ON A REFINEMENT OF STRONG STARK CONJECTURE FOR $\mathbb{Z}A_4$

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Let K/k be a finite Galois extension of number fields with $\mathrm{Gal}(K/k) = G$. For an irreducible complex character χ of G, let $L(\chi, s)$ denote the Artin L-function and let $L^*(\chi, 0)$ denote the leading coefficient in the Taylor series expansion of $L(\chi, s)$ at $s = 0$. Then one can view $L(s) := (L(\chi, s))_{\chi \in \hat{G}}$, as a function with values in $\prod_{\chi \in \hat{G}} \mathbb{C} = \zeta(\mathbb{C}[G])$, where ζ is the centre and $\hat{G}$ is the set of irreducible complex characters of G. Further, $L^*(0) = (L^*(\chi, 0))_{\chi \in \hat{G}}$ lies in $\zeta(\mathbb{R}[G])^\times$.

There exists a natural map arising from a long exact sequence of K-theory

$$\hat{\delta} : \zeta(\mathbb{R}[G])^\times \to K_0(\mathbb{Z}[G], \mathbb{R})$$

where $K_0(\mathbb{Z}[G], \mathbb{R})$ is the relative K_0 group (cf. [2]). The equivariant Tamagawa number conjecture (ETNC) [2, 3] for the motive $M = h^0(\mathrm{Spec}(K))_k$ states that $t(K/k) = \hat{\delta}(L^*(0)) - c(K/k) = 0$, where $c(K/k)$ is an element of $K_0(\mathbb{Z}[G], \mathbb{R})$ constructed using Tate sequence. The vanishing of $t(K/k)$ in $K_0(\mathfrak{M}, \mathbb{R})$, where $\mathfrak{M}$ is a maximal $\mathbb{Z}$-order containing $\mathbb{Z}[G]$, is equivalent to the strong Stark conjecture as stated in [5]. Also, the vanishing of $t(K/k)$ in $K_0(\mathbb{Z}[G])$ is equivalent to the central conjectures of [4]. ETNC also recovers several refinements of the Stark conjecture due to Gross, Rubin and others (cf. [1]).

The only known proof of the vanishing of $t(K/k)$ for a non-abelian extension is for an infinite family of quaternion extensions (cf. [3]). One of the key ideas in that proof is that the map

$$K_0(\mathbb{Z}[G], \mathbb{R}) \to K_0(\mathfrak{M}, \mathbb{R}) \times K_0(\mathbb{Z}[G^{\mathrm{ab}}], \mathbb{R}) \times K_0(\mathbb{Z}[G])$$

is injective for $G = Q_8$, the quaternion group (here $\mathfrak{M}$ is the maximal $\mathbb{Z}$-order containing $\mathbb{Z}[G]$). However, this fails to hold for $G = A_4$.

In this presentation, we shall restrict to a particular extension $K/\mathbb{Q}$ with $\mathrm{Gal}(K/\mathbb{Q}) \simeq A_4$, and develop some techniques for computing $c(K/\mathbb{Q})$. We use the ideas of Chinburg ([4]) to construct the Tate sequence and explicitly compute the determinants arising from this sequence to evaluate $c(K/\mathbb{Q})$, and verify the vanishing of $t(K/\mathbb{Q})$. One could possibly extend these techniques to verify the conjecture for an infinite family of A_4-extensions.

References

[1] Burns, D., *Equivariant Tamagawa Numbers and refined abelian Stark conjectures*, J. Math. Sci. Univ. Tokyo, 10 (2003).

[2] Burns, D., Flach, M., *Tamagawa numbers for motives with (noncommutative) coefficients I*, Doc. Math. 6 (2001), 501-570.

[3] Burns, D., Flach, M., *Tamagawa numbers for motives with (noncommutative) coefficients II*, Amer. Jour. of Math., 125 (2003), no. 3, 475-512.

[4] Chinburg, T., *The Analytic Theory of Multiplicative Galois Structure*, Memoirs of the Am. Math. Soc., No. 395 (1989).

[5] Chiburg, T., *On the Galois structure of algebraic integers and S-units*, Invent. Math., 74, 321-349 (1983).