The grid works! Why do we want to change it?

The reason? The way demand and supply of electricity are matched on the grid.
Buying and Selling Power

- **Context:** let us take a quick look at how electricity is generated, delivered and traded today
- **Disclaimer:** a lot of details are missing

1. Wholesale electricity market → greater reliance on **competition**
2. Retail electricity market → public utility
   - It is a utility that distributes a commodity, electrical power
Whole Sale Market competition

- How does a competitive commodity market generally work?
  - Numerous small producers and consumers who cannot affect price
  - They want to maximize their profit and benefit respectively

- *The invisible hands* of the market lead the price to equilibrium

- **Important feature of perfect competition** (not always the case): maximizes social surplus
The Curse of Constant Balance

- We want the same wonderful outcome for the electricity market
- **Balance Requirement:** \( \sum_i G_i(t) = \sum_i L_i(t) + \text{Losses} \), or else....

- Too much pressure for the *invisible hands* of the market
- To ensure reliable operations power systems use a **Central Coordinator** for scheduling and **Ancillary Services** (spinning and non-spinning reserves and regulation)
Nonprofit entity called the **Independent System Operator (ISO)**

- Common scenario: ISO runs a bid-based auction settling how much each individual should generate or consume
- It is done as a central optimization
- Lagrange multipliers of constraints = marginal price of resource $
Multi-settlement structure

When are the decisions made?:
Match \( random L_i(t) \) with day ahead plan + real time corrections

- Unit Commitment and Day-ahead Market (hourly intervals)
- Hour-ahead Adjustment Market (hourly intervals)
- Real-time Market (5-10 minute intervals)

Day-ahead planning

Real-time operations at the beginning of hour 12:00

- Expected minimum cost scheduled load (optimized)
The ISO’s Objective in a Pool Market

- Criterion: maximize social welfare $\rightarrow$ emulate perfect competition

Social welfare $= \sum$ welfare of individual participants

- Consumer’s welfare $=$
  
  Benefit from consuming power $U_j(L_j)$ $-$ consumption expense

- Supplier’s welfare $=$
  
  Revenue from selling power $-$ production cost $C_i(G_i)$

$\sum$ expenses paid by consumers $= \sum$ revenues received by suppliers

$\Rightarrow$ Social Welfare $= \sum_j U_j(L_j) - \sum_i C_i(G_i)$

The optimization solved should approximately be...

$$\text{Max Social Welfare} \equiv \max_{G_i, L_j} \sum_j U_j(L_j) - \sum_i C_i(G_i)$$

$$\text{s.t.} \quad \sum_i L_i = \sum_i G_i$$

See [Weber, Overbye, 2002]
The ISO’s Objective in a Pool Market

- Criterion: \textbf{maximize social welfare} \rightarrow \textbf{emulate perfect competition}

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\]

s.t \[ \sum_i L_i = \sum_i G_i \]

See [Weber, Overbye, 2002]
What is the customers $U_j(L_j)$? With few exceptions, inelastic

$$\text{Max Social Welfare} \equiv \min_{G_i} \sum_i C_i(G_i)$$

s.t. Kirckhoff and Ohm’s laws
Capacity Limits

- Flows and prices are based on inputs from these stakeholders: *Generators, Transmission and Distribution utilities, Retailers*
- Customers can draw power at will - not really given a choice
- Costs for reserves to deal with random injections are socialized
- Solar and Wind power free-ride these additional costs....
- How do Solar and Wind get paid?
Variable Energy Resources (VERs)

Wind and solar are random. Can VERs participate to the market?

- In the US FERC ruling for the main settlement (day ahead) market, implies that offers are financially binding
- **Reason:** shortage of control to hedge risk (*fast ramping* $G_i(t)$)
- VERs forgo day-ahead market risks and instead sell into real-time markets (*can fill only scraps of demand!!*)
Solutions? Flexible Demand

- Remember the 90’s debates on voice, video over wireless or IP?
- **Ingredients**: Multiple-Description Coding + Queue Management + Dynamic Channel Access (Opportunistic Rate Adaptation)

\[ q(t) = \text{Queue} \]
\[ q(t) = 0 \quad \text{if } R(t) < C(t) \]
\[ \dot{q}(t) = R(t) - C(t) \quad \text{if } 0 < q(t) < q_{\text{max}} \]
\[ q(t) = q_{\text{max}} \quad \text{if } R(t) > C(t) \]
\[ \dot{q}(t) = 0 \quad \text{if } R(t) > C(t) \]

\[ R(t) = \text{Random Source Rate} \]
\[ C(t) = \text{Random Channel Rate} \]

- **Note**: The demand is diced and spliced and held back to follow the supply (also spliced), not the other way around.
What could be opportunistic on the grid?

- Thermostatically Controlled Loads (TCLs), Heating Ventilation Air Conditioning (HVACs), Pumped Water

- (Tomorrow if successful) Electrical Vehicles

- Other: Deferrable appliances, Lighting control (dimmable)
**Aggregators** are entities that interact with the wholesale market providing an interface to manage populations of flexible demand.

Research on both sides of the aggregator (wholesale, retail)

- **Challenge 1:** Heterogenous population
- **Challenge 2:** Real time control of the appliances
- **Challenge 3:** Modeling the flexibility ex-ante in the market
Aggregators are entities that interact with the wholesale market providing an interface to manage populations of flexible demand. Research on both sides of the aggregator (wholesale, retail)

- **Challenge 1:** Heterogenous population
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- **Challenge 3:** Modeling the flexibility ex-ante in the market
Responsive Appliance Models

Research on Home Energy Management Systems (HEMS) (Chal. 1-2)

- Appliance is like a battery that has to be filled [Mohsenian-rad, Wong, Schober, Leon Garcia]

\[
\sum_{\ell=\text{arrival time}} L^{(m)}(t) = \text{constant known value} \ E^{(m)}
\]

- Define concave utility, [Li, Chen, Low] / convex dis-utility functions for consumption, [Gatsis, Giannakis]

\[
U_{i}^{(m)}(L_{i}^{(m)}(t))
\]

Research on flexible loads populations (Chal. 2-3)

- Consumption \( L_{i}^{(m)}(t) \in \mathcal{L}_{i}^{(m)} \) gives constant utility for user \( i \) within a certain horizon. e.g. [Koch, Matieu, Callaway], [Papavasiliou, Oren], [Foster, Caramanis], [Ilic, Xie, Joo], [Alizadeh, Scaglione, Thomas], [Chen, Mount, Tong ], [Nayyar, Taylor, Subramanian, Poolla, Varaiya ], [Domininguez-Garcia]...
Example of $\mathcal{L}_i^{(m)}$ we explored in [Cheng, Alizadeh, Scaglione’13]
- Appliances go from IDLE to WAIT (non-stationary process)
- $\alpha_{m,q,t} =$ Prob. that appl. $m$ goes from idle to wait in mode $q \in \{1, \ldots, Q_m\}$
- **Class 1** – A deferrable appliance decision is captured by an activation process $d_{m,q}(t) \in \{0, 1\}$ of a certain load injection profile $g_{m,q}(t)$ s.t. deadline and causality constraints

$$L_q^{(m)}(\ell) = \sum_{\ell} [d_{m,q}(\ell) - d_{m,q}(\ell - 1)] g_{m,q}(t - \ell).$$

- **Class 2** – For TCLs, $d_m(t) \in \{0, 1\}$ s.t. user comfort constraints
- Class 1 and 2 both Linear, with integer $d_m(t)$

Decision to optimize a time $t$? The $d_{m,q}(t) \in \{0, 1\}$. How? Depends on the Aggregator value proposition...
The Aggregator Retail Side

What is the right interface?

Today’s tools: Time of Use (TOU) tariffs and Emergency Interruptible Load (EIL) programs

None suitable for daily, non-stationary risk of VER power

What everybody seems to want is: Dynamic Pricing
What is the right interface?

- Today’s tools: Time of Use (TOU) tariffs and **Emergency Interruptible Load (EIL) programs**
- None suitable for daily, non-stationary risk of VER power
- What everybody seems to want is: **Dynamic Pricing**
The Aggregator Retail Side in Dynamic Pricing

Dynamic Pricing Interface

- The aggregator communicates an ex-ante retail price for power
- Customers HEMS find a selfish utility maximizing schedule - HEMS design problem
- The aggregator meters consumption to learn individual responses and adjust the retail price to maximize its profit
Passing Real Time Pricing: any risk?

If we get the prices right, profit maximizing individuals will produce/consume the welfare maximizing $G_i^*/L_j^*$ on their own.

- **Marginal pricing** = Lagrange multiplier of the balance equation (simple to show why)
  \[
  \max_{G_i, L_j} \sum_j U_j(L_j) - \sum_i C_i(G_i)
  \]
  \[
  \text{s.t. } \sum_j L_j = \sum_i G_i \quad \rightarrow \quad \text{marginal price} \quad (1)
  \]

- Great decentralized control strategy!

True if the aggregator model for $U_j(L_j)$ correctly described the flexibility of demand. [Roozbehani and Dahleh, 2011]

- If not → can cause economic and physical instabilities in the whole sale market
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$$
\max_{G_i,L_j} \sum_j U_j(L_j) - \sum_i C_i(G_i)
$$

subject to

$$
\sum_j L_j = \sum_i G_i \rightarrow \text{marginal price} \quad (1)
$$

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True if the aggregator model for $U_j(L_j)$ correctly described the flexibility of demand.

- If not $\rightarrow$ can cause economic and physical instabilities in the whole sale market [Roozbehani and Dahleh, 2011]
Any risk for the customer?

- A really good reason to anonymize AMI readers...

I am the power retail monopoly, I can guess from AMI what each user wants and can design my profit maximizing prices!!
Coordinated demand management strategies

**Alternative:** Customers cooperate in minimizing the Aggregator whole-sale real time cost via (Network Utility Maximization or Mechanism design) → Aggregator could sell ancillary services

\[
\text{Real Time Cost} = \pi_s(t) \left( P(t) - \sum_i L_i(t) \right)^+ + \pi_p(t) \left( P(t) - \sum_i L_i(t) \right)^- \\
\text{s.t. } L_i(t) = \sum_{m=1}^{M_i} L_i^{(m)}(t) \quad L_i^{(m)}(t) \in \mathcal{L}_i^{(m)}(t)
\]

\( P(t) \) = power scheduled in previous market clearing, \( \pi_s(t), \pi_p(t) \) market prices e.g., [Caron and Kesidis,2010], [Mohsenian-rad, Wong, Schober, Leon Garcia,2010],[Kefayati,Caramanis,2010],[Li,Chen,Low,2011],[Foster, Caramanis 2011],[Chen, Kishore’12], [Gatsis, Giannakis,2012] [Chen, Ji, Tong,2012], [Saad, Han, Poor, Bas,’12][Cheng, Alizadeh, Scaglione 2012]...

- **Tenet:** Minimize cost without revealing the individual constraints
- **Pros:** Decentralized protocol to compute the optimum policy
  The Aggregator could hedge risk in buying wind or solar
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- **Tenet:** Minimize cost without revealing the individual constraints
- **Pros:** Decentralized protocol to compute the optimum policy
  The Aggregator could hedge risk in buying wind or solar
Sounds good but does it work?...

Because of the integrality constraints there is a duality gap in the decentralized solution

It is not clear how to compensate customers for being flexible

The Aggregator ex-ante model for hedging risk in the whole-sale market with this kind of control is an open problem (not clear how to profit)
What is the value proposition for the Aggregator in the Market? Research on Large-scale Ex-Ante Modeling of Responsive Demand focuses on this question.

In some cases the research suggests online scheduling strategies.

This opens the door for a different economic model.
Aggregate Models (The Tank)

- The power that is flexible replenishes a tank that has to be emptied by the end of the day

  [Lambert, Gilman, Lilienthal, 2006], [Lamadrid, Mount, Zimmerman, Murillo-Sanchez, 2011], [Papavasiliou, Oren ’10]

- Very coarse, does not suggest how to schedule or value each contribution, but very easy to use to quantify economic benefits
Aggregate Models (The Detailed Model)

- Requires modeling each appliance constraints individually
  
  
  \[ c_i(t) = \text{capacity to fill}, \quad \rho_i = \text{charge rate} \]
  
  \[ \chi_i = \text{deadline}, \quad v_i = (\chi_i - t) - c_i(t)/\rho_i = \text{laxity} \]

  → Prioritize tasks as in Processor-Time Allocation [Liu 1973]: by deadline (Earliest Deadline First) or laxity (Least Laxity First)

- Generally computationally infeasible
  
  Simpler for EVs with deadlines: [S Chen, T Mount, L Tong, 2013], [A. Nayyar, J. Taylor, A. Subramanian, K. Poolla, and P. Varaiya 2013]
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Aggregate Models (Quantized Population Model)

- **State Space Model for TCL:** [Mathieu, Koch, Callaway, 2013]...
  
  Quantize the temperature range in bins, track the dynamics of the population histogram $x(t)$, control what to turn off $u(t)$

\[ x(t) = Ax(t-1) + Bu(t) \]

- **Queuing Deferrable Loads:** [Alizadeh, Thomas, Scaglione, 2012]...
  
  Cluster based on load shape $g_q(t)$, deadline $\chi_q$ and arrival
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- **Queuing Deferrable Loads:** [Alizadeh, Thomas, Scaglione, 2012],...
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\[ L(t) \approx Q \sum_{q=1}^{\infty} t \sum_{\ell=0}^{\infty} [d_q(\ell) - d_q(\ell-1)] g_q(t-\ell) \]

\[ 0 \leq d_q(\ell-1) \leq d_q(\ell) \leq a_q(\ell) \]

\[ a_q(\ell) \geq a_q(\ell-1) \]
Aggregate Models (Quantized Population Model)

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## Population Aggregate Constrains

$$\mathcal{L} : \quad L(t) \approx \sum_{q=1}^{Q} \sum_{\ell=0}^{t} [d_q(\ell) - d_q(\ell - 1)] g_q(t - \ell)$$

$$0 \leq d_q(\ell - 1) \leq d_q(\ell) \leq a_q(\ell), \quad d_q(\ell) \geq a(\ell - \chi_q)$$
Peak shaving with the Tank Model

- Tank model used to describe deferrable loads
- Large deviations from day-ahead dispatch

15000 PHEVs
1.1 kW home charge
5 service queues
IEEE 9 bus test case
3 generators 3 loads
Quadratic gen. costs
One aggregator
Uncontrollable load peak of 25 MWs

[Alizadeh, Cheng, Scaglione 2013]
Peak shaving - Quantized population model

- Quantized Deferrable EV model
- Load following dispatch very closely when using our model

![Graph showing load curve with different scenarios]

Same setting
5 service queue
Maximum charge 5 hours

[Alizadeh, Cheng, Scaglione 2013]
Benefits of Quantized Population Models

**Online central scheduling**

1. Refresh $Q$ inter-arrival variables each time unit $t$
2. Decides the profit maximizing schedule
3. Broadcasts an activation message (a time stamp)

**Ex-ante Models**

1. From daily events learn statistics of non-stationary population process (e.g. [Alizadeh, Scaglione, Kurani, Davies 2013] for PHEVs)
2. Use a Model Predictive Control (MPC) framework with a Certainty Equivalent Controller to forecast profits
Can this help solve the retail side?

- Using MPC the Aggregator can quantify what is the profit maximizing population management and bid
- The model is suitable for on-line direct scheduling
- Why would customers sign up for it though?
- Rather than giving a price per kW, the Aggregator could post an incentive for a certain appliance flexibility
Dynamically Designed Cluster-specific Incentives

- The incentive depends on the appliance cluster and mode
- Recruitment could be anonymous transaction, price would be fair
- After looking at table, customers 1) decide whether to participate or not; 2) choose the laxity → **How?**

---

[Alizadeh, Xiao, Scaglione, Van Der Schaar 2013], similar to deadline differentiated pricing [Bitar, Xu 2013]
What kind of problem are we solving?

- Each appliance in cluster \( q \) can sell to the aggregator one of \( M_q \) services (modes)
- The Aggregator posts prices it is willing to pay for each service
- Customers valuation for different services correlated
- Optimal posted prices? The closest approximation is the “optimal unit demand pricing” problem (but is the opposite direction of trade and has independent valuations)
Modeling the aggregator’s payoff

- $I_q^t(m) =$ Incentive paid to appliance in cluster $q$ recruited under mode $m$ at time $t$

  
  $I_q^t = [I_q^t(1), I_q^t(2), \ldots, I_q^t(M_q)]$.

- $U_q^t(m)$: the aggregator market benefit of recruiting an appliance in cluster $q$ under mode $m$ at time $t$, e.g.

  
  $U_q^t(m) = \sum_\ell \pi^e(\ell) L_{q,0}^t(\ell) - \min_{L_{q,m}(\ell) \in L_{q,m}^t} \sum_\ell \pi^e(\ell) L_{q,m}^t(\ell)$.

- If an appliance in cluster $q$ in mode $m$ is recruited

  
  Aggregator Payoff $= U_q^t(m) - I_q^t(m)$

- Remember: Mode endogenously picked by customers

- $E_{q,m}(I_q^t) =$ the event that a customer in cluster $q$ picks mode $m$

- $N_q^t =$ expected payoff when interacting with customer in cluster $q$

  
  Aggregator Profit $= \max_{I_q^t} \sum_t N_q^t = \sum_{t,m} P(E_{q,m}(I_q^t))(U_q^t(m) - I_q^t(m))$
Modeling the customer’s decision

Approaches to find $P(E_q, m(I^t_q))$? (probability that the aggregator posts $I^t_q$ and customer picks mode $m$)

1. **Bayesian model-based method**: rational customer - Risk-averseness captured by *types*

2. **Model-free learning method**: customers may only be boundedly rational and modeling them as rational may cause more harm than benefit
Model-based method: the rational customer’s utility

- Customer’s initial choice to use appliance: inelastic
- The customer’s utility has three terms:

1. Owner of appliance $i$’s level of random risk (disutility) for committing to operate appliance in mode $m$
   \[ R^t_i(m) : \mathcal{M}_{q_i} \to \mathbb{R}^+, \]

2. Incentive for appliance in cluster $q$ under mode $m$ at time $t$
   \[ I^t_q = [I^t_q(1), I^t_q(2), \ldots, I^t_q(M_q)]. \]

3. Utility of electricity use (constant, always happens) → ignore

Customer’s decision

- Assuming a quasi-linear utility for the customer
  \[ V^t_i(m_i) = I^t_{q_i}(m_i) - R^t_i(m_i). \]

- Utility maximization: \( \max_{m_i \in \mathcal{M}_{q_i}} V^t_i(m_i) \)
When does appliance $i$’s owner choose to 1) participate in the program and 2) pick mode $m$ for their appliance?

1. Individual rationality constraint

$$I_q^t(m) - R_i^t(m) \geq 0,$$

2. Incentive compatibility constraints

$$I_q^t(m) - R_i^t(m) \geq I_q^t(m') - R_i^t(m'), \quad \forall m' \in M_q.$$

- The $R_i^t(m)$’s are only privately known to the customer
- The aggregator has access to statistics of $R_i^t(m)$
  - For appliance cluster $q$, we choose $R_i^t(m) = \gamma_i r_q^t(m)$
  - $\gamma_i \rightarrow$ user type, statistics of $\gamma_i$ available
This formulation, the incentive design optimization is not convex.

We impose a design constraint that ensures that \textit{local incentive compatibility}, i.e, the ratios will be ordered.

**Design constraint** (Single-Crossing Incentive Profile):

\[ \forall m \in \mathcal{M}_q, \text{the ratio } \frac{I_q^t(m+1) - I_q^t(m)}{r_q^t(m+1) - r_q^t(m)} \text{ is non-increasing} \]

It can be shown that the min and max can be now removed.

Then, customer will pick mode \( m \geq 1 \) simply iff

\[ \frac{I_q^t(m + 1) - I_q^t(m)}{r_q^t(m + 1) - r_q^t(m)} \leq \gamma_i \leq \frac{I_q^t(m) - I_q(m - 1)}{r_q^t(m) - r_q^t(m - 1)} \]
Quadratic Program formulation

- $F_{\gamma}(g)$ is a uniform distribution

$$\max_{I_{q}^t \geq 0} \sum_{t=\ell}^{\infty} N_{q}^t = \frac{1}{\gamma_{\text{max}}} \sum_{t=\ell}^{\infty} \sum_{m=1}^{M_{q}} \left( U_{q}^t(m) - I_{q}^t(m) \right) \times$$

$$\left( \frac{I_{q}^t(m) - I_{q}^t(m-1)}{r_{q}^t(m) - r_{q}^t(m-1)} - \frac{I_{q}^t(m+1) - I_{q}^t(m)}{r_{q}^t(m+1) - r_{q}^t(m)} \right),$$

s.t.  

$$0 \leq I_{q}^t(m) \leq I_{q}^{t-1}(m+1), \quad 1 \leq m \leq M_{q} - 1,$$

$$\frac{I_{q}^t(m+1) - I_{q}^t(m)}{r_{q}^t(m+1) - r_{q}^t(m)} \leq \frac{I_{q}^t(m) - I_{q}^t(m-1)}{r_{q}^t(m) - r_{q}^t(m-1)}, \quad m \geq 1,$$

$$I_{q}^t(m) \geq I_{q}^t(m-1), \quad m \geq 1,$$

$$\frac{I_{q}^t(1)}{r_{q}^t(1)} \leq \gamma_{\text{max}}, \quad \ell \leq t < \infty.$$
How do we recruit?

- 620 PHEV charging record, LMPs from ISO NE’s Maine load zone on Sept 1st 2013
- How many EVs did we recruit (out of 620) and with how much laxity?

More savings in the evening: more arrivals, more laxity, peak prices
Welfare Effects

- Welfare generate via Direct Load Scheduling (DLS) of recruited appliances versus idealized Dynamic Pricing program
- Savings summed up across the 620 events (shown as a function of time of plug-in)
Conclusion

- We have discussed the possible ways flexible loads could become responsive.
- We left out how to sell more renewables power. See work on Risk Limiting Dispatch (RLD) [Varaiya, Wu, Bialek, 2011], [He, Murugesan, Zhang 2011], [Rajagopal, Bitar, Varaiya, Wu, 2013], ...
- Can one hedge the risk of VER random flexibility levels? What is the effect of errors and failures? ...many questions left
We have discussed the possible ways flexible loads could become responsive.

We left out how to sell more renewables power. See work on Risk Limiting Dispatch (RLD) [Varaiya, Wu, Bialek, 2011], [He, Murugesan, Zhang 2011], [Rajagopal, Bitar, Varaiya, Wu, 2013], ...

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