ABSTRACT
LINEAR EQUALITIES IN FIBONACCI NUMBERS

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We study the equation $F_b = F_{x(1)} + F_{-x(3)} + F_{-x(4)} + ... + F_{-x(m)}$ with $m \geq 3$, $0 < x(1) < b$, and $0 < x(3) < x(4) < ... < x(m)$. This equation naturally arises in the generalization of several problems that have appeared in the Fibonacci Quarterly in the problem sections. This equation also has intrinsic interest in its own right. The main theorem--the Accident theorem--states, that under very mild conditions, solutions to this equation cannot happen by accident; that is, there are no singular solutions but rather every solution belongs to a parametrizable class of solutions. Furthermore if $m \geq 4$, then b must be even and there are exactly 9 parametrizable solutions. The solutions of this equation for $m \geq 4$ are naturally described by a transformational grammar with 8 classes of production rules. This is the first major theorem in the literature on identities in Fibonacci numbers with an arbitrary number of summands whose subscripts have mixed signs. There are only 2 hypothesii of the accident theorem: We require that for all i, $x(i) > 2$ and that no proper subset of summands on the right hand side of the equation has a sum of zero. While the proof of the accident theorem requires many sub-theorems and lemmas, the basic proof method is to exploit *Fibonacci Telescoping* lemmas. An example of *Fibonacci Telescoping* is illustrated by the following identity which is one of the 9 parametrizable solutions for $m \geq 4$: $F_b = F_{b-o-3} + F_{-(b-o-2)} + F_{-(b-o)} + ... + F_{-(b-1)}$ with b even and o an arbitrary positive odd integer.