Design of User Codes in QS-CDMA Systems for MUI Elimination in Unknown Multipath

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***Abstract*—Low-complexity quasi-synchronous CDMA systems are derived for eliminating MUI completely in the presence of unknown and even rapidly varying multipath. They rely on judiciously designed precomputable user codes and offer a generalization of orthogonal frequency division multiplexing, especially valuable when carrier frequency errors and Doppler effects are present.**

I. Introduction and System Modeling

THE RECOGNITION of code-division multiple access (CDMA) as a promising multiple access scheme was followed by efforts to design simple linear receivers (zero-forcing or mean-square error) capable of handling asynchronous users, near-far effects, and multipath (see, e.g., [3], [6], [7], [10]). Because wireless communications entail unknown multipath channels, blind (SVD-based and adaptive inverse filtering) approaches have been proposed recently, in order to obviate bandwidth-consuming training sequences [5], [9]. They rely on the codes and the received data, but have relatively high complexity when subspace channel estimation is employed, or reduced capability to suppress MUI especially when simple (e.g., LMS-type) receivers are adopted to equalize fast changing channels.

In this letter, we introduce novel low-complexity CDMA transceivers capable of eliminating MUI deterministically in the presence of *unknown* and even rapidly varying multipath. More important, the resulting receivers do not rely on the received data because they are precomputable from appropriately designed user codes. Unlike existing blind CDMA systems that mitigate MUI at the *receiver*, the proposed system selects user codes at the *transmitter* so that MUI is suppressed *irrespective* of the frequency-selective multipath which is turned into flat fading. The latter is analogous to channel effects encountered with orthogonal frequency-division multiplexing (OFDM), which is the standard for digital audio broadcasting in Europe [1], [2], and has been studied recently for multiuser quasi-synchronous (QS) CDMA systems in [4] and [8]. Multiuser OFDM (or OFDMA) follows as a special case of our general code design algorithm. In QS-CDMA systems, users attempt to transmit following the base-station's clock waveform (or a GPS as suggested in [4]). However, the received waveforms at the base-station are still asynchronous by a few chips due to oscillator drifts and relative motion between the mobiles and the base-station.

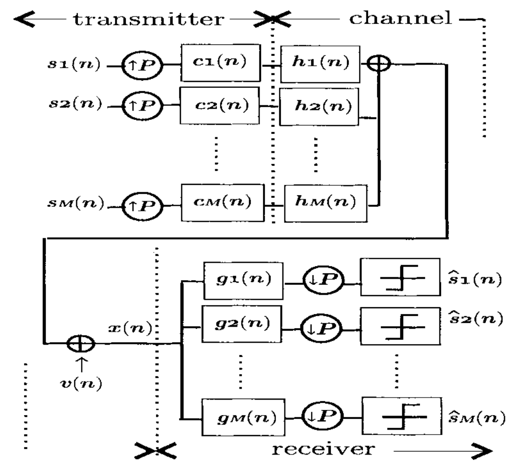

Fig. 1. Multirate discrete-time CDMA model.

The block diagram in Fig. 1 represents a CDMA system, described in terms of its equivalent discrete-time baseband model, where signals, codes, and channels are represented by samples of their complex envelopes taken at the chip rate (see also [9], [10]). Upsamplers and downsamplers serve the purpose of multiplexing and demultiplexing (spreading and despreading) by a factor P. Each of the M users spreads the information sequence $s_m(n)$ with the upsampler and encodes it using the code $c_m(n)$ of length P, before transmission through the *unknown* Lth-order channel $h_m(l)$ which, in addition to multipath, includes the transmit spectral-shaping pulse and the mth user's asynchronism in the form of delay factors (note that if user 1 is taken as reference, then in the absence of multipath, asynchronism of user m relative to user 1 by d_m chips, corresponds to a pure delay channel $h_m(l) = \alpha_m \delta(l - d_m)$). The multiplexed data are received in AGN $v(n)$, filtered, and sampled at the chip rate. Subsequent processing by the receive-filter $g_m(k)$, is followed by despreading (downsampling) by P and decision, to obtain the estimated $\hat{s}_m(n)$. We adopt the following assumptions:

- a1) $P - M \geq L$ and $M > L$, where L is the maximum expected order of all channels $\{h_m(l)\}_{m=1}^{M}$;
- a2) codes have $L_g \geq L$ trailing zeros ("guard chips"); if $L_g = P - M$, then $\{c_m(n) = 0\}_{n=M}^{P-1}$;
- a3) $h_1(0) \neq 0$ which is guaranteed when the receiver is synchronized to the user of interest only.

The number of guard chips L_g in a2) may be large in the completely asynchronous case (e.g., in IS-95) because $L = L_d + L_s$ consists of the maximum delay ($L_d \leq P$ chips) within a symbol among all users relative to the user

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of interest, plus the maximum delay-spread of the multipath (L_s nominally <5 chips). As L (and hence L_g) increases, our system's information rate decreases. In our QS-CDMA system, $L_d < 2, 3$ chips and hence, a1) is satisfied with a small $L_g \leq 7$. Note that no power control is assumed.

Under a1)–a3), the received samples are $x(n) = \sum_{m=1}^{M}\sum_i s_m(i)\sum_l h_m(l)c_m(n-l-iP)+v(n)$, or, in vector (block) form:[1, 2]

$$\boldsymbol{x}(n) = \sum_{m=1}^{M} \boldsymbol{C}_m \boldsymbol{h}_m s_m(n) + \boldsymbol{v}(n) \qquad (1)$$

where the nth block is the $P \times 1$ data vector $\boldsymbol{x}(n) := [x(nP)\ x(nP+1)\cdots x(nP+P-1)]^T$, and $\boldsymbol{v}(n)$ is defined similar to $\boldsymbol{x}(n)$. Note that thanks to the guard chips in a2), successive data blocks do not interfere with each other; thus, convolution of the mth user's code with the channel is represented as multiplication of the mth user's $P \times (L+1)$ Toeplitz matrix $\boldsymbol{C}_m$ (with first column $[c_m(0)\cdots c_m(M-1)0\cdots 0]$ and first row $[c_m(0)0\cdots 0]^T$) by the channel vector, $\boldsymbol{h}_m^T := [h_m(0)\quad h_m(1)\cdots h_m(L)]$. If $\boldsymbol{g}_k := [g_k^*(P-1)\cdots g_k^*(1)g_k^*(0)]^T$ is the weight vector for receiver k, its output takes the form:

$$\boldsymbol{g}_k^{\mathcal{H}}\boldsymbol{x}(n) = \boldsymbol{g}_k^{\mathcal{H}}\boldsymbol{C}_k\boldsymbol{h}_k s_k(n) + \sum_{m=1, m\neq k}^{M} \boldsymbol{g}_k^{\mathcal{H}}\boldsymbol{C}_m\boldsymbol{h}_m s_m(n) + \boldsymbol{g}_k^{\mathcal{H}}\boldsymbol{v}(n). \qquad (2)$$

Our problem is to design codes $\{c_m(n)\}_{n=0}^{M-1}$, or equivalently, their polynomials $C_m(z) := \sum_{n=0}^{M-1} c_m(n)z^{-n}$, and rely only on these codes to obtain $\boldsymbol{g}_k$ so that the MUI [described by the sum in (2)] is eliminated completely.

II. Lagrange–Vandermonde CDMA Transceivers

To eliminate MUI irrespective of the channels $\{h_m(l)\}_{m=2}^{M}$, receiver $\boldsymbol{g}_k$ must satisfy: $\boldsymbol{g}_k^{\mathcal{H}}\boldsymbol{C}_m = \boldsymbol{0}^{\mathcal{H}}$ for $m \neq k$; hence, $\boldsymbol{g}_k$ in (2) must lie in the intersection of null spaces $\bigcap_{m=1, m\neq k}^{M} \mathcal{N}(\boldsymbol{C}_m)$. On the other hand, $\boldsymbol{g}_k$ should not annihilate user k, implying that $\boldsymbol{g}_k^{\mathcal{H}} \notin \mathcal{N}(\boldsymbol{C}_k)$. Following the same logic for all users, we arrive at the necessary and sufficient conditions for *channel-independent* MUI elimination:

$$\forall\, i \in [1, M]\, \exists \boldsymbol{g}_i\colon \boldsymbol{g}_i^{\mathcal{H}} \in \mathcal{N}(\boldsymbol{C}_m) \quad \forall\, m \neq i \quad \text{and} \quad \boldsymbol{g}_i^{\mathcal{H}} \notin \mathcal{N}(\boldsymbol{C}_i). \qquad (3)$$

With arbitrary codes $\{c_m(n)\}_{m=1}^{M}$, the existence of vectors $\{\boldsymbol{g}_i\}_{i=1}^{M}$ satisfying (3) is not guaranteed. For example, the Walsh–Hadamard codes do not satisfy (3) for $M > 2$ and thus perfect MUI suppression is impossible without relying on channel knowledge. The key idea behind this letter is to exploit the Toeplitz structure of $\boldsymbol{C}_m$ in designing codes that satisfy (3). It turns out that if ρ is a root of $C_m(z)$, the Vandermonde vector $\boldsymbol{g}^{\mathcal{H}} := [1\,\rho^{-1}\cdots\rho^{-L}]$ is a left null-vector of the corresponding matrix $\boldsymbol{C}_m$; i.e., $\boldsymbol{g}^{\mathcal{H}}\boldsymbol{C}_m = \boldsymbol{0}^{\mathcal{H}}$. Hence, the null spaces of matrices $\{\boldsymbol{C}_m\}_{m=1}^{M}$ can be controlled by the roots of the corresponding polynomials $\{C_m(z)\}_{m=1}^{M}$. Based on these ideas, our code design algorithm follows these steps: 1) select a set of M distinct nonzero points $\{\rho_i\}_{i=1}^{M}$ on the complex plane; 2) use ρ_i's to construct M *Lagrange* polynomials (each of order $M-1$):

$$C_m(z) = \mathcal{K}_m \frac{\prod_{i=1, i\neq m}^{M}(1-\rho_i z^{-1})}{\prod_{i=1, i\neq m}^{M}(1-\rho_i\rho_m^{-1})}, \quad m \in [1, M] \qquad (4)$$

and 3) build the M *Vandermonde* receive filters: $\boldsymbol{g}_i^{\mathcal{H}} := [1\rho_i^{-1}\rho_i^{-2}\ \cdots\ \rho_i^{-P+1}]$. By direct substitution, it follows that $C_m(\rho_m) = \mathcal{K}_m$ and $C_m(\rho_i) = 0$ for $i \neq m$; hence,

$$\boldsymbol{g}_i^{\mathcal{H}}\boldsymbol{C}_m = \mathcal{K}_m[C_m(\rho_i),\, \rho_i^{-1}C_m(\rho_i),\, \cdots,\, \rho_i^{-L}C_m(\rho_i)] = \boldsymbol{0}^{\mathcal{H}} \qquad (5)$$

for $i \neq m$, whereas $\boldsymbol{g}_m^{\mathcal{H}}\boldsymbol{C}_m = \mathcal{K}_m[1\rho_i^{-1}\rho_i^{-2}\cdots\rho_i^{-L}] \neq \boldsymbol{0}^{\mathcal{H}}$; therefore, $\forall\, m \in [1, M]$ we have

$$\boldsymbol{g}_m^{\mathcal{H}}\boldsymbol{C}_m\boldsymbol{h}_m = \mathcal{K}_m \sum_{l=0}^{L} h_m(l)\rho_m^{-l} := \mathcal{K}_m H_m(\rho_m). \qquad (6)$$

Root ρ_m is not a root of $C_m(z)$, but is a "signature root" for user m because the Vandermonde receiver $\boldsymbol{g}_m^{\mathcal{H}}$ employs ρ_m in order to recover $s_m(n)$ and eliminate MUI interference that users $k \neq m$ introduce to user m [according to (4), ρ_m is a root of $\{C_i(z)\}_{i=1, i\neq m}^{M}$]. Note also that the pairs of Lagrange–Vandermonde (LV) transceivers obey by construction (3), and upon substituting (5) and (6) into (2) we obtain

$$\boldsymbol{g}_k^{\mathcal{H}}\boldsymbol{x}(n) = \mathcal{K}_k H_k(\rho_k)s_k(n) + \boldsymbol{g}_k^{\mathcal{H}}\boldsymbol{v}(n). \qquad (7)$$

The main features of our LV-CDMA transceivers are as follows.

1) Under a1)–a3), MUI is eliminated deterministically, irrespective of the multipath channel—a property that holds true also for time-varying channels, provided that the channels' coherence time is greater than the symbol interval.
2) Equation (7) shows that the LV system converts frequency-selective channels to flat-fading channels. Factor $H_k(\rho_k)$ can be removed using automatic gain control (AGC) for the amplitude and differential encoding for phase insensitivity. Channel nulls can be handled via channel encoding and "root-hopping"—a code switching strategy from symbol-to-symbol that generalizes the frequency-hopping concept and together with error-correcting codes allows recovery from deep fades.
3) With our LV-CDMA system, one does not have to estimate (via training sequences or blindly) the decorrelating (or MMSE) receivers needed when multipath propagation and quasi-synchronism are present. In addition, LV transceivers have very low complexity because filters are precomputed and each user is demodulated using a simple inner-product $\boldsymbol{g}_m^{\mathcal{H}}\boldsymbol{x}(n)$.
4) OFDMA is a special case of our LV CDMA system. Selecting as ρ_i's the roots of $1 - z^{-M} = \prod_{i=1}^{M}(1-\exp(j2\pi i/M)z^{-1})$, we obtain Lagrange codes with coefficients $c_m(n) = \exp(j2\pi mn/M)$ and Vandermonde filters with taps $g_m(n) = \exp(-j2\pi mn/M)$, for $n = 0, \cdots, M-1$, $m = 1, \cdots, M$. Within the class of

[1] Upper (lower) boldface notation will be used for matrices (column vectors); *, ^{T}, and $^{\mathcal{H}}$ will denote complex conjugation, transposition, and Hermitian transposition, respectively.

[2] Detailed derivations and performance analysis will be included in the full journal submission.

LV-CDMA systems, OFDMA plays a prominent role because in the presence of AWN, perfect synchronism, and absence of multipath, OFDM codes are orthogonal LV codes that possess optimality in terms of maximizing signal-to-noise-ratio at the mth receiver's output ($:=\mathrm{SNR}_m$). Maximizing SNR_m, we find that in addition to being Vandermonde, $\boldsymbol{g}_m$ must be a filter matched to the code vector $\boldsymbol{c}_m^T := [c_m(0) \cdots c_m(M-1)]$; hence, $\boldsymbol{g}_m^{\mathcal{H}} \boldsymbol{c}_i = \mathcal{K}_m \delta(m-i)$ and $\boldsymbol{g}_m^{\mathcal{H}} = \boldsymbol{c}_m^T$, which is equivalent to requiring that $\boldsymbol{c}_m$ (and thus $\boldsymbol{g}_m$) are orthogonal vectors. It follows easily that the only orthogonal Vandermonde vectors are the Fourier basis vectors with entries $c_m(n) = g_m^*(P-1-n) = \mathcal{K}_m \exp(j2\pi mn/M)$ which leads us to OFDMA. However, for frequency-selective channels $\mathrm{SNR}_m = |\mathcal{K}_m|^2 |H_m(\rho_m)|^2 \; / E\{|\mathbf{g}_m^{\mathcal{H}} \boldsymbol{v}(n)|^2\}$, and our current research for optimizing SNR_m (and thus BER since $\boldsymbol{v}(n)$ is Gaussian) leads to user codes that take into account channel transfer function, under transmit power constraints.

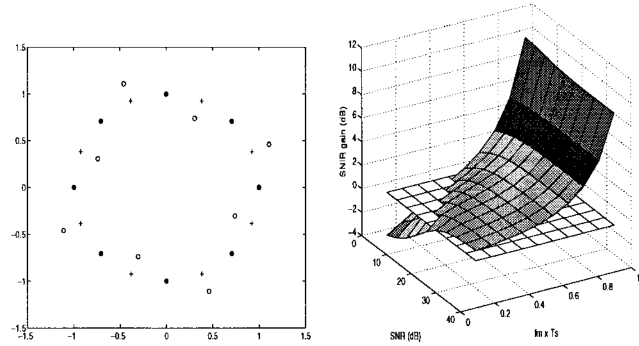
5) In contrast to OFDMA where code roots are fixed and equispaced around the unit circle, root locations in LV-CDMA can be optimized to improve performance when dealing with frequency- or time-selective channels. Time selectivity arising due to Doppler effects and/or carrier offsets manifests itself as multiplicative $\exp(j\omega_m n)$ factors modulating each of the channels $h_m(l)$. The kth receiver output then becomes

$$\begin{aligned}\boldsymbol{g}_k^{\mathcal{H}} \boldsymbol{x}(n) = s_k(n) e^{j\omega_k n P} H_k(\rho_k e^{-j\omega_k}) C_k(\rho_k e^{-j\omega_k}) \\ + \boldsymbol{g}_k^{\mathcal{H}} \boldsymbol{v}(n) + \sum_{m \neq k} s_m(n) e^{j\omega_m n P} \\ \cdot H_m(\rho_k e^{-j\omega_m}) C_m(\rho_k e^{-j\omega_m})\end{aligned}$$

which shows that three undesired effects arise: i) symbols are modulated by $\exp(j\omega_m nP)$; ii) channel and code roots rotate by the *unknown* angle ω_m on the complex plane; and iii) MUI is superimposed to the user of interest because ρ_k is no longer a common root of $\{C_m(ze^{-j\omega_m})\}_{m=1,\, m\neq k}^{M}$ (roots of the latter have rotated by ω_m). The signal-to-noise and interference ratio (SNIR) for the kth user turns out to be $|H_k(\rho_k e^{-j\omega_k})|^2 |C_k(\rho_k e^{-j\omega_k})|^2 \, / [\sum_{m\neq k}^{M} |H_m(\rho_k e^{-j\omega_m})|^2 \; |C_m(\rho_k e^{-j\omega_m})|^2 \; + \sigma_v^2 (1 - |\rho_k|^{-2P}) \; / (1 - |\rho_k|^{-2})]$. Therefore, at least in principle, user codes can be designed to maximize SNIR_k, $\forall\, k$, by optimizing code root locations. The optimization is challenging because SNIR_k depends nonlinearly on ρ_k. However, suboptimal codes increasing robustness against frequency drifts can be devised by designing polynomials $C_m(z)$ such that: i) $C_k(\rho_k e^{-j\omega_k})$ is as flat as possible, $\forall\, k$ and ii) $C_m(\rho_k e^{-j\omega_m})$ is as small as possible, $\forall\, m \neq k$. Because $C_m(\rho_k) = 0$ by construction, the Taylor series expansion of $C_m(\rho_k e^{-j\omega_m})$ around ω_m, leads to (primes denote differentiation):

$$\begin{aligned}&C_m(\rho_k e^{-j\omega_m}) \\ &\quad = -j\rho_k \omega_m C_m'(\rho_k) - \frac{\rho_k^2 \omega_m^2}{2} C_m''(\rho_k) + o(\omega_m^2)\end{aligned}$$

which suggests that resistance to Doppler effects can be gained by selecting polynomials $C_m(z)$ with multiple roots at $z = \rho_k$.



(a) (b)

Fig. 2. (a) Roots of user codes. (b) SNIR gain: Lagrange versus OFDMA.

To test Lagrange versus OFDMA coding strategies, we simulated $M = 16$ users with roots, $\{\rho_m = [1 + 0.1\cos(\pi m/2)] \exp(j2\pi m/M)\}$, with $m = 0, \cdots, M-1$, shown in Fig. 2(a) with circles, along with OFDMA's root constellation ("+"). Fig. 2(b) depicts the ratio between $\mathrm{SNIR}_m^{LV} / \mathrm{SNIR}_m^{\mathrm{OFDMA}}$ obtained with the LV and OFDMA systems, as a function of SNR and the frequency shift, normalized to one symbol interval, $f_m T_s$. Taps $h_m(n)$ were generated as independent Gaussian to simulate Rayleigh fading channels of order $L = 5$. Fig. 2(b) depicts the SNIR gain (in decibels) averaged over 400 independent channel realizations and shows that OFDMA yields better performance for very small frequency shifts and at low SNR. However, as soon as the frequency shift exceeds a small fraction of the OFDMA filter bandwidth $1/T_s$, the LV scheme built from the roots of Fig. 2(a) clearly outperforms OFDMA.

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