*, vol. 17, pp. 29–48, May 2000.
[25] ——, "Space-time diversity systems based on linear constellation precoding," *IEEE Trans. Wireless Commun.*, 2002, to be published.
[26] ——, "Space-time diversity systems based on unitary constellation-rotating precoders," in *Proc. Int. Conf. Acoust., Speech, Signal Process.*, Salt Lake City, UT, May 7–11, 2001, pp. 2249–2432.
[27] S. Zhou and G. B. Giannakis, "Space-time coding with maximum diversity gains over frequency-selective fading channels," *IEEE Signal Processing Lett.*, vol. 8, pp. 269–272, Oct. 2001.

Zhiqiang Liu received the B.S. degree from the Department of Radio and Electronics, Peking University, Beijing, China, in 1991 and the M.E. degree from the Institute of Electronics, Chinese Academy of Science, Beijing, in 1994, and the Ph.D. degree from the Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, in 2001.

From 1995 to 1997, he worked as a research scholar at the Department of Electrical Engineering, National University of Singapore. From 1999 to 2001, he was a research assistant with the University of Minnesota. He is now an assistant professor with the Department of Electrical and Computer Engineering, University of Iowa, Iowa City. His research interests lie in the areas of wireless communications and signal processing, including space-time coding, diversity techniques, wideband wireless communication systems, multiuser detection, multicarrier, and blind channel estimation algorithms.

Yan Xin (S'00) received his B.E. degree in electronics engineering from Beijing Polytechnic University, Beijing, China, in 1992 and the M.Sc. degree in mathematics and the M.Sc. degree in electrical engineering from University of Minnesota, Minneapolis, in 1998 and 2000, respectively. He is currently pursuing the Ph.D. degree with the Department of Electrical and Computer Engineering at the University of Minnesota.

His research interests include space-time coding, diversity techniques, and multicarrier transmissions.

Georgios B. Giannakis (F'97) received the Diploma degree in electrical engineering from the National Technical University of Athens, Athens, Greece in 1981. From September 1982 to July 1986, he was with the University of Southern California (USC),Los Angeles, where he received the M.Sc. degree in electrical engineering in 1983, the M.Sc. degree in mathematics in 1986, and the Ph.D. in electrical engineering in 1986.

After lecturing for one year at USC, he joined the University of Virginia, Charlotteville, in 1987, where he became a Professor of electrical engineering in 1997. Since 1999, he has been a Professor with the Department of Electrical and Computer Engineering, University of Minnesota, Mineapolis, where he now holds an ADC Chair in Wireless Telecommunications. His general interests span the areas of communications and signal processing, estimation and detection theory, time-series analysis, and system identification—subjects on which he has published more than 140 journal papers, 270 conference papers, and two edited books. Current research topics focus on transmitter and receiver diversity techniques for single- and multiuser fading communication channels, precoding and space-time coding for block transmissions, multicarrier, and wideband wireless communication systems. He is a frequent consultant to the telecommunications industry.

Dr. Giannakis was the (co-) recipient of four best paper awards from the IEEE Signal Processing (SP) Society in 1992, 1998, 2000, and 2001. He also received the Society's Technical Achievement Award in 2000. He co-organized three IEEE-SP Workshops, and guest (co-) edited four special issues. He has served as Editor-in-Chief for the IEEE SIGNAL PROCESSING LETTERS, as Associate Editor for the IEEE TRANSACTIONS ON SIGNAL PROCESSING and the IEEE SIGNAL PROCESSING LETTERS, as secretary of the SP Conference Board, as member of the SP Publications Board, as member and vice chair of the Statistical Signal and Array Processing Technical Committee, and as chair of the SP for Communications Technical Committee. He is a member of the Editorial Board for the PROCEEDINGS OF THE IEEE and the steering committee of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. He is a member of the IEEE Fellows Election Committee and of the IEEE-SP Society's Board of Governors.